Tamil Nadu Board SSLC Class 10 Maths 2019 Question Paper Solutions

SECTION - I

1. The function f: N → R is defined by $f(n) = 2^n$. The range of the function is:
(a) the set of all even positive integers
(b) N
(c) R
(d) a subset of set of all even positive integers

Solution:
Correct answer: (d)

Given,
$f(n) = 2^n$
The value of 2^n is an even positive integer for all natural numbers.
Hence, the range of the function is a subset of all even possible integers.

2. If a, b, c, l, m are in AP, then the value of a - 4b + 6c - 4l + m is:
(a) 1
(b) 2
(c) 3
(d) 0

Solution:
Correct answer: (d)

a - 4b + 6c - 4l + m
= a + 6c + m - 4(b + l)
= a + m + 6c - 4(2c) [since b + l = 2c]
= 2c + 6c - 4(2c) [since a + m = 2c]
= 8c - 8c
= 0

3. The common ratio of the GP a^{m-n}, a^m, a^{m+n} is:
(a) a^m
(b) a^{-m}
(c) a^n
(d) a^{-n}
Solution:
Correct answer: (c)

Given ,
a^{m-n}, a^m, a^{m+n} are in GP.
Common ratio = a^m / a^{m-n}

$= a^{m - (m - n)}$
$= a^{m - m + n}$
$= a^n$

4. The number of polynomials having zeroes 2 and 1 is:
(a) 1
(b) 2
(c) 3
(d) more than 3

Solution:
Correct answer: (d)

Let $p(x) = ax^2 + bx + c$ be the quadratic polynomial.
Given, 2 and 1 are the zeroes of the polynomial.
Sum of zeroes = -b/a
2 + 1 = -b/a
⇒ -b/a = 3/1
⇒ b/a = -3/1
Thus, b = -3 and a = 1
Product of zeroes = c/a
2 × 1 = c/a
⇒ c/a = 2/1
Thus, c = 2 and a = 1
The possible expressions which satisfies the above conditions are:
$p(x) = x^2 - 3x + 2$
$p(x) = kx^2 - 3kx + 2k$, where k is a real number
$p(x) = (x^2/k) + (3/k)x + (2/k)$, where k is any non-zero real number
and so on.
Therefore, there exist more than 3 polynomials with zeroes 2 and 1.

5. The common root of the equations $x^2 - bx + c = 0$ and $x^2 + bx - a = 0$ is:
(a) (c + a)/2b
(b) (c - a)/2b
(c) (c + b)/2a
(d) (a + b)/2c

Solution:
Correct answer: (a)

$x^2 - bx + c = 0$ and $x^2 + bx - a = 0$ have a common root.
⇒ $x^2 - bx + c = x^2 + bx - a$
⇒ c + a = bx + bx
⇒ c + a = 2bx
⇒ x = (c + a)/2b

6. Which of the following statements is incorrect?
(a) A unit matrix is a scalar matrix
(b) A scalar matrix is a diagonal matrix
(c) A unit matrix is a diagonal matrix

(d) For any two matrices, the addition of matrices exists

Solution:
Correct answer: (d)

We know that,
The addition of two matrices exists, if they have the same order.

7. If the line segment joining the points A(3, 4) and B(14, -3) meets the x-axis at P, then the ratio in which P divides the segment AB is:
(a) 4 : 3
(b) 3 : 4
(c) 2 : 3
(d) 4 : 1

Solution:
Correct answer: (a)

Let P(x, 0) divide the line segment joining the points A(3, 4) and B(14, -3) in the ratio m : n.
Using section formula:
Y-coordinate of P is:
0 = [m(-3) + n(4)]/(m + n)
⇒ -3m + 4n = 0
⇒ 4n = 3m
⇒ m/n = 4/3
Therefore, the required ratio is 4 : 3.

8. The distance of the point (-2, -3) from x-axis is:
(a) -2
(b) 2
(c) -3
(d) 3

Solution:
Correct answer: (d)

Given point is (-2, -3).
We know that the distance of any point from the x-axis is the value of y-coordinate.
Hence, the required distance is 3 units.

9. The sides of two similar triangles are in the ratio 2 : 3, then their areas are in the ratio:
(a) 9 : 4
(b) 4 : 9
(c) 2 : 3
(d) 3 : 2

Solution:
Correct answer: (b)

Given,

Ratio of sides of two similar triangles = 2 : 3
Ratio of their areas = Square the ratio of sides
$= 2^2 : 3^2$
= 4 : 9

10. The perimeters of two similar triangles are 24 cm and 18 cm respectively. If one side of the first triangle is 8 cm, then the corresponding side of the other triangle is:
(a) 4 cm
(b) 3 cm
(c) 9 cm
(d) 6 cm

Solution:
Correct answer: (d)

Given,
Perimeters of two similar triangles are 24 cm and 18 cm respectively.
Length of side of the first triangle = 8 cm
Let x be the side of another triangle.
Ratio of perimeters = Ratio of the corresponding sides
⇒ 24/18 = 8/x
⇒ 4/3 = 8/x
⇒ x = 24/4
⇒ x = 6
Therefore, the corresponding side of the second triangle is 6 cm.

11. $1 - [\sin^2\theta / (1 + \cos\theta)] =$
(a) $\cos\theta$
(b) $\tan\theta$
(c) $\cot\theta$
(d) $\text{cosec}\,\theta$

Solution:
Correct answer: (a)

$1 - [\sin^2\theta / (1 + \cos\theta)]$
$= 1 - \{[\sin^2\theta) / (1 + \cos\theta)] \times [(1 - \cos\theta) / (1 - \cos\theta)]\}$
$= 1 - [\sin^2\theta(1 - \cos\theta) / (1 - \cos^2\theta)]$
$= 1 - [\sin^2\theta(1 - \cos\theta)/\sin^2\theta]$
$= 1 - (1 - \cos\theta)$
$= 1 - 1 + \cos\theta$
$= \cos\theta$

12. $\sin^2\theta + 1/(1 + \tan^2\theta) =$
(a) $\text{cosec}^2\theta + \cot^2\theta$
(b) $\text{cosec}^2\theta - \cot^2\theta$
(c) $\cot^2\theta - \text{cosec}^2\theta$
(d) $\sin^2\theta - \cos^2\theta$

Solution:

Correct answer: (b)

$\sin^2\theta + 1/(1 + \tan^2\theta)$
$= \sin^2\theta + 1/\sec^2\theta$
$= \sin^2\theta + \cos^2\theta$
$= 1$
$= \text{cosec}^2\theta - \cot^2\theta$ [since $1 + \cot^2\theta = \text{cosec}^2\theta$]

13. If the total surface area of a solid right circular cylinder is 200π cm^2 and its radius is 5 cm, then the sum of its height and radius is:
(a) 20 cm
(b) 25 cm
(c) 30 cm
(d) 15 cm

Solution:
Correct answer: (a)

Given,
Total surface area of cylinder = 200π cm^2
Radius = r = 5 cm
Let h be the height of the cylinder.
$\Rightarrow 2\pi r(r + h) = 200\pi$
$\Rightarrow 5(5 + h) = 100$
$\Rightarrow 5 + h = 100/5$
$\Rightarrow 5 + h = 20$
$\Rightarrow h = 20 - 5 = 15$ cm
Sum of the radius and height = r + h = 5 + 15 = 20 cm

14. Variance of the first 11 natural numbers is:
(a) $\sqrt{5}$
(b) $\sqrt{10}$
(c) $5\sqrt{2}$
(d) 10

Solution:
Correct answer: (d)

We know that variance of first n natural numbers = V = $(n^2 - 1)/12$
VAriance for first 11 natural numbers = $[(11)^2 - 1]/12$
= (121 - 1)/12
= 120/12
= 10

15. There are 6 defective items in a sample of 20 items. One item is drawn at random. The probability that it is a non-defective item is:
(a) 7/10
(b) 0
(c) 3/10
(d) 2/3

Solution:
Correct answer: (a)

Given,
Total number of items = 20
Defective items = 6
Non-defective items = 20 - 6 = 14
P(non-defective item) = 14/20 = 7/10

SECTION - II

16. Define a set.

Solution:
A set is defined as the collection of well defined objects which can be separated distinctly.
For example, S = {1, 3 , 5 , 7} is a collection of the odd numbers from 1 to 7.

17. Find the 18th and 25th terms of the sequence defined by
$a_n = n(n + 3)$, in $n \in N$ and n is even
$a_n = 2n/(n^2 + 1)$, if $n \in N$ and n is odd

Solution:
18th term of the sequence:
n = 18 (even)
$a_n = n(n + 3)$
$a_{18} = 18(18 + 3)$
$= 18 \times 21$
$= 378$
25th term of the sequence:
n = 25 (odd)
$a_n = 2n/(n^2 + 1)$
$a_{25} = (2 \times 25)/ [(25)^2 + 1]$
$= 50/ (625 + 1)$
$= 50/626$
$= 25/313$

18. Using cross multiplication rule, solve:
3x + 5y = 25
7x + 6y = 30

Solution:
Given,
3x + 5y = 25
7x + 6y = 30
Comparing with the standard form,
$a_1 = 3, b_1 = 5, c_1 = -25$
$a_2 = 7, b_2 = 6, c_2 = -30$
Using cross multiplication method,
$x/(b_1c_2 - b_2c_1) = y/(c_1a_2 - c_2a_1) = 1/(a_1b_2 - a_2b_1)$

x/(-150 + 150) = y/(-175 + 90) = 1/(18 - 35)
x/0 = y/(-85) = 1/(-17)
x/0 = -1/17, y/(-85) = -1/17
x = 0, y = -85/-17
x = 0, y = 5

19. Form a quadratic equation whose roots are (4 + √7)/2, (4 - √7)/2.

Solution:
Let α and β be the roots of the quadratic equation.
Given, (4 + √7)/2, (4 - √7)/2 are the roots of the quadratic equation.
α + β = [(4 + √7)/2] + [(4 - √7)/2]
= [(4 + √7 + 4 - √7)/2]
= 8/2
= 4
αβ = [(4 + √7)/2] × [(4 - √7)/2]
= $[(4)^2 - (\sqrt{7})^2]/4$
= (16 - 7)/4
= 9/4
Therefore, the required quadratic equation is
$x^2 - 4x + (9/4) = 0$
⇒ $4x^2 - 16x + 9 = 0$

20. Define a diagonal matrix.

Solution:
Diagonal matrix is a square matrix in which every element except the principal diagonal elements is zero.
Example:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

21.

Find the product of the matrices $\begin{bmatrix} 6 \\ -3 \end{bmatrix} \begin{bmatrix} 2 & -7 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} 6 \\ -3 \end{bmatrix} \begin{bmatrix} 2 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \times 2 & 6 \times (-7) \\ -3 \times 2 & -3 \times (-7) \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -42 \\ -6 & 21 \end{bmatrix}$$

22. Find the equation of the straight line passing through the points (-1, 1) and (2, -4).

Solution:
Let the given points be:
$(x_1, y_1) = (-1, 1)$
$(x_2, y_2) = (2, -4)$
Equation of the line passing through the given points is:
$(y - y_1)/(y_2 - y_1) = (x - x_1)/(x_2 - x_1)$
(y - 1)/(-4 - 1) = (x + 1)/(2 + 1)
(y - 1)/(-5) = (x + 1)/3
3(y - 1) = -5(x + 1)
3y - 3 = -5x - 5
5x + 3y - 3 + 5 = 0
5x + 3y + 2 = 0

23. Prove the identity (sin θ/cosec θ) + (cos θ/sec θ) = 1.

Solution:
LHS = (sin θ/cosec θ) + (cos θ/sec θ)
= sin θ . sin θ + cos θ . cos θ
= $\sin^2\theta + \cos^2\theta$
= 1
= RHS
Hence proved.

24. A solid right circular cylinder has radius 7 cm and height 20 cm. Find its total surface area. (take π = 22/7)

Solution:
Given,
Radius of right circular cylinder = r = 7 cm
Height = h = 20 cm
Total surface area = 2πr(r + h)
= 2 × (22/7) × 7 × (7 + 20)
= 2 × 22 × 27
= 1188 cm^2

25. The volume of a cone with a circular base is 216π cm3. If the base radius is 9 cm, then find the height of the cone.

Solution:

Let h be the height of the cone.
Given,
Radius of cone = r = 9 cm
Volume = 216π cm^3
$(1/3)\pi r^2 h = 216\pi$
$(1/3) \times 9 \times 9 \times h = 216$
$27h = 216$
$h = 216/27$
h = 8 cm
Therefore, the height of the cone is 8 cm.

26. The standard deviation of 20 observations is √5. If each observation is multiplied by 2, find the standard deviation and variance of the resulting observations.

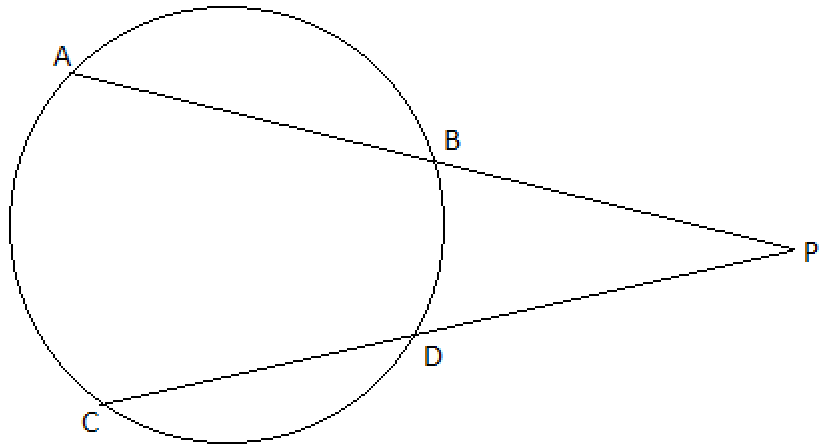
Solution:
Given,
Standard deviation = √5
Each observation is multiplied by 2, then new standard deviation = 2√5
Variance = (standard deviation)2
$= (2\sqrt{5})^2$
$= 4 \times 5$

27. A die is thrown twice. Find the probability of getting a total of 9.

Solution:
Total number of outcomes = $n(S) = 6^2 = 36$
Let E be the event of getting a sum 9.
E = {(4, 5), (5, 4), (3, 6), (6, 3)}
Number of outcomes favourable to E = n(E) = 4
P(E) = n(E)/n(S)
= 4/36
= 1/9
Hence, the required probability is 1/9.

28. AB and CD are two chords of a circle which intersect each other externally at P. If AB = 4 cm, BP = 5 cm and PD = 3 cm, then find CD.

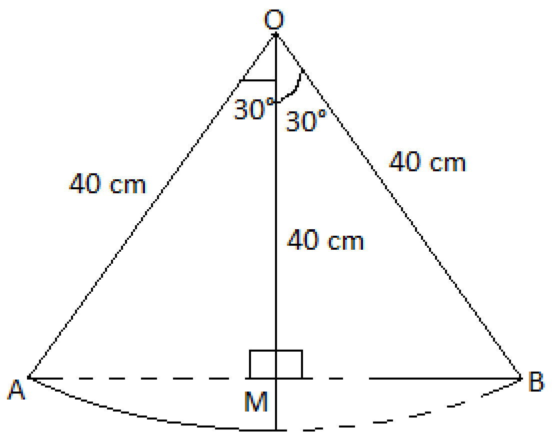
Solution:
Given,
AB and CD are two chords of a circle which intersect each other externally at P.
AB = 4 cm, BP = 5 cm and PD = 3 cm
Two chords AB and CD meet at P when produced.



⇒ PA × PB = PC × PD
⇒ (PB + AB) × PB = (PD + CD) × PD
⇒ (5 + 4) × 5 = (3 + CD) × 3
⇒ 9 × 5 = (3 + CD) × 3
⇒ (3 + CD) = 45/3
⇒ 3 + CD = 15
⇒ CD = 15 - 3
⇒ CD = 12 cm

29. A simple pendulum of length 40 cm subtends 60° at the vertex in one full oscillation. What will be the shortest distance between the initial position and the final position of the bob?

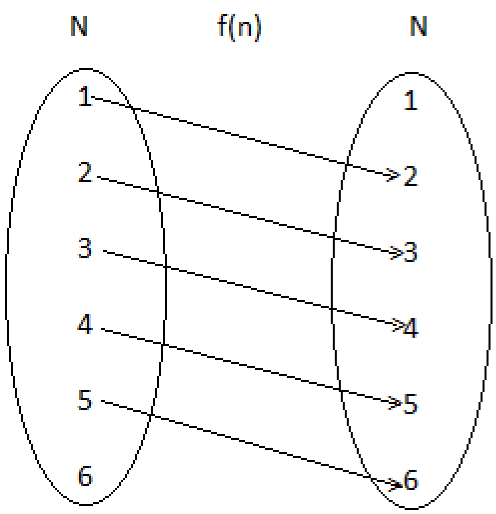
Solution:
Given,
Length of the pendulum = 40 cm



In triangle OMA,
sin 30° = AM/OA
1/2 = AM/40
AM = 40/2
AM = 20 cm
AB = 2AM = 2(20) = 40 cm
Hence, the required shortest distance is 40 cm.

30. (a) Show that the function f: N → N defined by f(n) = n + 1 is not onto, by drawing arrow diagrams.

Solution:
Given,
f(n) = n + 1
f(1) = 1 + 1 = 2
f(2) = 2 + 1 = 3
f(3) = 3 + 1 = 4
f(4) = 4 + 1 = 5



Hence, the given function is not on-to.

OR

(b) Find the equation of the perpendicular to the line x = 5, passing through the point (5, 8).

Solution:
Given,
x = 5 is a vertical line.
Thus, it doesn't have a slope.
The line perpendicular to a vertical line will be a horizontal line. Hence, its slope is 0 (zero).
Therefore, the equation of the line ia:
$y - y_1 = 0(x - x_1)$ (point slope form)
y - 8 = 0(x - 5)
y - 8 = 0
y = 8

SECTION - III

31. A function f : [-7, 6) → R is defined as

$$f(x)=\begin{cases}x^2+2x+1; & -7\le x<-5\\ x+5; & -5\le x<2\\ x-1; & 2\le x<6\end{cases}$$

Find [4f(-3) + 2f(4)]/ [f(-6) - 3f(1)].

Solution:
f(-3) = -3 + 5 = 2
f(4) = 4 - 1 = 3
f(-6) = (-6)2 + 2(-6) + 1
= 36 - 12 + 1
= 25
f(1) = 1 + 5 = 6
[4f(-3) + 2f(4)]/ [f(-6) - 3f(1)] = [4 × 2 + 2 × 3]/ [25 - 3 × 6]
= (8 + 6)/(25 - 18)
= 14/7
= 2

32. Find the sum of the series $5^2 + 7^2 + 9^2 + ... + 39^2$.

Solution:
$5^2 + 7^2 + 9^2 + ... + 39^2$
$= (1^2 + 2^2 + ... + 39^2) - (2^2 + 4^2 + 6^2 + ... + 38^0) - (1^2 + 3^2)$
= (39 × 40 × 79)/6 - [4 × (19 × 20 × 39)/6] - 10
= 20540 - 9880 - 10
= 10650

33. The 4th term of a geometric sequence is 2/3 and its seventh term is 16/81. Find the geometric sequence.

Solution:
Let a be the first term and r be the common ratio of GP.
Given,
$a_4 = 2/3$
$a_7 = 16/81$
Now,
$a_7/a_4 = (16/81)/ (2/3)$
$ar^6/ar^3 = (16 × 3)/ (81 × 2)$
$r^3 = 8/27$
$r^3 = (2/3)^3$
⇒ r = 2/3
Thus, $ar^3 = 2/3$
$a(2/3)^3 = 2/3$
$a = (3/2)^2$
a = 9/4
Hence, the required GP is: $9/4, (9/4)(2/3), (9/4)(2/3)^2$,....

34. Show that the roots of the equation $x^2 + 2(a + b)x + 2(a^2 + b^2) = 0$ are not real.

Solution:

Given,

$x^2 + 2(a + b)x + 2(a^2 + b^2) = 0$

$\Delta = B^2 - 4AC$

$= [2(a + b)]^2 - 4(1)[2(a^2 + b^2)]$

$= 4(a^2 + b^2 + 2ab) - 8a^2 - 8b^2$

$= 4a^2 + 4b^2 + 8ab - 8a^2 - 8b^2$

$= -4a^2 + 8ab - 4b^2$

$= -4(a^2 - 2ab + b^2)$

$= -4(a - b)^2 < 0$

Therefore, the roots of the given equation are imaginary,i.e. not real.

35. The GCD of $x^4 + 3x^3 + 5x^2 + 26x + 56$ and $x^4 + 2x^3 - 4x^2 - x + 28$ is $x^2 + 5x + 7$. Find their LCM.

Solution:

$$
\begin{array}{r|rrrrr}
 & x^2 & -2x & +8 & & \\
\hline
x^2 + 5x + 7 & x^4 & +3x^3 & +5x^2 & +26x & +56 \\
- & x^4 & +5x^3 & +7x^2 & & \\
\hline
 & & -2x^3 & -2x^2 & +26x & +56 \\
- & & -2x^3 & -10x^2 & -14x & \\
\hline
 & & & 8x^2 & +40x & +56 \\
- & & & 8x^2 & +40x & +56 \\
\hline
 & & & & & 0
\end{array}
$$

Quotient = $x^2 - 2x + 8$

LCM = $(x^2 - 2x + 8)(x^4 + 2x^3 - 4x^2 - x + 28)$

36. Solve:

$$(x \quad 1)\begin{pmatrix} 1 & 0 \\ -2 & -3 \end{pmatrix}\begin{pmatrix} x \\ 5 \end{pmatrix} = 0$$

Solution:

Given,

$$\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

$$\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} x + 0 \\ -2x - 15 \end{bmatrix} = 0$$

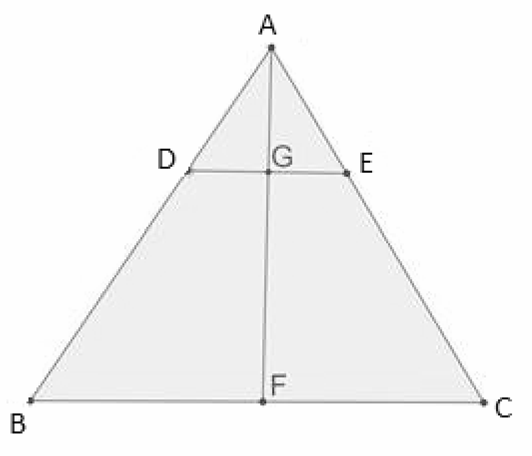
(x)(x) + (1)(-2x - 15) = 0
x^2 - 2x - 15 = 0
x^2 - 5x + 3x - 15 = 0
x(x - 5) + 3(x - 5) = 0
(x - 5)(x + 3) = 0
x = 5, -3

37. If C is the midpoint of the line segment joining A(4, 0) and B(0, 6) and if O is the origin, then show that C is equidistant from all the vertices of ΔOAB.

Solution:
Given,
A(4, 0) and B(0, 6)
C = Midpoint of AB
= [(4 + 0)/2, (0 + 6)/2]
= (4/2, 6/2)
= (2, 3)
Let O(0, 0) be the origin.
Now,
OC = $\sqrt{(2^2 + 3^2)}$ = √(4 + 9) = √13
AC = $\sqrt{[(2 - 4)^2 + (3 - 0)^2]}$
= √(4 + 9)
= √13
BC = $\sqrt{[(2 - 0)^2 + (3 - 6)^2]}$
= √(4 + 9)
= √13
Therefore, OC = AC = BC
Hence, C is equidistant from all the vertices of ΔOAB.

38. The points D and E are on the sides AB and AC of ΔABC respectively, such that DE || BC. If AB = 3AD and the area of ΔABC is 72 cm^2, then find the area of the quadrilateral DBCE.

Solution:
Given,
The points D and E are on the sides AB and AC of ΔABC respectively, such that DE || BC.
AB = 3AD



In ΔADE and ΔABC

∠ADE = ∠ABC (corresponding angles)

∠DEA = ∠BCA (corresponding angles)

∴ ΔAED ~ ΔACB

Similarly,

ΔAGD ~ ΔAFB (where AF ⊥ BC)

⇒ AF = 3AG (∵ given AB = 3AD) ....(i)

Also,

BC = 3 × DE ....(ii)

Now,

(1/2) × BC × AF = (1/2) × (3 × DE) × (3 × AG) [From (i) and (ii)]

(1/2) × BC × AF = 9 × (1/2) × DE × AG

⇒ Area of triangle ABC = 9 × Area of triangle ADE

Thus, area of triangle ADE = 72/9 = 8 cm^2

Area of quadrilateral BCED = Area of triangle ABC - Area of triangle ADE

= 72 - 8

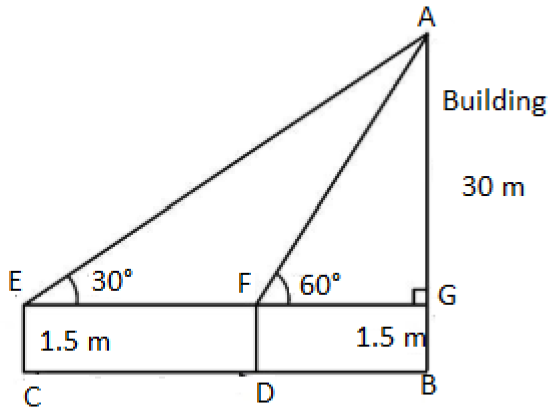
= 64 cm^2

39. A boy is standing at some distance from a 30 m tall building and his eye level from the ground is 1.5 m. The angle of elevation from his eye to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Solution:

Let AB be the building.

C and D be the points of observation.



AG = 30 - 1.5 = 28.5 m
In right triangle AGF,
tan 60° = AG/FG
√3 = 28.5/FG
FG = 28.5/√3
FG = 9.5√3 m
In right triangle AGE,
tan 30° = AG/GE
1/√3 = 28.5/(EF + 9.5√3)
EF + 9.5√3 = 28.5√3
EF = 28.5√3 - 9.5√3
EF = 19√3 m
Hence, the boy walked 19√3 m towards the building.

40. The diameter of a road roller of length 120 cm is 84 cm. If it takes 500 complete revolutions to level a playground, then find the cost of levelling it at the cost of 75 paise per square meter. (take π = 22/7)

Solution:
Given,
Diameter = 84 cm
Length = h = 120 cm
Radius = r = 84/2 = 42 cm
Surface area of roller = 2πrh
= 2 × (22/7) × 42 × 120
= 44 × 6 × 120
= 31680 cm^2
Area covered for 500 revolutions = 31680 × 500
= 15840000 cm^2
= 1584 sq.m
Cost of levelling 1 sq.m = 75 paise = Rs. 0.75
Cost of levelling the ground = 1584 × Rs. 075 = Rs. 1188

41. For a collection of data, if ∑x = 35, n = 5, $\sum(x - 9)^2 = 82$, then find $\sum x^2$ and $\sum(x - \bar{x})^2$.

Solution:
Given,
∑x = 35, n = 5, $\sum(x - 9)^2 = 82$

$\bar{x} = \sum x/n = 35/5 = 7$

$\sum(x - 9)^2 = 82$

$\sum(x^2 - 18x + 81) = 82$

$\sum x^2 - 18\sum x + 81\sum 1 = 82$

$\sum x^2 - 18(35) + 81(5) = 82$

$\sum x^2 - 630 + 405 = 82$

$\sum x^2 = 630 + 82 - 405$

$\sum x^2 = 307$

Similarly,

$\sum(x - 9)^2 = 82$

$\sum(x - 7 - 2)^2 = 82$

$\sum[(x - 7) - 2]^2 = 82$

$\sum(x - 7)^2 - 4\sum(x - 7) + 4\sum 1 = 82$

$\sum(x - \bar{x})^2 - 4\sum(x - \bar{x}) + 4(5) = 82$

$\sum(x - \bar{x})^2 - 4(0) + 20 = 82$

$\sum(x - \bar{x})^2 = 82 - 20$

$\sum(x - \bar{x})^2 = 62$

42. The probability that A, B and C can solve a problem are 4/5, 2/3 and 3/7 respectively. The probability of the problem being solved by A and B is 8/15, B and C is 2/7, A and C is 12/35. The probability of the problem being solved by all the three is 8/35. Find the probability that the problem can be solved by at least one of them.

Solution:

Given,

P(A) = 4/5

P(B) = 2/3

P(C) = 3/7

$P(A \cap B) = 8/15$

$P(B \cap C) = 2/7$

$P(A \cap C) = 12/35$

$P(A \cap B \cap C) = 8/35$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= (4/5) + (2/3) + (3/7) - (8/15) - (2/7) - (12/35) + (8/35)$

$= (84 + 70 + 45 - 56 - 30 - 36 + 24)/105$

$= 101/105$

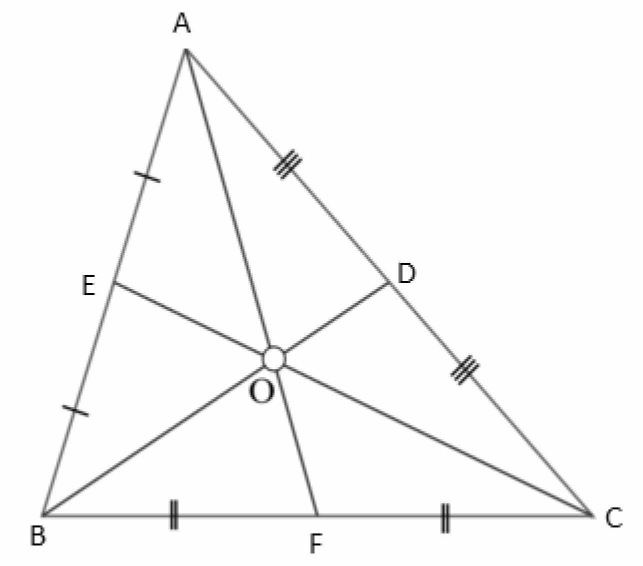
Hence, the probability of the problem can be solved by at least one of them is 101/105.

43. A triangle has vertices at (6, 7), (2, -9) and (-4, 1). Find the slopes of its medians.

Solution:

Let A(6, 7), B(2, -9) and C(-4, 1) be the vertices of triangle ABC.

BD, CE and AF be the medians of AC, AB and BC respectively.



D = Midpoint of AC
= [(6 - 4)/2, (7 + 1)/2]
= (2/2, 8/2)
= (1, 4)
E = Midpoint of AB
= [(6 + 2)/2, (7 - 9)/2]
= (8/2, -2/2)
= (4, -1)
F = Midpoint of BC
= [(2 - 4)/2, (-9 + 1)/2]
= (-2/2, -8/2)
= (-1, -4)
Slope of median AF = (-4 - 7)/(-1 - 6) = 11/7
Slope of median BD = (4 + 9)/ (1 - 2) = -13
Slope of median CE = (-1 - 1)/ (4 + 4) = -2/8 = -¼

44. A hollow cylinder pipe is of length 40 cm. Its internal and external radii are 4 cm and 12 cm respectively. It is melted and cast into a solid cylinder of length 20 cm. Find the radius of the new cylinder.

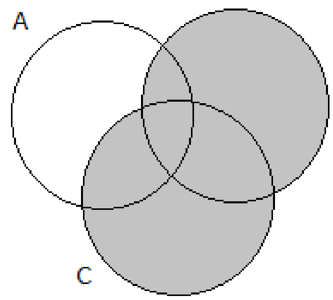
Solution:
Given,
Length of hollow cylinder = H = 40 cm
Internal radius = r = 4 cm
External radius = R = 12 cm
Length of the solid cylinder = h = 20 cm
Let r_2 be the radius of the solid cylinder.
Volume of solid cylinder = Volume of hollow cylinder
$\pi r_2^2 h = \pi h(R^2 - r^2)$
$r_2^2 \times 20 = 40[(12)^2 - (4)^2]$
$r_2^2 = 2[144 - 16]$
$r_2^2 = 2 \times 128$
$r_2^2 = 256$
$r_2 = 16$ cm

Hence, the required radius is 16 cm.

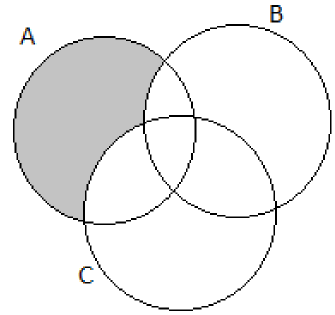
45. (a) Prove that A\(B ∪ C) = (A\B) ∩ (A\C) by using Venn diagrams.

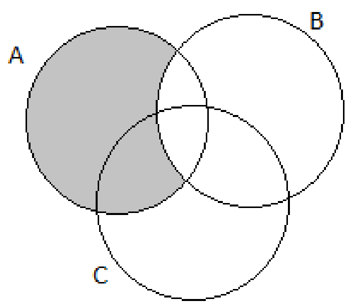
Solution:

B ∪ C

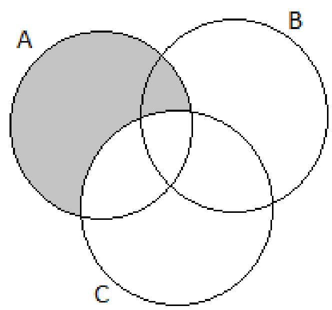


A\ (B ∪ C)

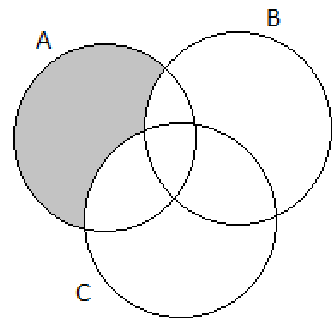




A\B



A\C



(A\B) ∩ (A\C)

Hence, proved that: A\(B ∪ C) = (A\B) ∩ (A\C)

OR

(b) If the remainder on division $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the value of k and hence find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

Solution:

Given,

$p(x) = x^3 + 2x^2 + kx + 3$

$g(x) = x - 3$

$r(x) = 21$

Thus, $p(3) = 21$

$(3)^3 + 2(3)^2 + k(3) + 3 = 21$

$27 + 18 + 3k + 3 = 21$

$48 + 3k = 21$

$3k = 21 - 48$

$k = -27/3$

$k = -9$

Therefore, the cubic polynomial is $f(x) = x^3 + 2x^2 + (-9)x - 18 = x^3 + 2x^2 - 9x - 18$

$$
\begin{array}{r|llll}
 & x^2 & +5x & +6 & \\
\hline
x-3 & x^3 & +2x^2 & -9x & -18 \\
- & x^3 & -3x^2 & & \\
\hline
 & & 5x^2 & -9x & -18 \\
 & - & 5x^2 & -15x & \\
\hline
 & & & 6x & -18 \\
 & & - & 6x & -18 \\
\hline
 & & & & 0
\end{array}
$$

Consider,

$x^2 + 5x + 6 = 0$

$x^2 + 2x + 3x + 6 = 0$

$x(x + 2) + 3(x + 2) = 0$

$(x + 2)(x + 3) = 0$

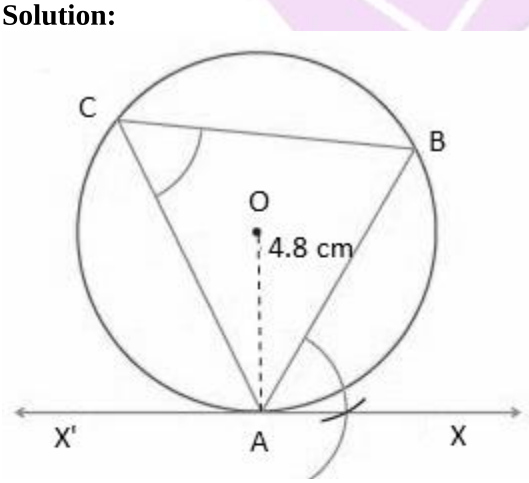
$x = -2, x = -3$

Hence, the required zeroes of the cubic polynomial are 3, -2, -3.

SECTION - IV

46. (a) Draw a circle of radius 4.8 cm. Take a point on the circle, draw the tangent at that point using the tangent-chord theorem.

Solution:

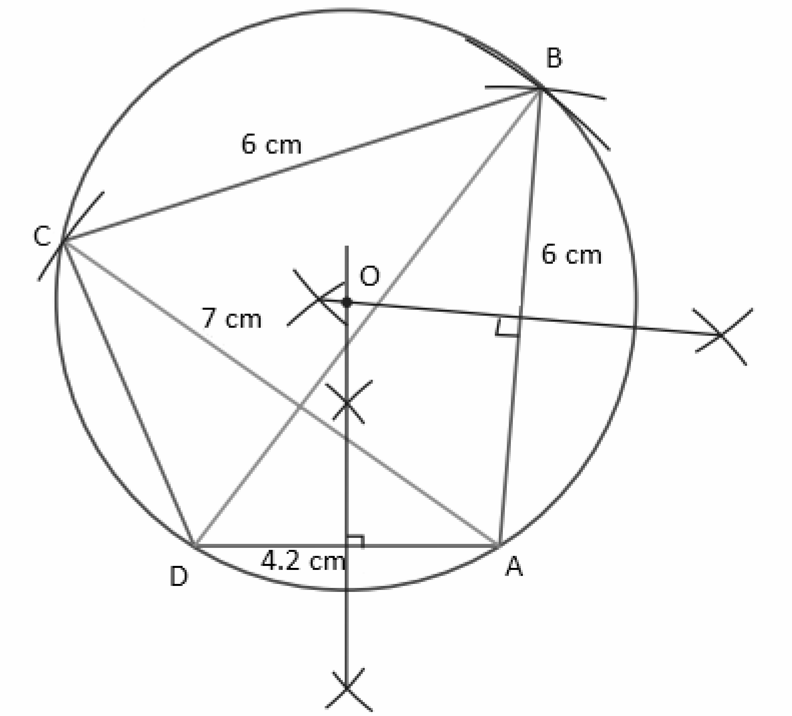


Hence, XAX' is the required tangent at A on the circle of radius 4.8 cm.

OR

(b) Construct a cyclic quadrilateral ABCD in which AB = 6 cm, AC = 7 cm, BC = 6 cm and AD = 4.2 cm.

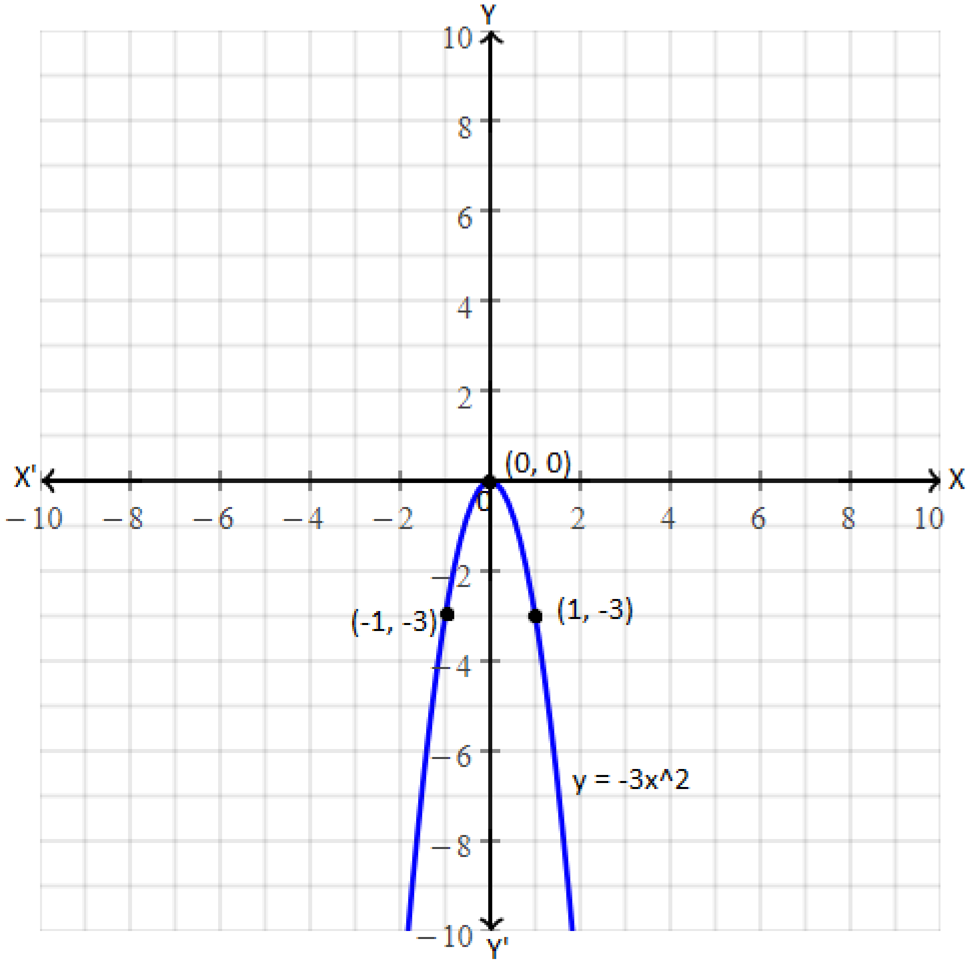
Solution:



47. (a) Draw the graph of $y = -3x^2$.

Solution:
Given,
$y = -3x^2$
Table for the corresponding x and y values:

x	-1	0	1
y	-3	0	-3

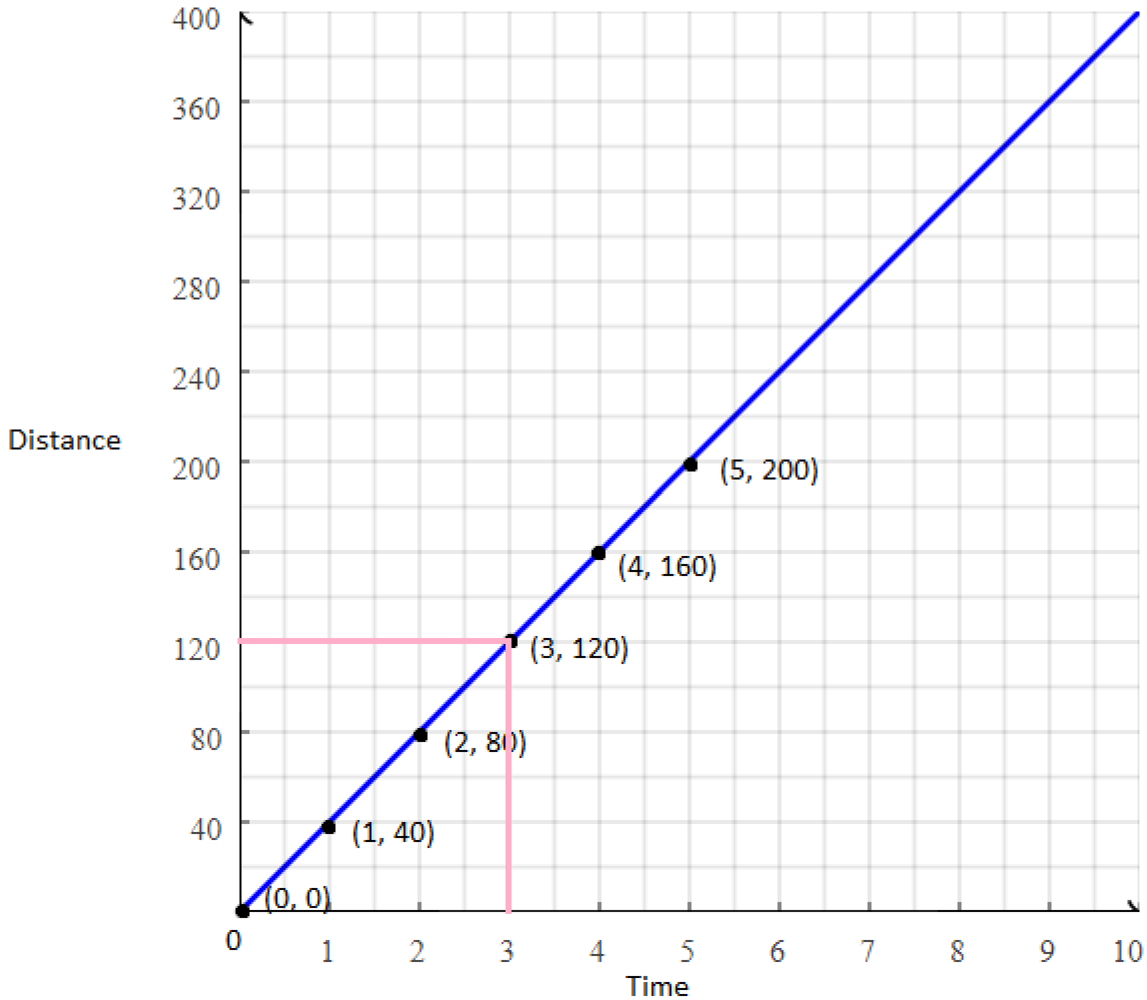


OR

(b) A bus travels at a speed of 40 km/hr. Write the distance - time formula and draw the graph of it. Hence, find the distance travelled in 3 hours.

Solution:
Given,
Speed = 40 km/hr
Let x be the time taken.
Distance = Speed × time
y = 40x

x	0	1	2	3	4	5
y	0	40	80	120	160	200



From the graph,
Distance travelled in 3 hours is 120 km.