RBSE Class 10th Maths Question Paper With Solution 2015

QUESTION PAPER CODE S–09–Mathematics

PART - A

Question 1: Write the common difference of the A.P. 7, 5, 3, 1, – 1, – 3, ...

Solution:

The given AP: 7, 5, 3, 1, – 1, – 3, ...
The first term a = 7
Common difference [d] = $a_2 - a_1$
= 5 - 7
= -2
Hence, the common difference is -2.

Question 2: Write the distance of the point (– 5, 4) from the x-axis.

Solution:

The distance of the point (-5, 4) from x-axis means the distance of the point (-5,4) from (-5,0).
Therefore, the required distance is given by distance formula

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5 + 5)^2 + (4 - 0)^2}$

$= \sqrt{0 + 16}$

$= \sqrt{4^2}$

= 4

Question 3: Write the solution of the pair of linear equations 4x + 2y = 5 and x – 2y = 0.

Solution:

4x + 2y = 5 ---- (1)

x – 2y = 0

x = 2y ---- (2)

Put equation (2) in (1),

4 * (2y) + 2y = 5

10y = 5

y = 5 / 10

y = 1 / 2

Putting the value of y in (2),

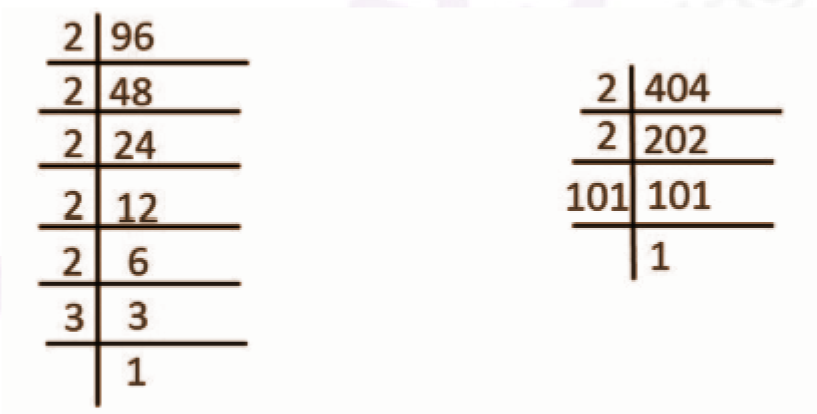
x = 2y

x = 2 (1 / 2)

x = 1

Question 4: Find the HCF of 96 and 404 by the Prime Factorisation Method.

Solution:



96 = 2 × 2 × 2 × 2 × 2 × 3

404 = 2 × 2 × 101

HCF of 96 and 404 = Product of common prime factors

= 2 × 2

= 4

Question 5: One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will not be an ace.

Solution:

The total number of favourable outcomes n (S) = 52.

Probability of getting an ace card = 4.

P (getting an ace card) = n(A) / n(S)

= 4 / 52

= 1 / 13

P (not getting an ace card) = 1 - P (getting an ace card)

= 1 - (1 / 13)

= 12 / 13

Question 6: If K (5, 4) is the midpoint of the line segment PQ and coordinates of Q are (2, 3), then find the coordinates of point P.

Solution:

Given that, PQ is a line segment with K its mid-point

Let, P ≡ (x_1, y_1) and Q ≡ (x_2, y_2)

The coordinates of Q are (2, 3) and that of mid-point K are (5, 4)

By the definition of midpoint,

x coordinates of midpoint = $[x_1 + x_2] / [2]$

y coordinates of midpoint = $[y_1 + y_2] / [2]$

$5 = [x_1 + 2] / [2]$

$10 = [x_1 + 2]$

$10 - 2 = x_1$

$x_1 = 8$

$4 = [y_1 + 3] / [2]$

$8 = [y_1 + 3]$

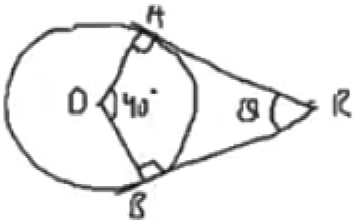
$8 - 3 = y_1$

$y_1 = 5$

Hence, the coordinates of point P are (8, 5).

Question 7: If tangents RA and RB from a point R to a circle with centre O are inclined to each other at an angle of θ and ∠AOB = 40° then find the value of θ.

Solution:



$\angle OAR = 90^{o}$

$\angle OBR = 90^{o}$

In the quadrilateral OARB, the sum of the angles of a quadrilateral is 360^{o}.

$\angle OAR + \angle OBR + \angle ARB + \angle AOB = 360^{o}$

$90^{o} + 90^{o} + \theta + 40^{o} = 360^{o}$

$220^{o} + \theta = 360^{o}$

$\theta = 360^{o} - 220^{o}$

$\theta = 140^{o}$

Question 8: How many tangents can be constructed to any point on the circle of radius 4 cm?

Solution:

One tangent passes through a given point irrespective of its radius.

At a point on the circle, only one tangent can be drawn as it is perpendicular to the normal at the point.

Question 9: Find the circumference of a circle whose diameter is 14 cm.

Solution:

The radius will be half of the diameter.

The diameter of the circle = 14 cm

Radius of the circle = 14 / 2 = 7 cm

Circumference = 2Πr

= 2 * Π * 7

= 2 * (22 / 7) * 7

= 43.98 cm

Question 10: Write the length of an arc of a sector of a circle with radius r and angle with degree measure θ.

Solution:

Circumference of a circle of radius r is equal to $2\pi r$

The angle subtended by circumference at the centre in radians is 360^o.

Hence, length of arc subtending an angle θ^o is

$l = [\theta / 360] \times 2\pi r$

$l = [\pi\theta r] / 180$

PART - B

Question 11: Show that sin 28° cos 62° + cos 28° sin 62° = 1.

Solution:

LHS = $\sin 28^\circ \cos 62^\circ + \cos 28^\circ \sin 62^\circ$

$= \sin 28^o \cos (90^o - 28^o) + \cos 28^o \sin (90^o - 28^o)$

$= \sin 28^o \sin 28^o + \cos (90^o - 68^o) \sin 68^o$

$= \sin^2 28^o + \cos^2 28^o$

$= 1$

Question 12: Find the value of tan 67° / cot 23°.

Solution:

$\tan 67^o / \cot 23^o$

$= \cot (90^o - 67^o) / \cot 23^o$

$= \cot 23^o / \cot 23^o$

$= 1$

Question 13: If 3 cot A = 4, then evaluate $[1 – \tan^2 A] / [1 + \tan^2 A]$.

Solution:

$\cot A = 4 / 3$

$B / P = 4 / 3$

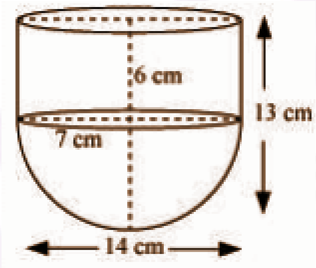
Let $B = 4k$ and $P = 3k$

So in a right angle triangle with angle,

$P^2 + B^2 = H^2$
H = 5k
Now tan A = 1 / cot A = 3 / 4
cos A = B / H = 4 / 5
sin A = P / H = 3 / 5
tan A = sin A / cos A = [3 / 5] / [4 / 5] = 3 / 4
Let us take the LHS
$[1 - \tan^2 A] / [1 + \tan^2 A]$
$= [1 - (3 / 4)^2] / [1 + (3 / 4)^2]$
= 7 / 25

Question 14: A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The radius of the hemisphere is 7 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

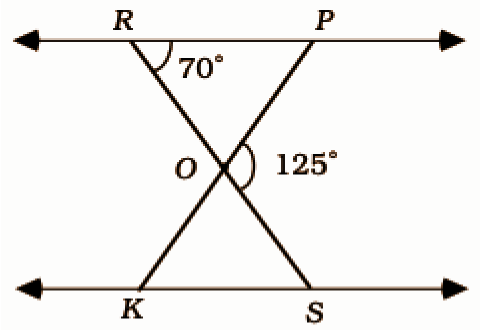
Solution:



Let H be the height of the material, R be the radius of hemisphere & cylinder and D be the diameter
H = 13 cm
D = 14 cm
R = 14 / 2 = 7cm
Curved surface area of hemisphere = $2\pi r^2$
= 2 × [22 / 7] × 7 × 7
= 44 × 7
= 308 cm^2
R = 7 cm
H = 13 cm
Height of cylinder = H - R = 6 cm
Curved surface area of cylinder = $2\pi rh$

$= 2 \times [22 / 7] \times 7 \times 6$
$= 44 \times 6$
$= 264 cm^2$
Surface area of the vessel = $(308 + 264)\ cm^2$
$= 572 cm^2$

Question 15: In the figure, Δ OPR ~ Δ OSK , ∠POS = 125˚ and ∠PRO = 70˚. Find the values of ∠OKS and ∠ROP.



Solution:
Given that,
Δ OPR ~ Δ OSK, ∠POS = 125˚ and ∠PRO = 70˚
Sum of the supplementary angles = 180^o
$\angle POS + \angle POR = 180^o$
$125^o + \angle ROP = 180^o$
$\angle ROP = 180^o - 125^o$
$\angle ROP = 55^o$
$\angle OKS = \angle PRO$ [Since triangle Δ OPR ~ Δ OSK]
$\angle OKS = 70^o$

PART - C

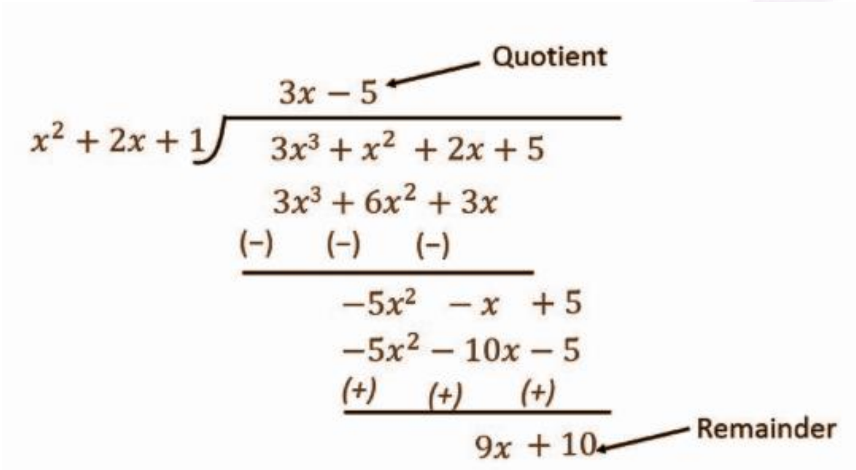
Question 16: Prove that $\{[1 - \tan A] / [1 + \cot A]\}^2 = \tan^2 A$.

Solution:
$(1 - \tan A / 1 - \cot A)^2$
$= (1 + \tan^2 A - 2 \tan A) / (1 + \cot^2 A - 2 \cot A)$

$= (\sec^2 A - 2 \tan A) / (\text{cosec}^2 A - 2 \cos A / \sin A)$
$= (\sec^2 A - 2 * \sin A / \cos A) / (\text{cosec}^2 A - 2 \cos A / \sin A)$
$= (1 / \cos^2 A - 2 \sin A / \cos A) / (1 / \sin^2 A - 2 \cos A / \sin A)$
$= [(1 - 2 \sin A \cos A) / \cos^2 A] / [(1 - 2 \cos A \sin A) \sin^2 A]$
$= (1 - 2 \sin A \cos A) / \cos^2 A * \sin^2 A / (1 - 2 \sin A \cos A)$
$= \sin^2 A / \cos^2 A$
$= \tan^2 A$

Question 17: Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.

Solution:



Quotient: 3x - 5
Remainder: 9x + 10

Question 18: Prove that √ 2 is an irrational number.

Solution:
Let √2 be a rational number.
Therefore, √2 = p / q [p and q are in their lowest terms i.e., HCF of (p, q) = 1 and q ≠ 0.
On squaring both sides,
$p^2 = 2q^2$ … (1)
Clearly, 2 is a factor of $2q^2$.
⇒ 2 is a factor of p^2 [since $2q^2 = p^2$]

⇒ 2 is a factor of p.

Let p = 2 m for all m (where m is a positive integer)

Squaring both sides,

$p^2 = 4m^2$ … (2)

From (1) and (2),

$2q^2 = 4m^2$

⇒ $q^2 = 2m^2$

Clearly, 2 is a factor of $2m^2$.

⇒ 2 is a factor of q^2 [since $q^2 = 2m^2$]

⇒ 2 is a factor of q.

Thus, both p and q have common factor 2 which is a contradiction that H.C.F. of (p, q) = 1.

Therefore, our supposition is wrong.

Hence $\sqrt{2}$ is not a rational number but an irrational number.

Question 19: How many terms of the A.P. 17, 15, 13, ... must be taken, so that their sum is 81?

Solution:

a = 17

d = - 2

$S_n = 81$

=> [n / 2] [2a + (n - 1) d] = 81

=> n [2 * 17 + (n - 1) (-2)] = 162

=> n (34 - 2n + 2) = 162

=> n (36 - 2n) = 162

=> $36n - 2n^2 - 162 = 0$

=> $2n^2 - 36n + 162 = 0$

=> $n^2 - 18n + 81 = 0$

=> $n^2 - 9n - 9n + 81 = 0$

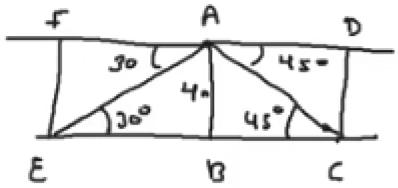
=> n (n - 9) - 9 (n - 9) = 0

=> (n - 9) (n - 9) = 0

=> $(n - 9)^2 = 0$
=> n - 9 = 0
=> n = 9
Number of terms = 9

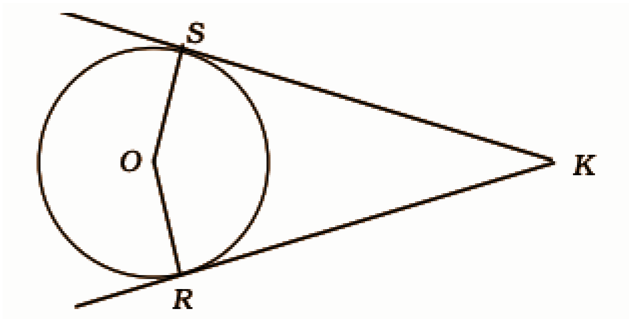
Question 20: From a point on a bridge across a river the angles of depression of the banks on opposite sides of the river are 30° and 45° respectively. If the bridge is at a height of 4 m from the banks, find the width of the river.

Solution:

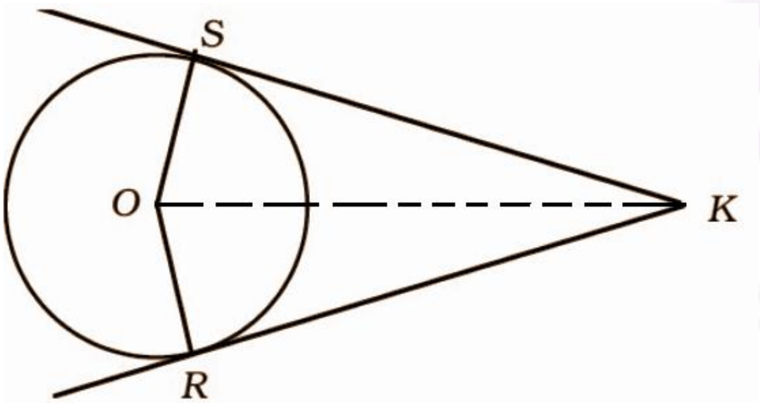


AB = 4cm = CD = EF
∠FAE = 30^o
∠DAC = 45^o
tan θ = perpendicular / base
tan (∠FAE) = FE / FA
tan 30^o = 4 / FA
1 / √3 = 4 / FA
FA = 4√3 m
Similarly,
tan (∠DAC) = CD / DA
tan 45^o = 4 / DA
DA = 4m
Width of river = W = EC = FD
= FA + AD
= 4√3 + 4
= 4 (√3 + 1)m

Question 21: In the given figure, O is the centre of a circle and two tangents KR, KS are drawn on the circle from a point K lying outside the circle. Prove that KR = KS.



Solution:



Construction: Join OK

OS = OR [radius of the circle]

cos (∠SOK) = OS / OK

cos (∠ROK) = OR / OK

cos (∠SOK) = cos (∠ROK)

∠SOK = ∠ROK = θ

sin (∠SOK) = SK / OK

sin (∠ROK) = RK / OK

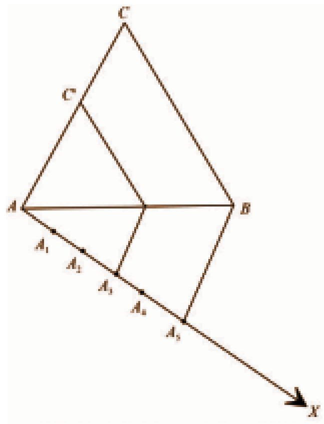
sin (∠SOK) = sin (∠ROK)

SK / OK = RK / OK

SK = RK

Question 22: Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are 3 / 5 time of the corresponding sides of the given triangle.

Solution:



Question 23: Find the area of the corresponding major sector of a circle with radius 7 cm and angle 120˚.

Solution:

Area of a sector of a circle of radius r subtending an angle θ^o at the centre is given by $A = \pi r^2\theta / 360$.

$A = (120^o) * 7^2 *\pi / 360$

$= 5880\pi / 360$

$= 18472.6 / 360$

$= 51.313\ cm^2$

Question 24: A copper rod of radius 1 cm and length 2 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

Solution:

The radius of the copper rod, $r_1 = 1$ cm.

The length of the copper rod $l_1 = 2$ cm.

Length of the wire = $l_2 = 18$m

To find the thickness of the wire (l_2)

Both the rod and wire are in the form of cylinder and volume of the cylinder = $\pi r^2 l$.

The volume of the copper rod is equal to the volume of wire as density and mass of the material does not change.

$V_1 = V_2$

$\pi r_1^2 l_1 = \pi r_2^2 l_2$

$r_2 = \sqrt{r_1^2 l_1 / l_2}$

$= \sqrt{1^2 * 2 / 18 * 100}$

= 1 / 30 cm

Thickness of the wire is equal to the diameter which is twice the radius.

$t_2 = 2r_2$

= 2 * (1 / 30)

= 1 / 15 cm

Question 25: Neeraj and Dheeraj are friends. Find the probability of their birthdays when

(i) birthdays are different.

(ii) birthdays are the same.

Solution:

(i) P (both having different birthdays)

= P (both will not have same birthday)

= 1 - P (both will have the same birthday)

= 1 - (1 / 365)

= 364 / 365

(ii) Number of days in a year = 365

Number of days when the birthday is possible = 1

P (both have same birthday) = 1 / 365

PART - D

Question 26: The cost of 5 apples and 3 oranges is Rs. 35 and the cost of 2 apples and 4 oranges is Rs. 28. Formulate the problem algebraically and solve it graphically.

Solution:

Let the cost of 1 apple be x and the cost of 1 orange be y.

According to question

5x + 3y = 35 - - - - - (i)

2x + 4y = 28 - - - - (ii)

Multiply by 2 in equation (i) and by 5 in equation (ii)

Now,

=> 10x + 6y = 70 - - - - (i)

=> 10x + 20y = 140 - - - (ii)

By subtraction equation (i) from (ii)

=> 14y = 70

=> y = 10 / 14

=> y = 5

Putting the value of y in equation (i)

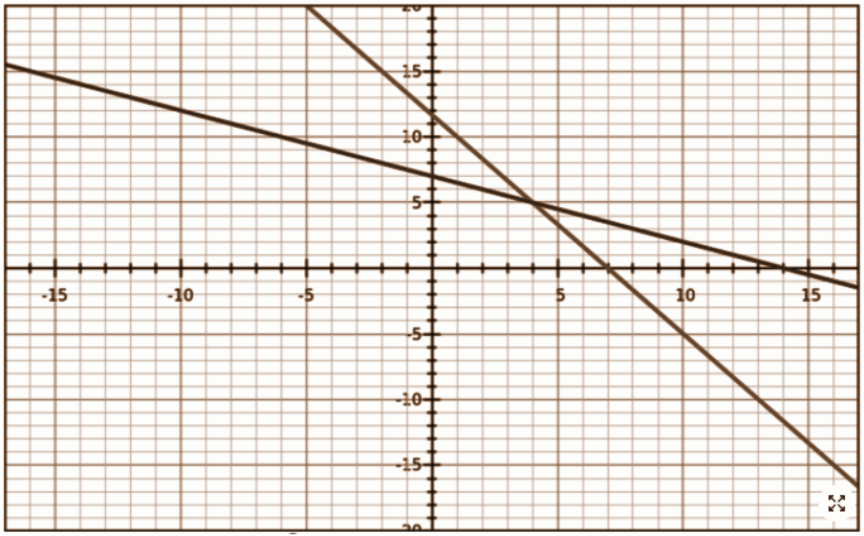
=> 5x + (3 * 5) = 35

=> 5x = 35 - 15

=> x = 20 / 5

=> x = 4

Hence, the cost of an apple = ₹4 and the cost of an orange = ₹5.



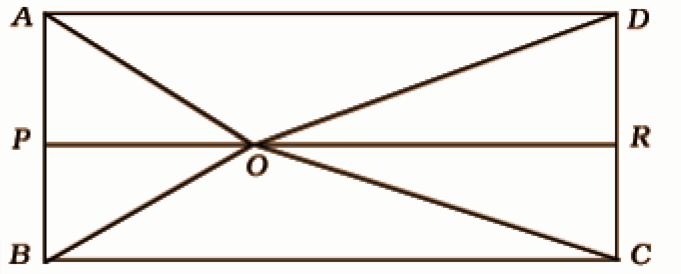
Question 27: The speed of a boat in still water is 18 km / h. It takes 1 / 2 an hour extra to go 12 km upstream instead of going the same distance downstream. Find the speed of the stream.

Solution:

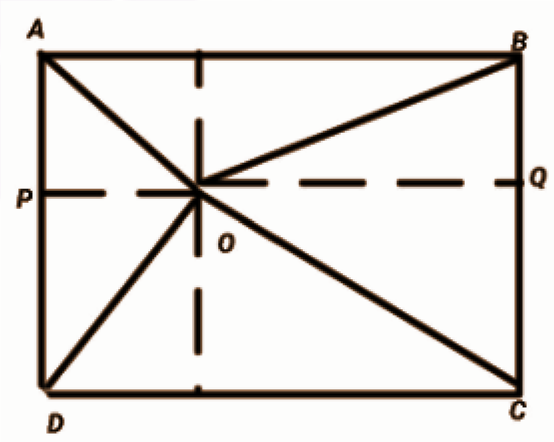
Speed of a boat in still water = 18 km / h

Distance = 12 km
Now, Let speed of stream = x km / h
Let up stream = 18 + x km / h
Downstream = 18 - x km / h
Time is downstream – the time is upstream = (1 / 2)h
12 / [18 - x] - 12 / [18 + x] = 1 / 2
Multiply by (18 - x) (18 + x)
216 + 12x - 216 + 12x = (1 / 2) (18^2 - x^2)
24x = (1 / 2) (324 - x^2)
x^2 + 48x - 324 = 0
x = 6
x = -54
Hence, the speed of stream = 6 km / h.

Question 28: O is any point inside the rectangle ABCD. Prove that $OB^2 + OD^2 = OA^2 + OC^2$.



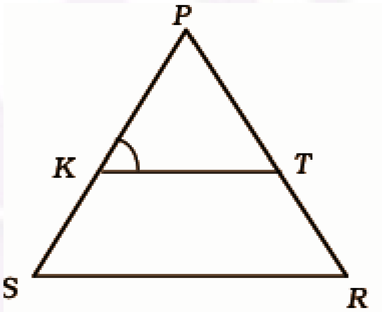
Solution:



We draw PQ ∥ AB ∥ CD as shown in the figure.
ABCD is a rectangle, it means ABPQ and PQDC are also rectangles
For, ABPQ,

AP = BQ [opposite sides are equal]
For, PQDC
PD = QC [opposite sides are equal]
Now, for ΔOPD,
$OD^2 = OP^2 + PD^2$ ------(1)
For, ΔOQB,
$OB^2 = OQ^2 + BQ^2$ --------(2)
Add equations (1) and (2),
$OB^2 + OD^2 = (OP^2 + PD^2)+ (OQ^2 + BQ^2)$
$= (OP^2 + CQ^2) + (OQ^2 + AP^2)$
ΔOPA and ΔOQC are also right-angled triangles
For, ΔOCQ $\Rightarrow OQ^2 + CQ^2 = OC^2$
For, ΔOPA $\Rightarrow OP^2 + AP^2 = OA^2$,
$OB^2 + OD^2 = OC^2 + OA^2$

Question 29: In the given figure, PK / KS = PT / TR and ∠PKT = ∠PRS . Prove that Δ PSR is an isosceles triangle.



Solution:
PK / KS = PT / TR
KS / KP = RT / PT
[KS / KP] + 1 = [RT / PT] + 1
[KS + KP] / KP = [RT + PT] / PT
PS / KP = PR / PT
∠KPT is a common angle to ΔKPT and ΔSPR.
Using SAS similarity of triangles, ΔKPT ~ ΔSPR
⟹∠PKT = ∠PSR
⟹∠PRS = ∠PSR [∵∠ PKT = ∠PRS]

Since two angles of ΔPSR are equal, it is an isosceles triangle.

Question 30: In the following distribution calculate mean from the assumed mean:

CI	**10 - 25**	**25 - 40**	**40 - 55**	**55 - 70**	**70 - 85**	**85 - 100**
Frequency	**2**	**3**	**7**	**5**	**6**	**7**

OR

Find the mode of the following distribution :

CI	**0 - 20**	**20 - 40**	**40 - 60**	**60 - 80**	**80 - 100**	**100 - 120**
Frequency	**10**	**35**	**52**	**61**	**38**	**20**

Solution:

CI	10 - 25	25 - 40	40 - 55	55 - 70	70 - 85	85 - 100
Frequency	2	3	7	5	6	7
Mid value	17.5	32.5	47.5	62.5	77.5	92.5
$d_i = x_i$ - A / h	-2	-1	0	1	2	3
f_id_i	-4	-3	0	5	12	21

$$mean(\overline{x}) = A + \left\{ h \times \frac{\sum (f_i \times u_i)}{\sum f_i} \right\}$$

= 47.5 + 15 * {[31 / 30]

= 47.5 + 15.5

= 63

OR

Here the maximum frequency is 61, and the class corresponding to this frequency is 60 – 80. So the modal class is 60 - 80.

Therefore, l = 60, h = 20, f_1 = 61, f_0 = 52 , f_2 = 38

Mode = l + [(f_1- f_0) / (2f_1 - f_0 - f_2)] × h

= 60 + [(61 - 52) / (2 × 61 - 52 – 38)] × 20

= 60 + [9 / 32] * 20

= 60 + [5.625]

= 65.625

Hence, the mode of the data is 65.625.