EXERCISE 27(A)

1. Two angles of a quadrilateral are 89^0 and 113^0. If the other two angles are equal; find the equal angles.

Solution:

Let us consider the other angle as x^0

As per the question, we have

$89^0 + 113^0 + x^0 + x^0 = 360^0$

$2x^0 = 360^0 - 202^0$

$2x^0 = 158$

$x^0 = 158 / 2$

We get,

$x = 79^0$

Therefore, the other two equal angles are 79^0 each.

2. Two angles of a quadrilateral are 68^0 and 76^0. If the other two angles are in the ratio 5: 7; find the measure of each of them.

Solution:

Given

Two angles are 68^0 and 76^0

Let us consider the other two angles as 5x and 7x

Hence,

$68^0 + 76^0 + 5x + 7x = 360^0$

$12x + 144^0 = 360^0$

$12x = 360^0 - 144^0$

$12x = 216^0$

$x = 216^0 / 12$

We get,

$x = 18^0$

Now, the other angles is calculated as below

$5x = 5 \times 18^0 = 90^0$

$7x = 7 \times 18^0 = 126^0$

Therefore, the values of the other angles are 90^0 and 126^0

3. Angles of a quadrilateral are $(4x)^0$, $5(x+2)^0$, $(7x – 20)^0$ and $6(x + 3)^0$. Find

(i) the value of x.

(ii) each angle of the quadrilateral.

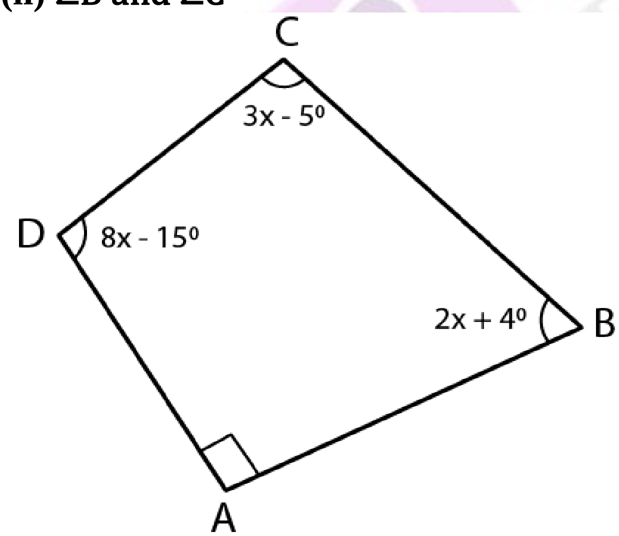
Solution:

Given

The angles of quadrilateral are,

$(4x)^0$, $5(x + 2)^0$, $(7x – 20)^0$ and $6(x + 3)^0$
We know that the sum of angles in a quadrilateral is 360^0
Hence,
$(4x)^0 + 5(x + 2)^0 + (7x – 20)^0 + 6(x + 3)^0 = 360^0$
$4x + 5x + 10^0 + 7x – 20^0 + 6x + 18^0 = 360^0$
$22x + 8^0 = 360^0$
$22x = 360^0 - 8^0$
$22x = 352^0$
$x = 352^0 / 22$
We get,
$x = 16^0$
Hence, the value of x is 16^0
Therefore, the angles are,
$(4x)^0 = (4 \times 16)^0$
$= 64^0$
$5(x + 2)^0 = 5(16 + 2)^0$
$= 90^0$
$6(x + 3)^0 = 6(16 + 3)^0$
$= 114^0$

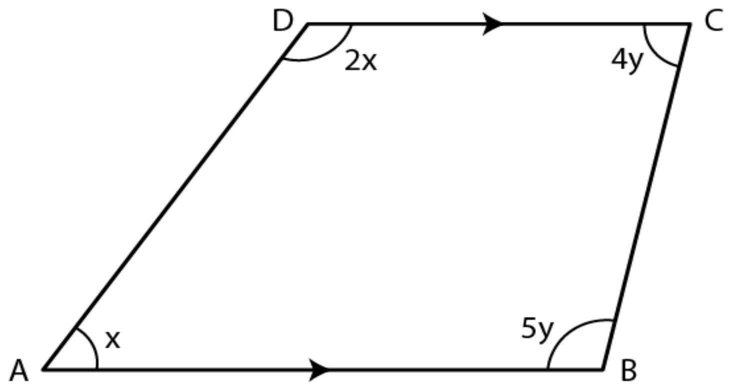
4. Use the information given in the following figure to find:
(i) x
(ii) ∠B and ∠C



Solution:
Here, given that,
$\angle A = 90^0$
$\angle B = (2x + 4)^0$

$\angle C = (3x - 5)^0$
$\angle D = (8x - 15)^0$
We know that,
All the angles in a quadrilateral is 360^0
So,
$\angle A + \angle B + \angle C + \angle D = 360^0$
$90^0 + (2x + 4)^0 + (3x - 5)^0 + (8x - 15)^0 = 360^0$
On further calculation, we get
$90^0 + 2x + 4^0 + 3x - 5^0 + 8x - 15^0 = 360^0$
$74^0 + 13x = 360^0$
$13x = 360^0 - 74^0$
$13x = 286^0$
$x = 286^0 / 13$
We get,
$x = 22^0$
The value of x is 22^0
Now,
$\angle B = 2x + 4 = 2 \times 22^0 + 4$
$= 48^0$
$\angle C = 3x - 5 = 3 \times 22^0 - 5$
$= 61^0$
Therefore, $\angle B = 48^0$ and $\angle C = 61^0$

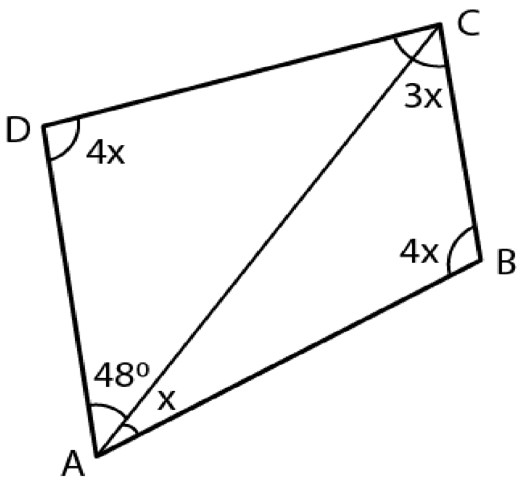
5. In quadrilateral ABCD, side AB is parallel to side DC. If ∠A: ∠D = 1: 2 and ∠C: ∠B = 4: 5
(i) Calculate each angle of the quadrilateral.
(ii) Assign a special name to quadrilateral ABCD.
Solution:



Given
∠A: ∠D = 1: 2

Let us consider ∠A = x and ∠D = 2x
∠C: ∠B = 4: 5
Let us consider ∠C = 4y and ∠B = 5y
Also, given
AB || DC and the sum of opposite angles of quadrilateral is 180^0
So,
$\angle A + \angle D = 180^0$
$x + 2x = 180^0$
$3x = 180^0$
We get,
$x = 60^0$
Therefore, $\angle A = 60^0$
$\angle D = 2x$
$= 2 \times 60^0$
$= 120^0$
Therefore, $\angle D = 120^0$
Now,
$\angle B + \angle C = 180^0$
$5y + 4y = 180^0$
$9y = 180^0$
We get,
$y = 20^0$
Now,
$\angle B = 5y = 5 \times 20^0$
$= 100^0$
$\angle C = 4y = 4 \times 20^0$
$= 80^0$
Therefore, $\angle A = 60^0$; $\angle B = 100^0$; $\angle C = 80^0$ and $\angle D = 120^0$

6. From the following figure find:
(i) x,
(ii) ∠ABC,
(iii) ∠ACD.
Solution:

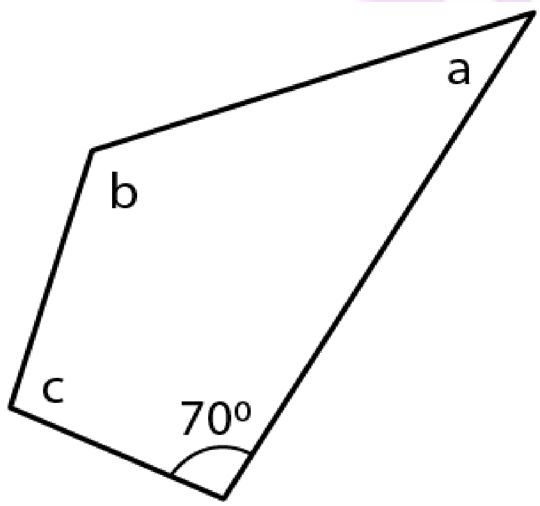


(i)We know that,
In quadrilateral the sum of angles is equal to 360^0
Hence,
$x + 4x + 3x + 4x + 48^0 = 360^0$
$12x = 360^0 – 48^0$
$12x = 312$
We get,
$x = 26^0$
Hence, the value of x is 26^0
(ii) $\angle ABC = 4x$
$4 \times 26^0 = 104^0$
Therefore, $\angle ABC = 104^0$
(iii) $\angle ACD = 180^0 – 4x - 48^0$
$= 180^0 – 4 \times 26^0 - 48^0$
$= 180^0 – 104^0 - 48^0$
We get,
$= 28^0$
Therefore, $\angle ACD = 28^0$

7. Given: In quadrilateral ABCD; $\angle C = 64^0$, $\angle D = \angle C – 8^0$; $\angle A = 5(a + 2)^0$ and $\angle B = 2(2a + 7)^0$.
Solution:
Given
$\angle C = 64^0$
$\angle D = \angle C - 8^0$
$= 64^0 - 8^0$
We get,
$\angle D = 56^0$

$\angle A = 5 (a + 2)^0$
$\angle B = 2(2a + 7)^0$
We know that, sum of all the angles in a quadrilateral = 360^0
So,
$\angle A + \angle B + \angle C + \angle D = 360^0$
$5(a + 2)^0 + 2(2a + 7)^0 + 64^0 + 56^0 = 360^0$
On further calculation, we get
$5a + 10^0 + 4a + 14^0 + 64^0 + 56^0 = 360^0$
$9a + 144^0 = 360^0$
$9a = 360^0 - 144^0$
$9a = 216^0$
We get,
$a = 24^0$
$\angle A = 5(a + 2)$
$= 5(24 + 2)$
We get,
$= 130^0$

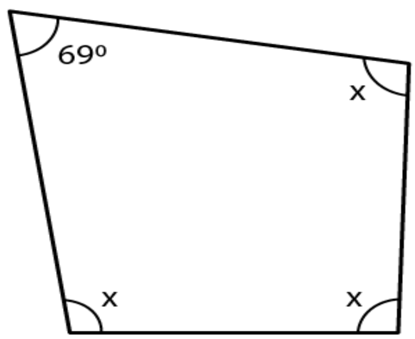
8. In the given figure
$\angle b = 2a + 15$
And $\angle c = 3a + 5$; find the values of b and c



Solution:
$\angle b = 2a + 15$ and
$\angle c = 3a + 5$
Sum of angles of a quadrilateral = 360^0
$70^0 + \angle a + \angle b + \angle c = 360^0$
$70^0 + a + (2a + 15) + (3a + 5) = 360^0$

$70^0 + a + 2a + 15 + 3a + 5 = 360^0$
$6a + 90^0 = 360^0$
$6a = 360^0 - 90^0$
$6a = 270^0$
We get,
$a = 45^0$
Hence, $\angle a = 45^0$
$b = 2a + 15 = 2 \times 45^0 + 15$
$= 90^0 + 15$
$= 105^0$
$c = 3a + 5 = 3 \times 45^0 + 5$
$= 135^0 + 5$
$= 140^0$
Therefore, $\angle a = 45^0$; $\angle$ b = 105^0 and $\angle c = 140^0$

9. Three angles of a quadrilateral are equal. If the fourth angle is 69^0; find the measure of equal angles.
Solution:

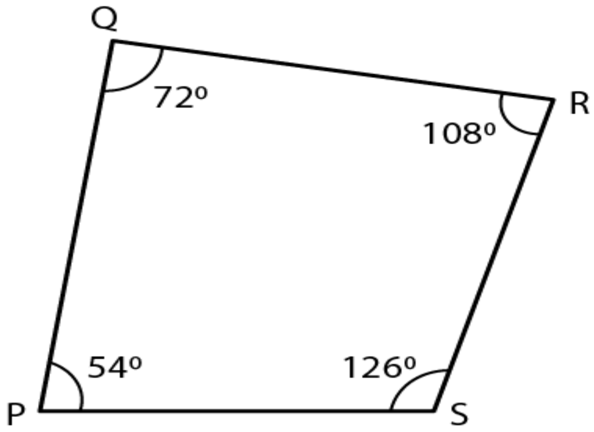


Given that,
Three angles of a quadrilateral are equal
Let us consider each angle as x^0
Hence,
$x^0 + x^0 + x^0 + 69^0 = 360^0$
$3x = 360^0 - 69^0$
$3x = 291^0$
$x = 291^0 / 3$
We get,
$x = 97^0$
Therefore, the measure of all the equal angles is 97^0

10. In quadrilateral PQRS, ∠P: ∠Q : ∠R: ∠S = 3: 4: 6: 7.

Calculate each angle of the quadrilateral and then prove that PQ and SR are parallel to each other. Is PS also parallel to QR?

Solution:



Given

∠P: ∠Q: ∠R: ∠S = 3: 4: 6: 7

Let ∠P = 3x

∠Q = 4x

∠R = 6x and

∠S = 7x

Hence,

∠P + ∠Q + ∠R + ∠S = 360^0

3x + 4x + 6x + 7x = 360^0

20x = 360^0

x = 360^0 / 20

We get,

x = 18^0

So,

∠P = 3x = 3 × 18^0

= 54^0

∠Q = 4x = 4 × 18^0

= 72^0

∠R = 6x = 6 × 18^0

= 108^0

∠S = 7x = 7 × 18^0

= 126^0

Now, adding two adjacent angles, we get

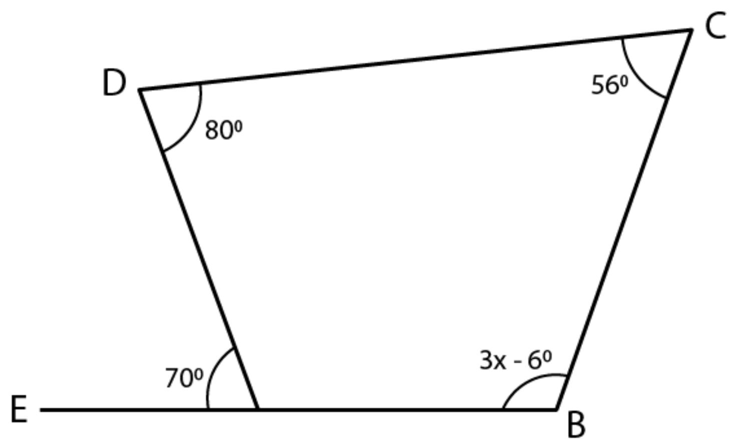
∠Q + ∠R = 72^0 + 108^0

= 180^0 and

∠P + ∠S = 54^0 + 126^0

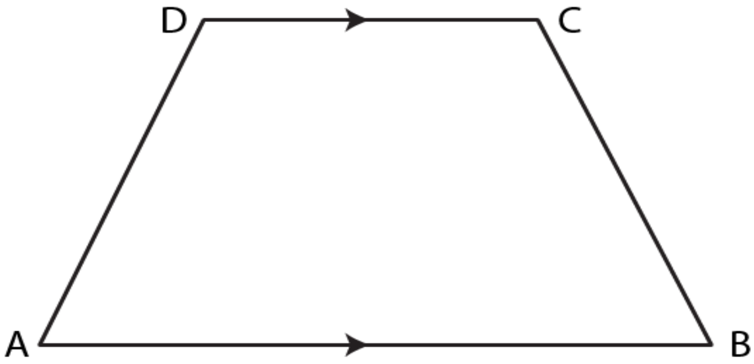
$= 180^0$
Therefore, PQ || RS
Since,
$\angle P + \angle Q = 54^0 + 72^0$
$= 126^0$
Which is not equal to180^0
Therefore, PS and QR are not parallel

11. Use the information given in the following figure to find the value of x.



Solution:
Given
A, B, C and D are the vertices of quadrilateral and BA is produced to E
Here,
$\angle EAD = 70^0$
Hence,
$\angle DAB = 180^0 - 70^0$ [By straight line]
$\angle DAB = 110^0$
Hence,
$\angle EAD + \angle DAB = 180^0$
The sum of angles of a quadrilateral is 360^0
$110^0 + 80^0 + 56^0 + 3x - 6^0 = 360^0$
$3x = 360^0 – 110^0 – 80^0 – 56^0 + 6^0$
$3x = 360^0 - 240^0$
$3x = 120^0$
$x = 120^0 / 3$
We get,
$x = 40^0$
Therefore, the value of x is 40^0

12. The following figure shows a quadrilateral in which sides AB and DC are parallel. If ∠A: ∠D = 4: 5, ∠B = $(3x – 15)^0$ and ∠C = $(4x + 20)^0$, find each angle of the quadrilateral ABCD.



Solution:

Let us consider ∠A = 4x and

∠D = 5x

Since, AB || DC

So,

∠A + ∠D = 180^0

Substituting the value of angle A and D, we get

$4x + 5x = 180^0$

$9x = 180^0$

$x = 20^0$

Now,

∠A = 4x = 4 × 20^0

= 80^0

∠D = 5x = 5 × 20^0

= 100^0

Similarly since, AB || DC

∠B + ∠C = 180^0

$3x – 15^0 + 4x + 20^0 = 180^0$

$7x + 5^0 = 180^0$

$7x = 180^0 - 5^0$

$7x = 175^0$

We get,

$x = 25^0$

∠B = 3x – 15^0 = 3 × 25^0 - 15^0

= $75^0 – 15^0$

= 60^0 and

$\angle C = 4x + 20^0 = 4 \times 25^0 + 20^0$

$= 100^0 + 20^0$

$= 120^0$

EXERCISE 27(B)

1. In a trapezium ABCD, side AB is parallel to side DC. If ∠A = 78^0 and ∠C = 120^0, find angles B and D.

Solution:

Given

AB || DC and BC is transversal

We know that,

The sum of co-interior angles of a parallelogram = 180^0

Hence,

∠B + ∠C = 180^0

∠B + 120^0 = 180^0

∠B = 180^0 - 120^0

We get,

∠B = 60^0

Also,

∠A + ∠D = 180^0

78^0 + ∠D = 180^0

∠D = 180^0 - 78^0

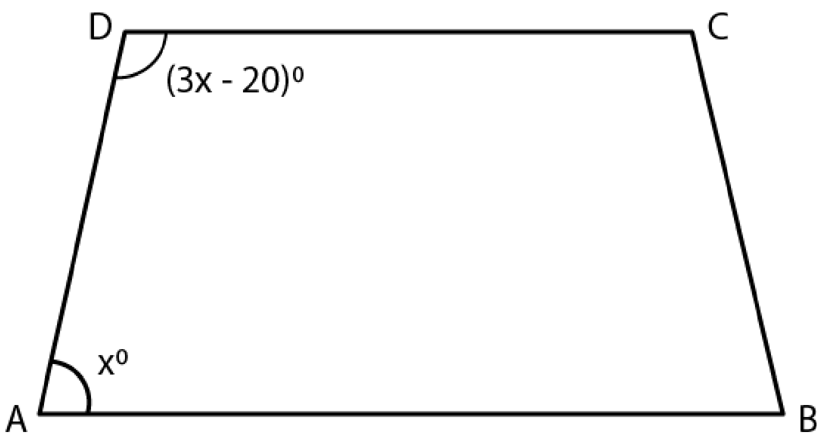
We get,

∠D = 102^0

Therefore, ∠B = 60^0 and ∠D = 102^0

2. In a trapezium ABCD, side AB is parallel to side DC. If ∠A = x^0 and ∠D = $(3x – 20)^0$; find the value of x.

Solution:



Given

AB || DC and BC is transversal

The sum of co-interior angles of a parallelogram = 180^0

Hence,
$\angle A + \angle D = 180^0$
$x^0 + (3x - 20)^0 = 180^0$
$x^0 + 3x - 20^0 = 180^0$
$4x^0 = 180^0 + 20^0$
$4x^0 = 200^0$
$x^0 = 200^0 / 4$
We get,
$x^0 = 50^0$
Hence, the value of x is 50^0

3. The angles A, B, C and D of a trapezium ABCD are in the ratio 3: 4: 5: 6. Let ∠A: ∠B: ∠C: ∠D = 3: 4: 5: 6. Find all the angles of the trapezium. Also, name the two sides of this trapezium which are parallel to each other. Give reason for your answer

Solution:

Let us consider the angles of a parallelogram ABCD be 3x, 4x, 5x and 6x
We know that,
The sum of angles of a parallelogram = 360^0
Hence,
$\angle A + \angle B + \angle C + \angle D = 360^0$
$3x + 4x + 5x + 6x = 360^0$
$18x = 360^0$
$x = 360^0 / 18$
We get,
$x = 20^0$
Now, the angles are,
$\angle A = 3x = 3 \times 20^0$
$\angle A = 60^0$
$\angle B = 4x = 4 \times 20^0$
$\angle B = 80^0$
$\angle C = 5x = 5 \times 20^0$
$\angle C = 100^0$
$\angle D = 6x = 6 \times 20^0$
$\angle D = 120^0$
Here,
The sum of ∠A and ∠D = 180^0
Therefore, AB is parallel to DC and the angles are co-interior angles whose sum = 180^0

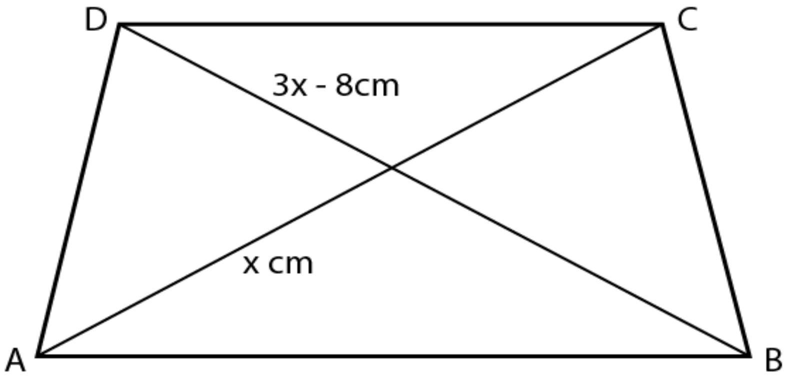
4. In a isosceles trapezium one pair of opposite sides are ………… to each other and the other pair of opposite sides are …………… to each other.

Solution:

In an isosceles trapezium one pair of opposite sides are **parallel** to each other and the other pair of opposite sides are **equal** to each other.

5. Two diagonals of an isosceles trapezium are x cm and (3x – 8) cm. Find the value of x.

Solution:



We know that,

The diagonals of an isosceles trapezium are of equal length

Figure

Hence,

$3x - 8 = x$

$3x - x = 8$

$2x = 8$

$x = 8 / 2$

We get,

$x = 4$

Therefore, the value of x is 4 cm

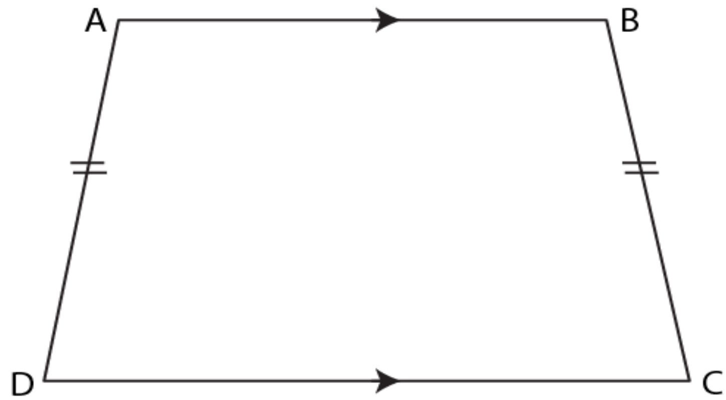
6. Angle A of an isosceles trapezium is 115^0; find the angles B, C and D.

Solution:

Since, the base angles of an isosceles trapezium are equal

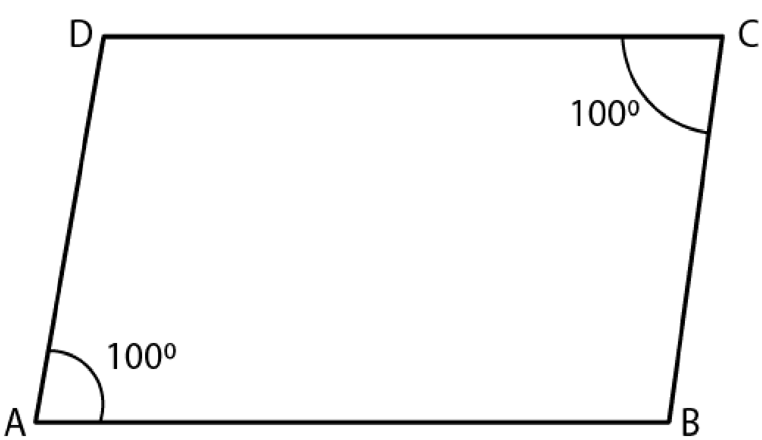
Hence,

$\angle A = \angle B = 115^0$



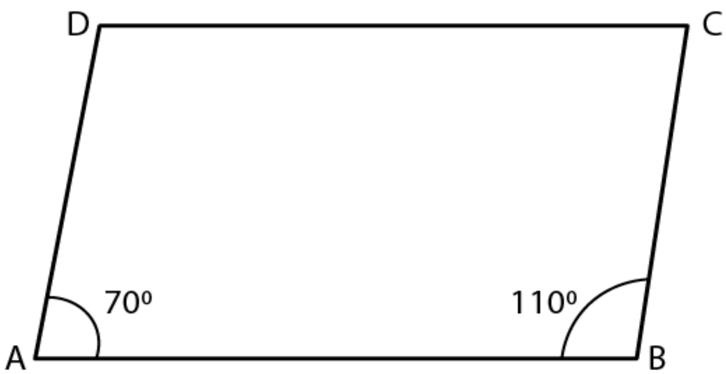
Also,
$\angle A$ and $\angle D$ are co-interior angles
The sum of co-interior angles of a quadrilateral is 180^0
So,
$\angle A + \angle D = 180^0$
$115^0 + \angle D = 180^0$
$\angle D = 180^0 - 115^0$
We get,
$\angle D = 65^0$
Hence,
$\angle D = \angle C = 65^0$
Therefore, the values of angles B, C and D are 115^0, 65^0 and 65^0

7. Two opposite angles of a parallelogram are 100^0 each. Find each of the other two opposite angles.
Solution:
Given
Two opposite angles of a parallelogram are 100^0 each



The sum of adjacent angles of a parallelogram = 180^0
Hence,
$\angle A + \angle B = 180^0$
$100^0 + \angle B = 180^0$
$\angle B = 180^0 - 100^0$
We get,
$\angle B = 80^0$
We know that,
The opposite angles of a parallelogram are equal
$\angle D = \angle B = 80^0$
Therefore, the other two opposite angles $\angle D = \angle B = 80^0$

8. Two adjacent angles of a parallelogram are 70^0 and 110^0 respectively. Find the other two angles of it.
Solution:
Given
Two adjacent angles of a parallelogram are 70^0 and 100^0 respectively

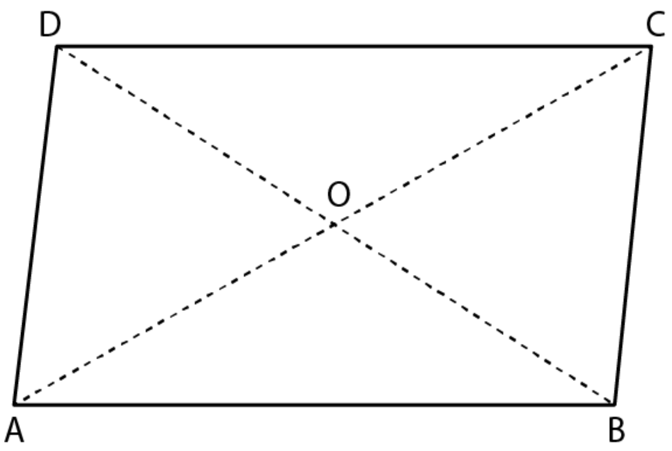


We know that,
Opposite angles of a parallelogram are equal.
Hence, $\angle C = \angle A = 70^0$ and $\angle D = \angle B = 110^0$

9. The angles A, B, C and D of a quadrilateral are in the ratio 2: 3: 2: 3. Show this quadrilateral is a parallelogram.
Solution:
Given
Angles of a quadrilateral are in the ratio 2: 3: 2: 3
Let us consider the angles A, B, C and D be 2x, 3x, 2x and 3x
We know that,
The sum of interior angles of a quadrilateral = 360^0
So,

$\angle A + \angle B + \angle C + \angle D = 360^0$
$2x + 3x + 2x + 3x = 360^0$
$10x = 360^0$
$x = 360^0 / 10$
We get,
$x = 36^0$
Hence, the measure of each angle is as follows
$\angle A = \angle C = 2x = 2 \times 36^0$
$\angle A = \angle C = 72^0$
$\angle B = \angle D = 3x = 3 \times 36^0$
$\angle B = \angle D = 108^0$
Since the opposite angles are equal and
The adjacent angles are supplementary
i.e $\angle A + \angle B = 180^0$
$72^0 + 108^0 = 180^0$
$180^0 = 180^0$ and
$\angle C + \angle D = 180^0$
$72^0 + 108^0 = 180^0$
$180^0 = 180^0$
Quadrilateral ABCD fulfills the condition
Therefore, a quadrilateral ABCD is a parallelogram

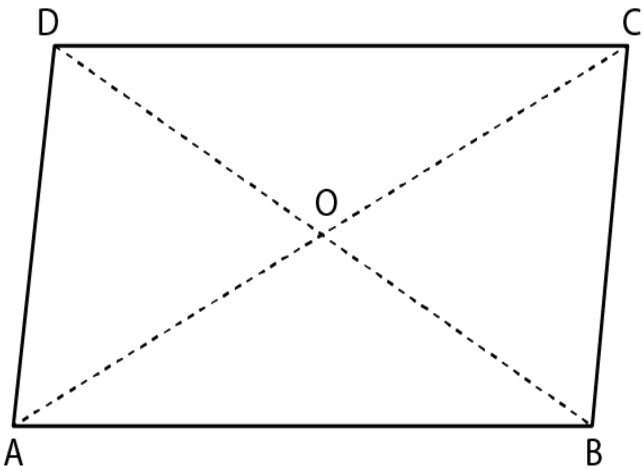
10. In a parallelogram ABCD, its diagonals AC and BD intersect each other at point O.



If AC = 12 cm and BD = 9 cm; find; lengths of OA and OB
Solution:
Given
AC and BD intersect each other at point O

So,
OA = OC = (1 / 2) AC and
Similarly,
OB = OD = (1 / 2) BD
Hence,
OA = (1 / 2) × AC
= (1 / 2) × 12
= 6 cm
OB = (1 / 2) × BD
= (1 / 2) × 9
= 4. 5 cm

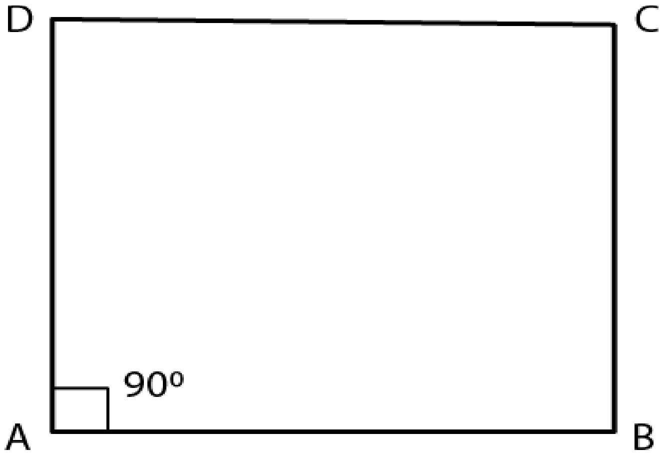
11. In a parallelogram ABCD, its diagonals intersect at point O. If OA = 6 cm and OB = 7.5 cm, find the lengths of AC and BD.



Solution:
The diagonals AC and BD intersect each other at point O
So,
OA = OC = (1 / 2) AC and
OB = OD = (1 / 2) BD
So,
OA = (1 / 2) × AC
AC = 2 × OA
AC = 2 × 6
We get,
AC = 12 cm and
OB = (1 / 2) × BD
BD = 2 × OB
BD = 2 × 7.5

We get,
BD = 15 cm

12. In a parallelogram ABCD, ∠A = 90^0
(i) What is the measure of angle B.
(ii) Write the special name of the parallelogram.
Solution:
Given
In a parallelogram ABCD, ∠A = 90^0

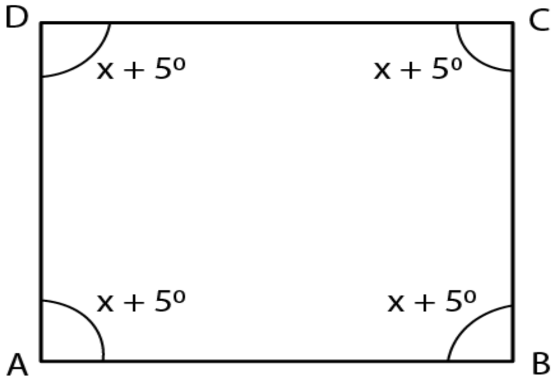


(i) We know that,
In a parallelogram, adjacent angles are supplementary
Hence,
∠A + ∠B = 180^0
90^0 + ∠B = 180^0
∠B = 180^0 - 90^0
We get,
∠B = 90^0
Therefore, the measure of ∠B = 90^0
(ii) Since all the angles of a given parallelogram is right angle.
Hence the given parallelogram is a rectangle

13. One diagonal of a rectangle is 18 cm. What is the length of its other diagonal?
Solution:
We know that,
In a rectangle, the diagonal are equal
Hence,
AC = BD
Given that one diagonal of a rectangle is 18 cm

Hence, the other diagonal of a rectangle will be = 18 cm
Therefore, the length of the other diagonal is 18 cm

14. Each angle of a quadrilateral is x + 5^0. Find:
(i) the value of x
(ii) each angle of the quadrilateral.
(iii) Give the special name of the quadrilateral taken.



Solution:
(i) We know that,
The sum of interior angles of a quadrilateral is 360^0
Hence,
$\angle A + \angle B + \angle C + \angle D = 360^0$
$x + 5^0 + x + 5^0 + x + 5^0 + x + 5^0 = 360^0$
$4x + 20^0 = 360^0$
$4x = 360^0 - 20^0$
$4x = 340^0$
$x = 340^0 / 4$
We get,
$x = 85^0$
Hence, the value of x is 85^0
(ii) Each angle of the quadrilateral ABCD = $x + 5^0$
$= 85^0 + 5^0$
We get,
$= 90^0$
Therefore, each angle of the quadrilateral = 90^0
(iii) The name of the taken quadrilateral is a rectangle

15. If three angles of a quadrilateral are 90^0 each, show that the given quadrilateral is a rectangle.
Solution:

If each angle of quadrilateral is 90^0, then the given quadrilateral will be a rectangle
We know that,
The sum of interior angles of a quadrilateral is 360^0
Hence,
$\angle A + \angle B + \angle C + \angle D = 360^0$
$90^0 + 90^0 + 90^0 + \angle D = 360^0$
$270^0 + \angle D = 360^0$
$\angle D = 360^0 - 270^0$
We get,
$\angle D = 90^0$
Since,
Each angle of the quadrilateral = 90^0
Therefore, the given quadrilateral is a rectangle.