RBSE Class 10th Maths Question Paper With Solution 2016

QUESTION PAPER CODE S–09–Mathematics

PART - A

Question 1: Find the HCF of integers 375 and 675 by the prime factorisation method.

Solution:

$375 = 5^3 \times 3$

$675 = 5^2 \times 3^3$

HCF of the two numbers is x = 5a × 3b where *a* and *b* are the smallest of the integers occurring in the prime factorization of each number.

Hence, $x = 5^2 \times 3^1 = 75$

Question 2: Find 11^{th} term of the A.P. -17, -12, -7 …………………

Solution:

AP: -17, -12, -7 …………………

a = -17

d = -12 + 17 = 5

a_{11} = a + (n - 1) d

= -17 + (11 - 1) * 5

= -17 + 55 - 5

= 55 - 22

= 33

Question 3: If cos A = 12 / 13, then calculate cot A.

Solution:

cos A = 12 / 13

Base / hypotenuse = 12 / 13

Base = 12 units, hypotenuse = 13 units

$(\text{perpendicular})^2 = (\text{hypotenuse})^2 - (\text{base})^2$

$= 13^2 - 12^2$

$=169 - 144$

$= 25$

Perpendicular = $\sqrt{25}$ = 5 units

cot A = base / perpendicular

= 12 / 5

Question 4: Express the trigonometric ratio tan A in terms of sec A.

Solution:

$\tan^2 A = \sec^2 A - 1$

$\tan A = \sqrt{\sec^2 A - 1}$

Question 5: The area of two similar triangles is in ratio 16:81. Find the ratio of its sides.

Solution:

The ratio of areas of two triangles is equal to the square of the ratio of sides of the triangles.

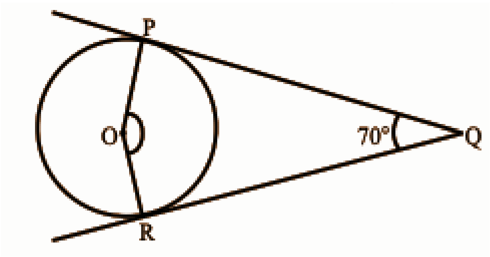
$A_1 / A_2 = (S_1 / S_2)^2$

$16 / 81 = (S_1 / S_2)^2$

$\sqrt{16 / 81} = (S_1 / S_2)$

$4 / 9 = (S_1 / S_2)$

Question 6: In the given figure, O is the centre of a circle and two tangents QP and QR are drawn on the circle from a point Q lying outside the circle. Find the value of angle POR.



Solution:

$\angle OPQ = 90^o$

$\angle ORQ = 90^o$

The sum of the angles of the quadrilateral OPQR is 360^o.

$\angle OPQ + \angle ORQ + \angle PQR + \angle POR = 360^o$

$90^o + 90^o + 70^o + \angle POR = 360^o$

$\angle POR = 360^o - 250^o$

$\angle POR = 110^o$

Question 7: How many tangents can be drawn on the circle of radius 5 cm from a point lying outside the circle at distance 9 cm from the centre.

Solution:

The number of tangents that can be drawn to a circle from a point lying outside the circle is two. Since the distance between the centre and the point (9 cm) is greater than the radius (5 cm), the point lies outside the circle.

Hence, two tangents can be drawn on the circle.

Question 8: Find the radius of that circle whose area is 616 cm^2.

Solution:

The area of the circle is 616 cm^2.

$\pi r^2 = 616$

$r = \sqrt{616 / \pi}$

$= \sqrt{196}$

= 14 cm

Question 9: If the angle of the major sector of a circle is 250°. Then find the angle of the minor sector.

Solution:

The angle of the major sector is 250^o.

Let the angle of the minor sector be x.

Circle = 360^o

Circle = major sector + minor sector

$360^o = 250^o + x$

$360^o - 250^o = x$

$x = 110^o$

Question 10: A coin is tossed once. Find the probability that it is not a tail.

Solution:

The total number of possible outcomes are head (H) and tail (T) = 2.

P (not getting a tail) = 1 / 2

PART - B

Question 11: If the middle point of two points A (-2, 5) and B (-5, y) is (-7 / 2, 3), then find the distance between points A and B.

Solution:

Let the middle point be P.

Using the distance formula,

$AP = \sqrt{([-7/2] - (-2))^2 + (3 - 5)^2}$

$= \sqrt{(9/4) + 4}$

$= 5/2$

AB = 2AP as P is the mid point of AB.

AB = 5

Question 12: The total surface area of a solid hemisphere is 462 cm^2. Find its radius.

Solution:

Let r be the radius of the solid hemisphere.

The total surface area of solid hemisphere = $3\pi r^2$

$\Rightarrow 3\pi r^2 = 462$

$\Rightarrow \pi r^2 = 154$

$\Rightarrow r^2 = 154 \times [7/22]$

$\Rightarrow r^2 = 49$

$\Rightarrow r = 7$ cm

Question 13: Per day expenses of 25 families of the frequency distribution of a Dhani of a village is given as follows.

Per day expense (in Rs)	**25 - 35**	**35 - 45**	**45 - 55**	**55 - 65**	**65 - 75**
Number of families	**3**	**7**	**6**	**6**	**3**

Find the mean expense of families by Direct Method.

Solution:

Per day expense (in Rs)	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75
Number of families	3	7	6	6	3
Midpoint	30	40	50	60	70
$f_i x_i$	90	280	300	360	210

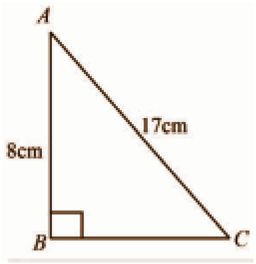
Mean = $\sum f_i x_i / \sum f_i$

= 1240 / 25

= 49.6

Question 14: For traffic control, a CCTV camera is fixed on an 8m straight pole. The camera can see 17m distance sightline from the top. Find the area visible by the camera around the pole?

Solution:



$AC^2 = AB^2 + BC^2$

$17^2 = 8^2 + BC^2$

$289 - 64 = BC^2$

$225 = BC^2$

BC = 15m

The area that is visible by the camera around the pole = πr^2

= π * (BC^2)

= π * (225)

= 706.86 m^2

Question 15: A Motor car travels 175 km distance from a place A to place B, at a uniform speed 70km / hr passes through all ten green traffic signals. Due to heavy traffic, it stops for one minute at the first signal, 3 minutes at the second signal, 5 minutes at the third signal and so on stops for 19 minutes at the tenth signal. How much total time it takes to reach the place B. Solve by suitable mathematical method.

Solution:

Time taken to travel from A to B excluding stops can be found using the formula:

Time = Distance / Speed

t_t = 175 / 70 hr

t_t = [175 / 70] × 60 min

t_t = 150 min

Now, stop time at first signal is t_1 = 1s and at the second signal is t_2 = 3s and so on.

This forms an arithmetic progression with first term a_1 = 1, common difference d = 2 , number of terms n = 10.

Total stop time is the sum of this series.

t_s = [n / 2] ($2a_1$ + (n - 1)d)

= [10 / 2] (2 × 1 + (10 - 1) × 2)

= 100 min

Total travel time equals the sum of stop time and travel time.

t = $t_s + t_t$

t = 100 + 150

t = 250 min

= 4 hr 10 min

PART - C

Question 16: Prove that √6 is an irrational number.

Solution:

Let √6 be a rational number, then

√6 = p÷q , where p,q are integers , q not = 0 and p,q have no common factors (except 1)

=> $6 = p^2 \div q^2$

=> $p^2 = 2q^2$ ....(i)

As 2 divides $6q^2$, so 2 divides p^2 but 2 is a prime number

=> 2 divides p

Let p = 2m, where m is an integer.

Substituting this value of p in (i),

$(2m)^2 = 6q^2$

=> $2m^2 = 3q^2$

As 2 divides $2m^2$, 2 divides $3q^2$

=> 2 divides 3 or 2 divide q^2

But 2 does not divide 3, therefore, 2 divides q^2

=> 2 divides q

Thus, p and q have a common factor 2.

This contradicts that p and q have no common factors (except 1).

Hence, the supposition is wrong.

Therefore, √6 is an irrational number.

Question 17: Find the zeroes of the quadratic polynomial $x^2 + x - 2$, and verify the relationship between the zeroes and coefficients.

Solution:

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2, 1$

$\alpha + \beta = (-b / a)$

$-2 + 1 = (-1 / 1)$

$-1 = -1$

$\alpha * \beta = (c / a)$

$-2 * 1 = (-2 / 1)$

$-2 = -2$

Hence, the relationship between zeros and coefficients of polynomial is verified.

Question 18: Find the sum of the first 15 terms of an A.P. whose 5th and 9th terms are 26 and 42 respectively.

Solution:

$a_5 = 26$

$a_9 = 42$

$a_n = a + (n - 1)d$

$a_5 = a + (5 - 1) d$ ---- (1)

$a_9 = a + (9 - 1)d$ ---- (2)

Subtracting (1) from (2)

$42 - 26 = a + (9 - 1)d - [(a + (5 - 1)d]$

$16 = a + 8d - a - 4d$

$16 = 4d$

$16 / 4 = d$

$d = 4$

Substitute the value of d in (1),

$26 = a + 4 * 4$

$26 - 16 = a$

$a = 10$

$S_n = [n / 2] [2a + (n - 1)d]$

$S_{15} = [15 / 2] [2 * 10 + 14 * 4]$

$= [15 / 2] [20 + 56]$

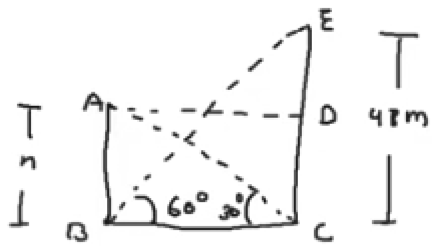
$= 7.5 * 76$

$= 570$

Question 19: The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot

of the building is 60°. If the tower is 48 meters high, find the height of the building.

Solution:



Given that,

CE = 48 m

∠EBC = 60^o

∠ACB = 30^o

To find: AB

Using trigonometric identity,

tan(∠EBC) = EC / BC ....(i)

tan(∠ACB) = AB / BC ....(ii)

Dividing (ii) by (i),

AB / EC = tan(∠ACB) / tan(∠EBC)

Substituting values,

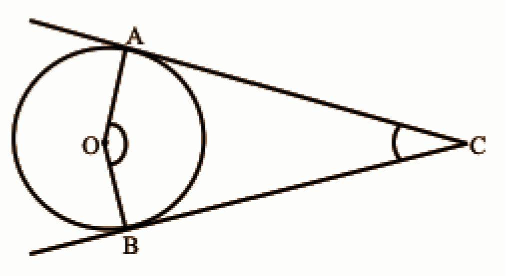
AB / 48 = tan 30^o / tan 60^o

AB / 48 = [1 / √3] / √3

AB = 48 / 3

AB = 16 m

Question 20: In the given figure, O is the centre of a circle and two tangents CA, CB are drawn on the circle from a point C lying outside the circle. Prove that ∠AOB and ∠ACB are supplementary.



Solution:

OA and OB are perpendicular to the tangents.

$\angle OBC = 90^o$

$\angle OAC = 90^o$

In the quadrilateral OABC, the sum of the angles is 360^o.

$\angle OAC + \angle OBC + \angle AOB + \angle ACB = 360^o$

$90^o + 90^o + \angle AOB + \angle ACB = 360^o$

$\angle AOB + \angle ACB = 360^o - 180^o$

$\angle AOB + \angle ACB = 180^o$

So, $\angle AOB$ and $\angle ACB$ are supplementary.

Question 21: Construct a triangle with sides 4cm, 5cm and 7cm and then another triangle whose sides are 3 / 4 of the corresponding sides of the first triangle.

Solution:

Steps of construction:

(i) Draw BC = 4cm.

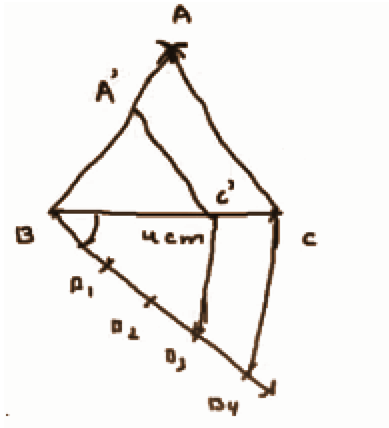
(ii) Cut an arc of 5 cm from B and an arc of 7 cm from C. Mark the point of intersection as A.

(iii) Draw BD making an acute angle with BC. Mark 4 equal arcs on BD.

(iv) Join B_4 to C, draw a line B_3C' parallel to B_4C.

(v) Draw a line C′A′ parallel to CA.

$\therefore \Delta A'B'C'$ is the required triangle.



Question 22: If an arc of a circle subtends an angle of 45° at the centre and if the area of the minor sector is 77cm², then find the radius of the circle.

Solution:

Area of the minor sector, A = 77 cm^2

The angle subtended by sector, θ = 45°

To find the radius r

Area of minor sector subtending an angle of θ° at the center of a circle of radius r is given by $A = [\theta / 360] \times \pi r^2$

Substituting values,

$77 = [45 / 360] \times \pi r^2$

$r \approx \sqrt{[77 \times 360] / [45 \times (22 / 7)]}$ ∵ $\pi \approx [22 / 7]$

$r = \sqrt{196}$

r = 14 cm

Question 23: Seven spheres of equal radii are made by melting a silver-cuboid of dimensions 8cm × 9cm × 11cm. Find the radius of a silver sphere.

Solution:

The total volume before melting equals the total volume after melting.

The volume of the cuboid is equal to the volume of seven spheres.

$V_c = 7V_s$

abc = 7 × [4 / 3] × πr^3

8 × 9 × 11 ≈ 7 × [4 / 3] × [22 / 7] × r^3

792 = $29.321r^3$

r^3 = 792 / 29.321

r^3 = 27

r = 3 cm

Question 24: The following table shows the marks obtained by 50 students in mathematics of class X in a school.

Obtained marks	**20 - 30**	**30 - 40**	**40 - 50**	**50 - 60**	**60 - 70**	**70 - 80**
Number of students	**5**	**9**	**8**	**12**	**13**	**3**

Find the median marks.

Solution:

Obtained marks	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Number of students	5	9	8	12	13	3
Cumulative frequency	5	14	22	34	47	50

N = 50

m = 50 / 2 = 25^{th} term

The cumulative frequency just greater than 25 is 34 and corresponding to 50 - 60 class.

Median = $l + \left[h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right]$

= 50 + {[25 - 22] / 12} * 10

= 50 + [30 / 12]

= 52.5

Question 25: A piggy bank contains a hundred coins of Rs. 1, twenty-five coins of Rs. 2, fifteen coins of Rs. 5 and ten coins of Rs. 10. If it is equally likely that one coin will fall when the bank is turned upside down, what is the probability that the coin -

(i) Will be a Rs. 2 coin?

(ii) Will not be a Rs. 5 coin?

Solution:

Probability of an event is defined as the ratio of favourable outcomes to the total outcomes.

(i) The probability that the fallen coin is Rs. 2 coin is the ratio of the number of Rs. 2 coins and the total number of coins.

P_2 = 25 / [100 + 25 + 15 + 10]

= 25 / 150

= 1 / 6

(ii) The probability that the fallen coin is Rs. 5 coin is the ratio of the number of Rs. 5 coins and the total number of coins.

P_5 = 15 / [100 + 25 + 15 + 10]

= 15 / 150

= 1 / 10

PART - D

Question 26: The cost of 2 exercise books and 3 pencils is Rs.17 and the cost of 3 exercise books and 4 pencils is Rs. 24. Formulate the problem algebraically and solve it graphically.

Solution:

Let the cost of exercise books be Rs. x.

And, let the cost of the pencil be Rs. y.

So,

$2x + 3y = 17$ (i)

$3x + 4y = 24$ (ii)

Multiplying 3 in equation (i) and multiplying 2 in equation (ii).

$6x + 9y = 51$ (iii)

$6x + 8y = 48$ (iv)

Subtracting equation (iii) and (iv)

$6x + 9y = 51$

$6x + 8y = 48$

$y = 3$

Putting the value y in equation (i)

$2x + 3y = 17$

$2x + 3 (3) = 17$

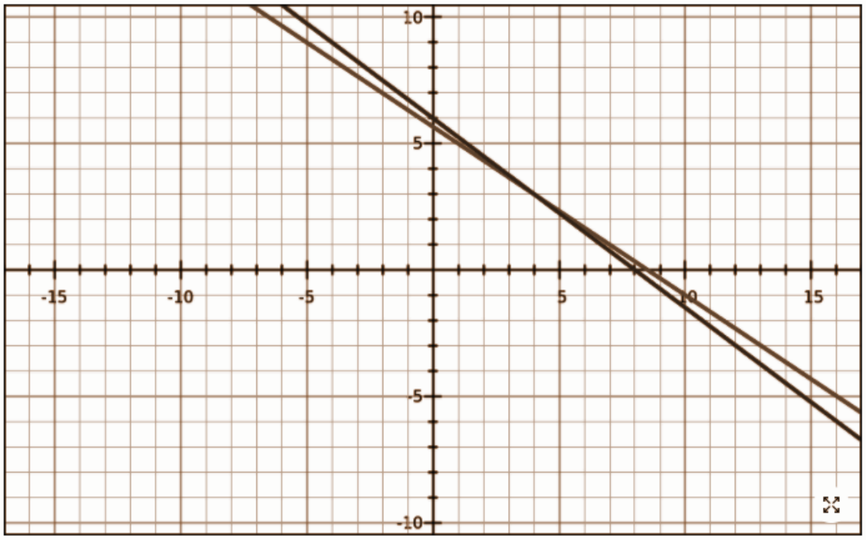
$2x + 9 = 17$

$2x = 17 - 9$

$2x = 8$

$x = 4$

Hence, the cost of exercise book is Rs.4 and that of a pencil is Rs.3.



Question 27: [i] The diagonal of a rectangular field is 40 meters more than the shorter side. If longer side is 20 meters more than the shorter side, find the sides of the field.

OR

[ii] A Pole has to be erected at a point on the boundary of a circular park of diameter 17 meters in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary are 7 meters. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

Solution:

[i] Let the shorter side of the rectangular field be 'x' meters.

Therefore the longer side will be (x + 20) meters.

And the length of the diagonal will be (x + 40) meters.

According to the question, the diagonal divides the rectangular into two right-angled triangles and the diagonal is the common side of the two triangles and it is also the longest side of the triangles i.e. the hypotenuse.

So, by Pythagoras Theorem,

$(\text{Diagonal})^2 = (\text{Smaller Side})^2 + (\text{Longer Side})^2$

$(x + 40)^2 = (x)^2 + (x + 20)^2$

$x^2 + 80x + 1600 = x^2 + x^2 + 40x + 400$

$x^2 + 80x - 40x + 1600 - 400 = 2x^2$

$x^2 - 40x - 1200 = 0$

$x^2 - 60x + 20x - 1200 = 0$

$x(x - 60) + 20(x - 60) = 0$

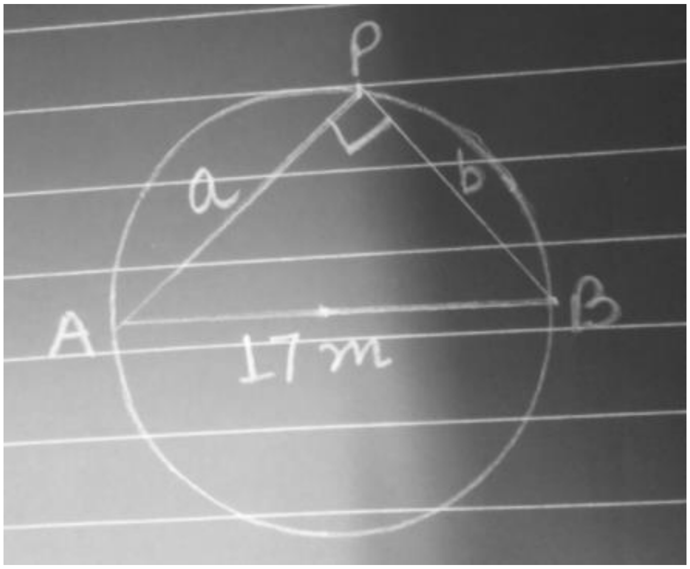
$(x + 20)(x - 60) = 0$

x = 60 because x = -20 as length cannot be negative.

So the length of the shorter side is 60 meters and the length of the longer side is 60 + 30 = 80 meters.

OR

[ii]



Let P be the position of the pole and A and B be the opposite fixed gates.

PA - PB = 7 m

⇒ a - b = 7

⇒ a = 7 + b ....(1)

In Δ PAB,

$AB^2 = AP^2 + BP^2$

$\Rightarrow (17) = (a)^2 + (b)^2$

$\Rightarrow a^2 + b^2 = 289$

⇒ Putting the value of a = 7 + b in the above,

$\Rightarrow (7 + b)^2 + b^2 = 289$

$\Rightarrow 49 + 14b + 2b^2 = 289$

$\Rightarrow 2b^2 + 14b + 49 - 289 = 0$

$\Rightarrow 2b^2 + 14b - 240 = 0$

Dividing the above by 2, we get.

$\Rightarrow b^2 + 7b - 120 = 0$

$\Rightarrow b^2 + 15b - 8b - 120 = 0$

$\Rightarrow b(b + 15) - 8(b + 15) = 0$

$\Rightarrow (b - 8)(b + 15) = 0$

$\Rightarrow b = 8$ or $b = -15$

Since this value cannot be negative, so b = 8 is the correct value.

Putting b = 8 in (1),

a = 7 + 8

a = 15 m

Hence PA = 15 m and PB = 8 m

So, the distance from the gate A to pole is 15 m and from gate B to the pole is 8 m.

Question 28: [i] If cos 3A = sin (A −34°), where A is an acute angle, find the value of A.

[ii] Prove the following identity, where the angles involved are acute angles for which the expression is defined.

$[1 + \cot^2 A] / [1 + \tan^2 A] = \{[1 - \cot A] / [1 - \tan A]\}^2$

Solution:

[i] cos 3A = sin (A − 34)

= cos (90 − A + 34)

= cos (124 − A)

So, 124 − A = 2nπ ± 3A

For positive, 124 − A = 2nπ + 3A

4A = 124 − 2nπ

As A is acute n = 0,

A = 124 / 4

A = 31°

For negative,

124 − A = 2nπ − 3A

124 + 2A = 2nπ

2A = 2nπ − 124

As A is acute, no n exists.

A = 31°

[ii] $[1 + \cot^2 A] / [1 + \tan^2 A] = \operatorname{cosec}^2 A / \cot^2 A$

$= [1 / \sin^2 A] / [1 / \tan^2 A]$

$= \cos^2 A / \sin^2 A$

$= \cot^2 A$

Taking the RHS,

$\{[1 - \cot A] / [1 - \tan A]\}^2$

$= \{[1 - (1 / \tan A)] / [1 - \tan A]\}^2$

$= \{(\tan A - 1) / [1 - \tan A]\}^2 * (1 / \tan^2 A)$

$= \cot^2 A$

$[1 + \cot^2 A] / [1 + \tan^2 A] = \{[1 - \cot A] / [1 - \tan A]\}^2$

Question 29: Find the area of that triangle whose vertices are (-5, 7), (4, 5) and (-4, -5).

Solution:

Area of the triangle having 3 vertices is : (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by the formula :

Area = [1 / 2] [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]

(x_1, y_1) = (-5 , 7)

(x_2, y_2) = (-4 , -5)

(x_3, y_3) = (4 , 5)

Substituting the values we get,

= [1 / 2] [(-5) (- 5 - 5) + (- 4) (5 - 7) + 4 (7 - 5)]

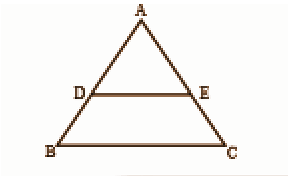
= [1 / 2] [50 + 8 + 48]

= [1 / 2] 106

= 53

Hence, the area of triangle ABC is 53 sq units.

Question 30: [i] In the given figure ABC is a triangle. If AD / AB = AE / AC, then prove that DE|| BC.



OR

[ii] The diagonals of a quadrilateral PQRS intersect each other at the point O such that PO / QO = RO / SO. Show that PQRS is a trapezium.

Solution:

[i] AD / AB = AE / AC

AD / AB = AE / AC

⇒ AB / AD = AC / AE

⇒ (AB / AD) - 1 = (AC / AE) - 1

⇒ (AB - AD) / AD = (AC - AE) / AE

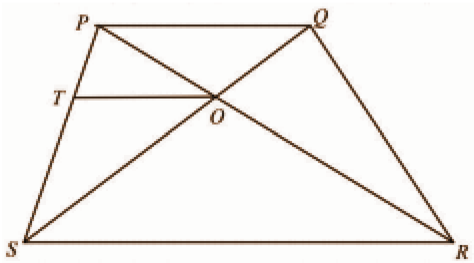
⇒ BD / AD = CE / AE

⇒ AD / BD = AE / CE

By BPT (Thales theorem), DE || BC.

OR

[ii]



Construction: Draw OT || PQ meeting PS at T.

In triangle PSR, OT || SR....(i)

ST / PT = RO / PO (By Basic proportionality theorem)..........(ii)

Also, SO / OQ = RO / PO (Given)...........(iii)

From (ii) and (iii),

ST / PT = SO / QO end fraction

SR || OT (Converse of Basic proportionality theorem)......(iv)

From (i) and (iv),

SR || PQ

Hence quadrilateral PQRS is trapezium.