Telangana Board SSC Class 10 Maths 2016 Question Paper 1 with Solutions

PART A
SECTION - I

1. Find the value of $\log_5 125$.

Solution:
Let $\log_5 125 = x$
⇒ 5x = 125
⇒ 5x = 53
⇒ x = 3
Therefore, $\log_5 125 = 3$

2. If A = {1, 1/4, 1/9, 1/16, 1/25}, then write A in set-builder form.

Solution:
A = {1, 1/4, 1/9, 1/16, 1/25}
= $\{1/1^2, 1/2^2, 1/3^2, 1/4^2, 1/5^2\}$
The set-builder form: $A = \{1/x^2 : x \leq 5, x \in N\}$

3. Write an example for a quadratic polynomial that has no zeroes.

Solution:
$x^2 + x + 11$ is one of the polynomials which do not have zeroes.

4. If $b^2 - 4ac > 0$ in $ax^2 + bx + c = 0$, then what can you say about roots of the equation? ($a \neq 0$)

Solution:
Given,
$ax^2 + bx + c = 0$
And
$b^2 - 4ac > 0$
Hence, the roots of the equation are real and unequal.

5. Find the sum of the first 200 natural numbers.

Solution:
First 200 natural numbers: 1, 2, 3, 4,...,200
n = 200
We know that,
Sum of first n natural numbers = n(n + 1)/2
Sum of the first 200 natural numbers = 200(200 + 1)/2
= 100 × 201
= 20100

6. For what values of m, the pair of equations 3x + my = 10 and 9x + 12y = 30 have a unique solution.

Solution:
Given.
3x + my = 10
9x + 12y = 30
Comparing with the standard form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,
$a_1 = 3, b_1 = m, c_1 = -10$
$a_2 = 9, b_2 = 12, c_2 = -30$
Condition for unique solution
$a_1/a_2 \neq b_1/b_2$
$3/9 \neq m/12$
$\Rightarrow m \neq 12/3$
$\Rightarrow m \neq 4$
Hence, m takes all the real values except 4.

7. Find the midpoint of the line segment joining the points (-5, 5) and (5, -5).

Solution:
Let the given points be:
$(x_1, y_1) = (-5, 5)$
$(x_2, y_2) = (5, -5)$
Midpoint = $[(x_1 + x_2)/2, (y_1 + y_2)/2]$
= [(-5 + 5)/2, (5 - 5)/2]
= (0/2, 0/2)
= (0, 0)

SECTION - II

8. If $x^2 + y^2 = 7xy$, then show that 2 log(x + y) = log x + log y + 2 log 3.

Solution:
Given,
$x^2 + y^2 = 7xy$
Adding 2xy on both the sides,
$x^2 + y^2 + 2xy = 7xy + 2xy$
$(x + y)^2 = 9xy$
$(x + y)^2 = (3)^2xy$
Taking log on both sides,
$\log (x + y)^2 = \log (3)^2xy$
$2 \log (x + y) = \log 3^2 + \log x + \log y$
$2 \log (x + y) = 2 \log 3 + \log x + \log y$

9. Length of a rectangle is 5 units more than its breadth. Express its perimeter in polynomial form.

Solution:
Let x be the breadth of a rectangle.
Length = (x + 5) units
Perimeter of rectangle = 2 (Length + Breadth)

= 2(x + 5 + x)
= 2(2x + 5)
= 4x + 10

10. Measures of sides of a triangle are in Arithmetic Progression. Its perimeter is 30 cm and the difference between the longest and shortest side is 4 cm, then find the measures of the sides.

Solution:
Let a - d, a, a + d be the measures of three sides of a triangle.
According to the given,
Perimeter = 30 cm
a - d + a + a + d = 30
3a = 30
a = 30/3
a = 10
Also,
a + d - (a - d) = 4
2d = 4
d = 4/2
d = 2
Thus, a - d = 10 - 2 = 8
a + d = 10 + 2 = 12
Hence, the measures of the triangle are 8 cm, 10 am and 12 cm.

11. Show that the points A(-3, 3), B(0, 0), C(3, -3) are collinear.

Solution:
If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then the area of the triangle formed by these vertices is 0.
That means $\frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$
Given,
A(-3, 3), B(0, 0), C(3, -3)
Area of triangle ABC = ½ [-3(0 + 3) + 0(-3 - 3) + 3(3 - 0)]
= ½ [-9 + 0 + 9]
= ½ (0)
= 0
Hence, the given points are collinear.

12. Solve the following pair of linear equations by substitution method.
2x - 3y = 19
3x - 2y = 21

Solution:
Given,
2x - 3y = 19….(i)
3x - 2y = 21….(ii)
From (i),
2x - 3y = 19
2x = 3y + 19
x = (3y + 19)/2….(iii)
Substituting (iii) in (ii),

$3[(3y + 19)/2] - 2y = 21$
$9y + 57 - 4y = 42$
$5y = 42 - 57$
$5y = -15$
$y = -15/5$
$y = -3$
Substituting $y = -3$ in (iii),
$x = [3(-3) + 19)]/2$
$= (-9 + 19)/2$
$= 10/2$
$= 5$
Hence, the solution of the given pair of linear equations is $x = 5$ and $y = -3$.

13. If $9x^2 + kx + 1 = 0$ has equal roots, find the value of k.

Solution:
Given,
$9x^2 + kx + 1 = 0$
Comparing with the standard form $ax^2 + bx + c = 0$,
$a = 9, b = k, c = 1$
Condition for equal roots:
$b^2 - 4ac = 0$
$k^2 - 4(9)(1) = 0$
$k^2 - 36 = 0$
$k^2 = 36$
$k = \sqrt{36}$
$k = \pm 6$

SECTION - III

14. Use Euclid's division lemma to show that the cube of any positive integer is of the form 7m or 7m + 1 or 7m + 6.

Solution:
Let a be any positive integer and b = 7.
By Euclid's division lemma,
$a = bq + r, 0 \le r < b$
$a = 7q + r; r = 0, 1, 2, 3, 4, 5, 6$
When r = 0,
$a = 7q$
$a^3 = (7q)^3$
$a^3 = 343q^3$
$a^3 = 7(49q^3)$
$a^3 = 7m$, where $m = 49q^3$
Also,in $(7q + r)^3$, consider r^3 and divide by 7. The remainder will give the result in each case.
When r = 1,
$1^3 = 1$ and $a^3 = 7m + 1$
When r = 2,
$2^3 = 8$, divided by 7, the remainder is 1. Therefore, $a^3 = 7m + 1$
When r = 3,

$3^3 = 27$ divided by 7, the remainder is 6. Therefore, $a^3 = 7m + 6$
When r = 4,
$4^3 = 64$ divided by 7, the remainder is 1. Therefore, $a^3 = 7m + 1$
When r = 5,
$5^3 = 125$ divided by 7, the remainder is 6. Therefore, $a^3 = 7m + 6$
When r = 6,
$6^3 = 216$ divided by 7, the remainder is 6. Therefore, $a^3 = 7m + 6$
Hence, the cube of any positive integer is of the form 7m or 7m + 1 or 7m + 6.

OR

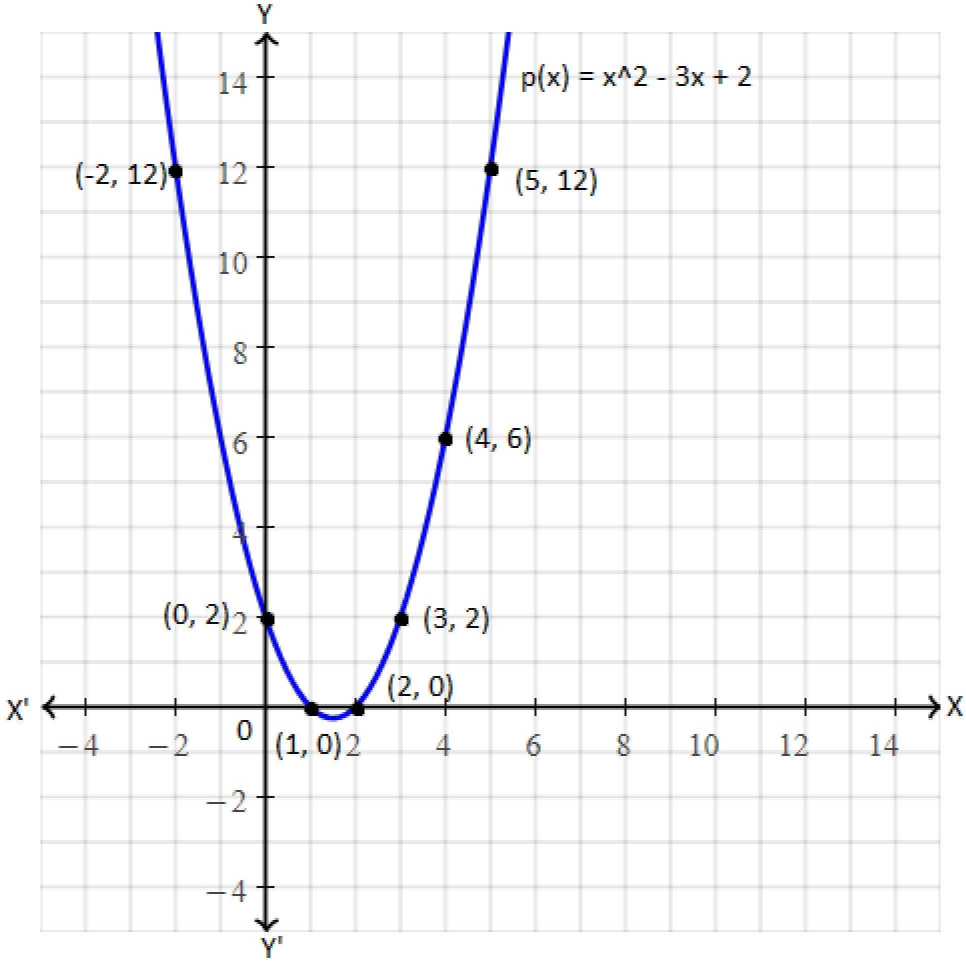
Prove that $\sqrt{2} - 3\sqrt{5}$ is an irrational number.

Solution:
Let $\sqrt{2} - 3\sqrt{5}$ be a rational number.
$\sqrt{2} - 3\sqrt{5} = a$, where a is an integer.
Squaring on both sides,
$(\sqrt{2} - 3\sqrt{5})^2 = a^2$
$(\sqrt{2})^2 + (3\sqrt{5})^2 - 2(\sqrt{2})(3\sqrt{5}) = a^2$
$2 + 45 - 6\sqrt{10} = a^2$
$47 - 6\sqrt{10} = a^2$
$-6\sqrt{10} = a^2 - 47$
$\sqrt{10} = (47 - a^2)/6$
$(47 - a^2)/6$ is a rational number since a is an integer.
Therefore, $\sqrt{10}$ is also an integer.
We know that integers are not rational numbers.
Thus, our assumption that $\sqrt{2} - 3\sqrt{5}$ is a rational number is wrong.
Hence, $\sqrt{2} - 3\sqrt{5}$ is an irrational number.

15. Draw the graph for the polynomial $p(x) = x^2 - 3x + 2$ and find the zeroes from the graph.

Solution:
Given,
$p(x) = x^2 - 3x + 2$
Let $y = x^2 - 3x + 2$

x	-2	0	1	2	3	4	5
Y = p(x)	12	2	0	0	2	6	12



OR

Draw the graph for the following pair of linear equations in two variables and find their solution from the graph.
3x - 2y = 2 and 2x + y = 6

Solution:
Given,
3x - 2y = 2
2x + y = 6

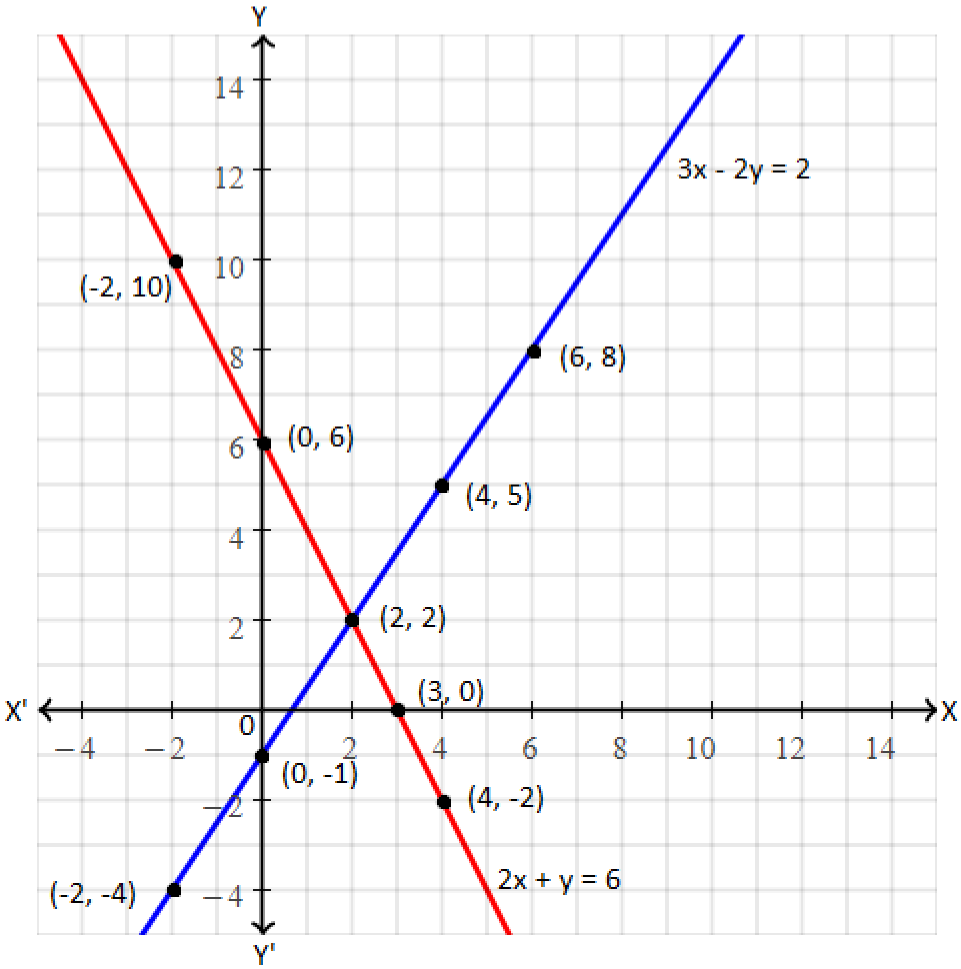
Consider the first equation:
3x - 2y = 2
2y = 3x - 2
y = (3/2)x - 1

x	-2	0	2	4	6
y	-4	-1	2	5	8

Now, consider another equation:
2x + y = 6

$y = -2x + 6$

x	-2	0	2	3	4
y	10	6	2	0	-2



The lines representing the given pair of equations intersecting each other at (2, 2).
Hence, the solution of the given pair of linear equations is x = 2 and y = 2.

16. Sum of the squares of two consecutive positive even integers is 100, find those numbers by using quadratic equations.

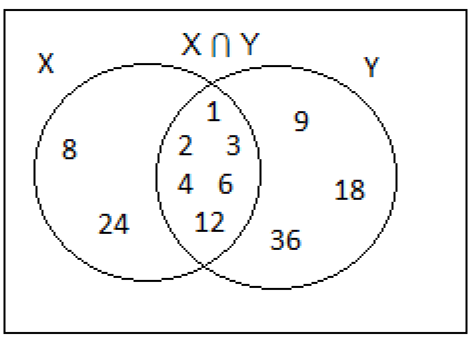
Solution:
Let x and x + 2 be the two consecutive even integers.
According to the given,
$x^2 + (x + 2)^2 = 100$
$x^2 + x^2 + 4 + 4x - 100 = 0$
$2x^2 + 4x - 96 = 0$
$2(x^2 + 2x - 48) = 0$
$x^2 + 2x - 48 = 0$

$x^2 + 8x - 6x - 48 = 0$
$x(x + 8) - 6(x + 8) = 0$
$(x + 8)(x - 6) = 0$
$x = -8, x = 6$
The value of x cannot be negative.
Thus, $x = 6$
$x + 2 = 6 + 2 = 8$
Hence, the required numbers are 6 and 8.

OR

X is a set of factors of 24 and Y is a set of factors of 36, then find sets X ∪ Y and X ∩ Y by using Venn diagram and comment on the answer.

Solution:
X = factors of 24
= {1, 2, 3, 4, 6, 8, 12, 24}
Y = factors of 36
= {1, 2, 3, 4, 6, 9, 12, 18, 36}
X ∪ Y = {1, 2, 3, 4, 6, 8, 12, 24} ∪ {1, 2, 3, 4, 6, 9, 12, 18, 36} = {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36}
X ∩ Y = {1, 2, 3, 4, 6, 8, 12, 24} ∩ {1, 2, 3, 4, 6, 9, 12, 18, 36} = {1, 2, 3, 4, 6, 12}



17. Find the sum of all the three digit numbers, which are divisible by 4.

Solution:
Three digit numbers which are divisible by 4 are:
100, 104, 108,...., 996
This is an AP with a = 100 and d = 4
nth term of AP
$a_n = a + (n - 1)d$
$996 = 100 + (n - 1)4$
$(n - 1)4 = 996 - 100 = 896$
$n - 1 = 896/4$
$n - 1 = 224$
$n = 225$
Sum of first n terms
$S_n = n/2(a + an)$
$S_{225} = (225/2) \times (100 + 996)$
$= (225/2) \times 1096$
$= 123300$

Hence, the required sum is 123300.

OR

Find the coordinates of the points of trisection of the line segment joining the points (-3, 3) and (3, -3).

Solution:
Let the given points be:
A(-3, 3) and B(3, -3)
Let C and D be the points of trisection of line joining the points A and B.

C divided AB in the ratio 1 : 2.
m : n = 1 : 2
Using section formula,
C = $[(mx_2 + nx_1)/ (m + n), (my_2 + ny_1)/ (m + n)]$
= [(3 - 6)/ (1 + 2), (-3 + 6)/(1 + 2)]
= (-3/3, 3/3)
= (-1, 1)
D is the midpoint of BC.
D = [(-1 + 3)/2, (1 - 3)/2]
= (2/2, -2/2)
= (1, -1)
Hence, the required points are (-1, 1) and (1, -1).