Telangana Board SSC Class 10 Maths 2017 Question Paper 1 with Solutions

PART A
SECTION - I

1. Write the nature of roots of the quadratic equation $2x^2 - 5x + 6 = 0$.

Solution:
Given,
$2x^2 - 5x + 6 = 0$
Comparing with the standard form $ax^2 + bx + c = 0$,
$a = 2, b = -5, c = 6$
Discriminant $= b^2 - 4ac$
$= (-5)^2 - 4(2)(6)$
$= 25 - 48$
$= -23 < 0$
Hence, the roots of the given quadratic equation are not real, i.e. the roots are imaginary.

2. Find the value of $\log_{\sqrt{2}} 256$.

Solution:
Let $\log_{\sqrt{2}} 256 = x$
$\Rightarrow (\sqrt{2})^x = 256$
$\Rightarrow (\sqrt{2})^x = 2^8$
$\Rightarrow (\sqrt{2})^x = (\sqrt{2})^{16}$
$\Rightarrow x = 16$
Therefore, $\log_{\sqrt{2}} 256 = 16$

3. In a GP, $t_n = (-1)^n$ 2017. Find the common ratio.

Solution:
Given,
nth term of GP is $t_n = (-1)^n$ 2017
$t_1 = (-1)^1$ 2017 = -2017
$t_2 = (-1)^2$ 2017 = 2017
Common ratio = t2/t1
= -2017/2017
= -1

4. Srikar says that the order of the polynomial $(x^2 - 5)(x^3 + 1)$ is 6. Do you agree with him? How?

Solution:
Given polynomial is
$p(x) = (x^2 - 5)(x^3 + 1)$
$= x^2(x^3 + 1) - 5(x^3 + 1)$

$= x^5 + x^2 - 5x^3 - 5$
$= x^5 - 5x^3 + x^2 - 5$
The highest degree of the polynomial is 5.
Hence, the order of the polynomial is not 6 and not agreeing with Srikar.

5. A(0, 3), B(k, 0) and AB = 5. Find the positive value of k.

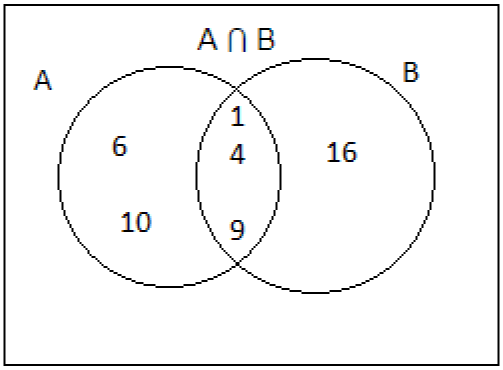
Solution:
Given,
A(0, 3), B(k, 0) and AB = 5
Using the distance formula,
$AB = \sqrt{[(k - 0)^2 + (0 - 3)^2]}$
$5 = \sqrt{(k^2 + 3^2)}$
Squaring on both sides,
$25 = k^2 + 9$
$\Rightarrow k^2 = 25 - 9$
$\Rightarrow k^2 = 16$
$\Rightarrow k = \sqrt{16}$
$\Rightarrow k = \pm 4$
Therefore, the positive value of k = 4.

6. Show that the pair of linear equations 7x + y = 10 and x + 7y = 10 are consistent.

Solution:
Given,
7x + y = 10
x + 7y = 10
Comparing with the standard form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,
$a_1 = 7, b_1 = 1, c_1 = -10$
$a_2 = 1, b_2 = 7, c_2 = -10$
$a_1/a_2 = 7/1$
$b_1/b_2 = 1/7$
$a_1/a_2 \neq b_1/b_2$
Hence, the given pair of linear equations are consistent and have a unique solution.

7. Represent A ∩ B through Venn diagram, where A = {1, 4, 6, 9, 10} and B = {perfect squares less than 25}.

Solution:
Given,
A = {1, 4, 6, 9, 10}
B = {perfect squares less than 25} = {1, 4, 9, 16}
A ∩ B = {1, 4, 6, 9, 10} ∩ {1, 4, 9, 16} = {1, 4, 9}



SECTION - II

8. Write any two three digit numbers. Find their LCM and GCD by prime factorisation method.

Solution:
Let 126 and 340 are the two three digit numbers.
Prime factorisation of 126:
$126 = 2 \times 3 \times 3 \times 7$
Prime factorisation of 340:
$340 = 2 \times 2 \times 5 \times 17$
$LCM = 2 \times 2 \times 3 \times 3 \times 7 \times 5 \times 17 = 21420$
GCD = 2

9. Find the sum of first 10 terms of an AP
3, 15, 27, 39,....

Solution:
Given AP:
3, 15, 27, 39,...
First term = a = 3
Common difference = d = 12
Sum of the first n terms
$S_n = n/2[2a + (n - 1)d$
$S_{10} = (10/2) [2(3) + (10 - 1)12]$
$= 5(6 + 108)$
$= 5 \times 114$
$= 570$
Hence, the sum of the first 10 terms of the given AP is 570.

10. Which of √2 and 2 is a zero of the polynomial $p(x) = x^3 - 2x$? Why?

Solution:
Given,
$p(x) = x^3 - 2x$
$p(\sqrt{2}) = (\sqrt{2})^3 - 2(\sqrt{2}) = 2\sqrt{2} - 2\sqrt{2} = 0$
$p(2) = (2)^3 - 2(2) = 8 - 4 = 4 \neq 0$
Therefore, √2 is the zero of the given polynomial.

11. The sum of a number and its reciprocal is 10/3. Find the number.

Solution:
Let x be any number.
According to the given,
$x + 1/x = 10/3$
$(x^2 + 1)/x = 10/3$
$3(x^2 + 1) = 10x$
$3x^2 + 3 - 10x = 0$
$3x^2 - 9x - x + 3 = 0$
$3x(x - 3) - 1(x - 3) = 0$
$(3x - 1)(x - 3) = 0$
$3x - 1 = 0, x - 3 = 0$
$x = 1/3, x = 3$
Hence, the required number is 3 or 1/3.

12. Two vertices of a triangle are (3, 2), (-2, 1) and its centroid is (5/3, -1/3). Find the third vertex of the triangle.

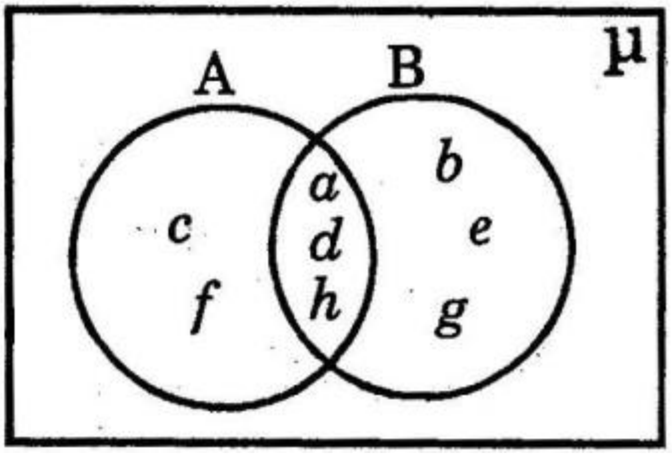
Solution:
Given,
Two vertices of a triangle are (3, 2), (-2, 1).
Centroid = (5/3, -⅓)
Let (x, y) be the third vertex.
Using centroid formula,
$(5/3, -1/3) = [(x_1 + x_2 + x_3)/2, (y_1 + y_2 + y_3)/3]$
$(5/3, -1/3) = [(3 - 2 + x)/3, (2 + 1 + y)/3]$
$(5/3, -1/3) = [(1 + x)/3, (3 + y)/3]$
$\Rightarrow (1 + x)/3 = 5/3, (3 + y)3 = -1/3$
$\Rightarrow 1 + x = 5, 3 + y = -1$
$\Rightarrow x = 4, y = -4$
Hence, the third vertex of the triangle is (4, -4).

13. Find the angle made by the line joining (5, 3) and (-1, -3) with the positive direction of X-axis.

Solution:
Let the given points be:
$(x_1, y_1) = (5, 3)$
$(x_2, y_2) = (-1, -3)$
Slope = $m = (y_2 - y_1)/(x_2 - x_1)$
$= (-3 - 3)/(-1 - 5)$
$= -6/-6$
$= 1$
Let θ be the angle made by the line with x-axis.
We know that,
$m = \tan \theta$
$1 = \tan \theta$
$\tan 45° = \tan \theta$
$\theta = 45°$
Hence, the required angle is 45°.

SECTION - III

14. From the following Venn diagram, write the elements of the sets of A and B. And verify $n(A \cup B) + n(A \cap B) = n(A) + n(B)$.



Solution:

$A = \{a, c, d, f, h\}$

$B = \{a, b, d, e, g, h\}$

$A \cup B = \{a, c, d, f, h\} \cup \{a, b, d, e, g, h\} = \{a, b, c, d, e, f, g, h\}$

$A \cap B = \{a, c, d, f, h\} \cap \{a, b, d, e, g, h\} = \{a, d, h\}$

$n(A) = 5$

$n(B) = 6$

$n(A \cup B\} = 8$

$n(A \cap B) = 3$

$n(A \cup B) + n(A \cap B) = 8 + 3 = 11$

$n(A) + n(B) = 5 + 6 = 11$

Therefore, $n(A \cup B) + n(A \cap B) = n(A) + n(B)$

OR

Use Euclid's division lemma to show that the square of any positive integer is of the form 5n or 5n + 1 or 5n + 4, where n is a whole number.

Solution:

Let a be any positive integer and b = 5.

By Euclid division lemma,

$a = bq + r$, where $0 \le r < b$

$\Rightarrow a = 5q + r; r = 0, 1, 2, 3, 4$

When r = 0,

$a = 5q$

$a^2 = (5q)^2$

$a^2 = 25q^2$

$a^2 = 5(5q^2)$

$a^2 = 5n$

where $n = 5q^2$

When r = 1,
$a = 5q + 1$
$a^2 = (5q + 1)^2$
$a^2 = 25q^2 + 10q + 1$
$a^2 = 5(5q^2 + 2q) + 1$
$a^2 = 5n + 1$
where $n = 5q^2 + 2q$
When r = 2,
$a = 5q + 2$
$a^2 = (5q + 2)^2$
$a^2 = 25q^2 + 20q + 4$
$a^2 = 5(5q^2 + 4q) + 4$
$a^2 = 5n + 4$
where $n = 5q^2 + 4q$
When r = 3,
$a = 5q + 3$
$a^2 = (5q + 3)^2$
$a^2 = 25q^2 + 30q + 5 + 4$
$a^2 = 5(5q^2 + 6q + 1) + 4$
$a^2 = 5n + 4$
where $n = 5q^2 + 6q + 1$
When r = 4,
$a = 5q + 4$
$a^2 = (5q + 4)^2$
$a^2 = 25q^2 + 40q + 15 + 1$
$a^2 = 5(5q^2 + 8q + 3) + 1$
$a^2 = 5n + 1$
where $n = 5q^2 + 8q + 3$
Hence, the square of any positive integer is of the form 5n or 5n + 1 or 5n + 4, where n is a whole number.

15. Find the sum of all three digit natural numbers, which are divisible by 3 and not divisible by 6.

Solution:
Three digit natural numbers that are divisible by 3:
102, 105, 108, 111, 114,...,993, 996, 999
Three digit natural numbers that are divisible by 3 but not by 6:
105, 111, 117,..., 993, 999
This is an AP with a = 105, d = 6 and $a_n = 999$
nth term of AP
$a_n = a + (n - 1)d$
$999 = 105 + (n - 1)6$
$\Rightarrow (n - 1)6 = 999 - 105$
$\Rightarrow n - 1 = 894/6$
$\Rightarrow n - 1 = 149$
$\Rightarrow n = 149 + 1$
$\Rightarrow n = 150$
Sum of the first n terms
$S_n = n/2(a + an)$
$S_{150} = (150/2) \times (105 + 999)$
$= 75 \times 1104$

= 82800
Hence, the required sum is 82800.

OR

Divide $3x^4 - 5x^3 + 4x^2 + 3x - 5$ by $x^2 - 3$ and verify the division algorithm.

Solution:
Let $p(x) = 3x^4 - 5x^3 + 4x^2 + 3x - 5$
$g(x) = x^2 - 3$

$$
\begin{array}{r|rrrrr}
 & 3x^2 & -5x & +13 & & \\
\hline
x^2 - 3 & 3x^4 & -5x^3 & +4x^2 & +3x & -5 \\
 & 3x^4 & & -9x^2 & & \\
\hline
 & & -5x^3 & +13x^2 & +3x & -5 \\
 & & -5x^3 & & +15x & \\
\hline
 & & & 13x^2 & -12x & -5 \\
 & & & 13x^2 & & -39 \\
\hline
 & & & & -12x & +34
\end{array}
$$

Quotient = $q(x) = 3x^2 - 5x + 13$
Remainder = $r(x) = -12x + 34$
By division algorithm,
$p(x) = [g(x)\ q(x)] + r(x)$
$g(x)\ q(x) + r(x) = (x^2 - 3)(3x^2 - 5x + 13) -12x + 34$
$= 3x^4 - 5x^3 + 13x^2 - 9x^2 + 15x - 39 - 12x + 34$
$= 3x^4 - 5x^3 + 4x^2 + 3x - 5$
$= p(x)$
Hence verified.

16. The perimeter of a right-angles triangle is 60 cm and its hypotenuse is 25 cm. Find the remaining two sides.

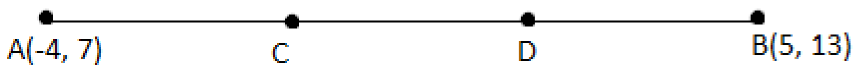
Solution:
Let ABC be the right triangle in which B is the right angle.
Given,
Hypotenuse = AC = 25 cm
Perimeter = 60 cm
AB + BC + AC = 60
AB + BC + 25 = 60
AB + BC = 35 cm
By Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$(25)^2 = AB^2 + (35 - AB)^2$
$625 = AB^2 + 1225 + AB^2 - 70AB$
$2AB^2 - 70AB + 1225 - 625 = 0$
$2AB^2 - 70AB + 600 = 0$
$AB^2 - 35AB + 300 = 0$
Let AB = x
$x^2 - 35x + 300 = 0$
$x^2 - 20x - 15x + 300 = 0$
$x(x - 20) - 15(x - 20) = 0$

$(x - 15)(x - 20) = 0$
$x = 15, x = 20$
Therefore, the remaining sides of the triangle are 15 cm and 20 cm.

OR

The points C and D are on the line segment joining A(-4, 7) and B(5, 13) such that AC = CD = DB. Then find coordinates of points C and D.

Solution:
Given,
The points C and D are on the line segment joining A(-4, 7) and B(5, 13) such that AC = CD = DB.

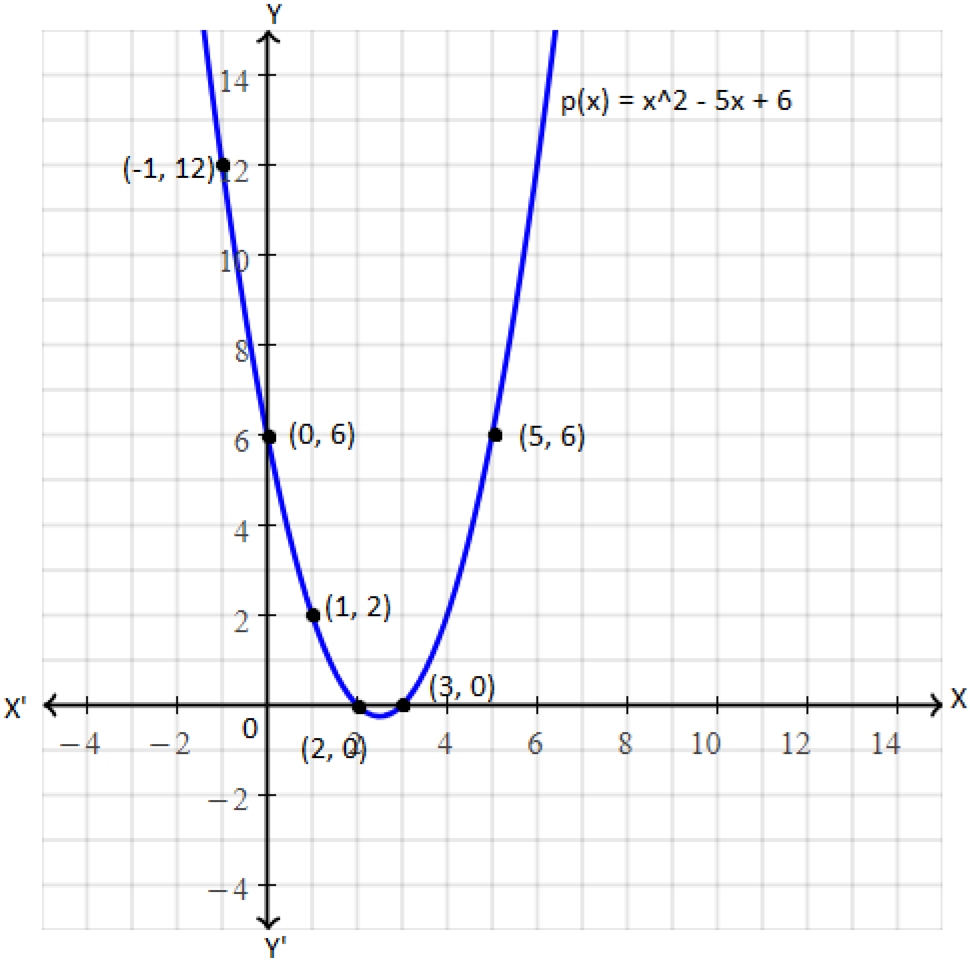


C divided AB in the ratio 1 : 2.
m : n = 1 : 2
Using section formula,
$C = [(mx_2 + nx_1)/ (m + n), (my_2 + ny_1)/ (m + n)]$
$= [(5 - 8)/ (1 + 2), (13 + 14)/(1 + 2)]$
$= (-3/3, 27/3)$
$= (-1, 9)$
D is the midpoint of BC.
$D = [(-1 + 5)/2, (9 + 13)/2]$
$= (4/2, 22/2)$
$= (2, 11)$
Therefore, C = (-1, 9) and D = (2, 11)

17. Draw the graph for the polynomial $p(x) = x^2 - 5x + 6$ and find the zeroes from the graph.

Solution:
Given,
$p(x) = x^2 - 5x + 6$

x	-1	0	1	2	3	5
p(x)	12	6	2	0	0	6



Parabola intersecting the x-axis at (2, 0) and (3, 0).
Hence, the zeroes of the given polynomial are 2 and 3.

OR

Draw the graph of $2x + y = 6$ and $2x - y + 2 = 0$ and find the solution from the graph.

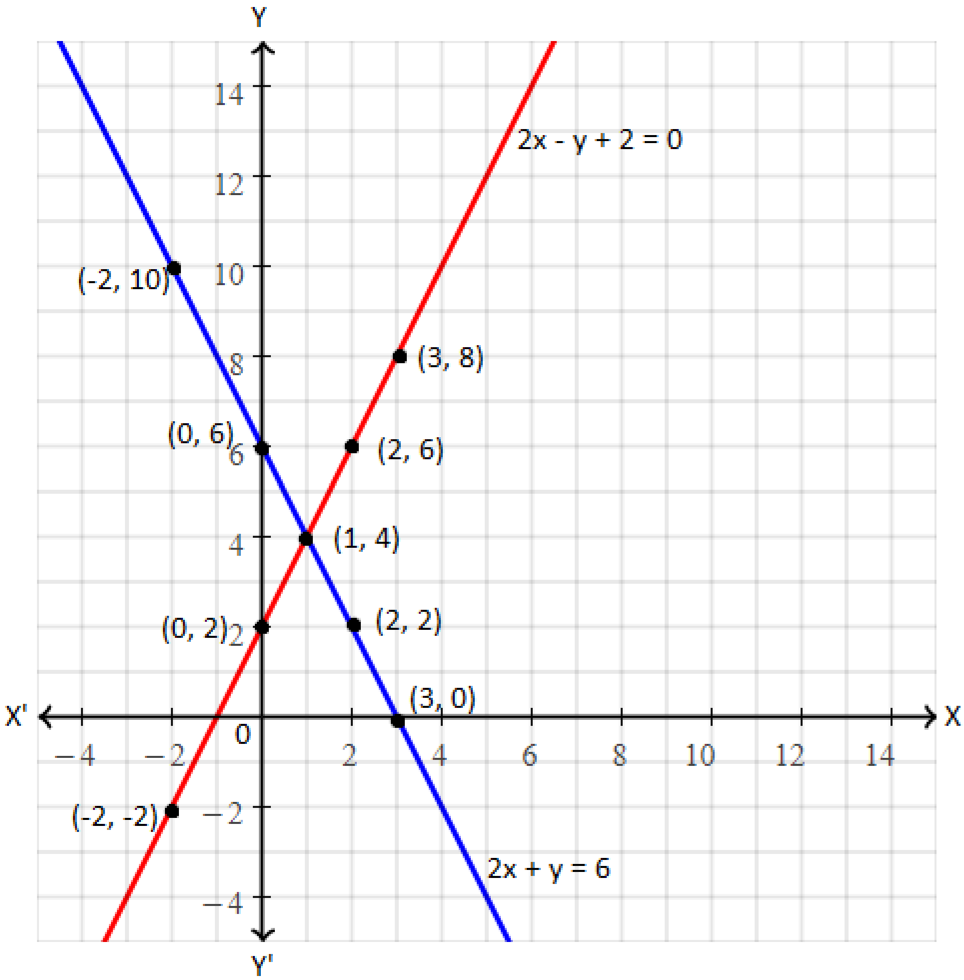
Solution:
Given,
$2x + y = 6$
$2x - y + 2 = 0$
Consider the first equation:
$2x + y = 6$
$y = -2x + 6$

x	-2	0	1	2	3
y	10	6	4	2	0

Now, consider another equation:

$2x - y + 2 = 0$
$y = 2x + 2$

Graph:



The lines representing the given pair of equations intersecting each other at (1, 4).
Hence, the solution is $x = 1$ and $y = 4$.