

Telangana Board SSC Class 10 Maths 2017 Question Paper 2 with Solutions

PART A SECTION - I

1. If $\sin A = 1/\sqrt{2}$ and $\cot B = 1$, prove that $\sin (A + B) = 1$, where A and B both are acute angles.

Solution:

Given,

A and B are acute angles.

$$\sin A = 1/\sqrt{2}$$

$$\sin A = \sin 45^\circ$$

$$\Rightarrow A = 45^\circ$$

$$\cot B = 1$$

$$\cot B = \cot 45^\circ$$

$$\Rightarrow B = 45^\circ$$

Now,

$$\sin (A + B) = \sin (45^\circ + 45^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

Hence proved.

2. The length of the minute hand of a clock is 3.5 cm. Find the area swept by a minute hand in 30 minutes. (use $\pi = 22/7$)

Solution:

Given,

Length of a minute hand = 3.5 cm

The minute hand forms a semicircle in 30 min.

Thus, radius = $r = 3.5$ cm

$$\text{Area} = \pi r^2/2$$

$$= (1/2) \times (22/7) \times 3.5 \times 3.5$$

$$= 19.25$$

Hence, the area swept by the minute hand of a clock is 19.25 cm².

3. Express $\cos \theta$ in terms of $\tan \theta$.

Solution:

$$\tan \theta = \sin \theta / \cos \theta$$

Squaring on both sides,

$$\tan^2 \theta = \sin^2 \theta / \cos^2 \theta$$

$$\tan^2 \theta = (1 - \cos^2 \theta) / \cos^2 \theta$$

$$\tan^2 \theta = (1 / \cos^2 \theta) - 1$$

$$\tan^2 \theta + 1 = 1 / \cos^2 \theta$$

$$\Rightarrow \cos^2 \theta = 1 / (1 + \tan^2 \theta)$$

$$\Rightarrow \cos \theta = 1/\sqrt{1 + \tan^2 \theta}$$

4. From the first 50 natural numbers, find the probability of a randomly selected number is a multiple of 3.

Solution:

Total number of outcomes = $n(S) = 50$

i.e. the first 50 natural numbers.

Let E be the event of getting a number which is multiple of 3.

$E = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48\}$

$n(E) = 16$

$P(E) = n(E)/n(S)$

$= 16/50$

$= 8/25$

Hence, the required probability is $8/25$.

5. Write the formula to find the curved surface area of a cone and explain each term in it.

Solution:

Curved surface area of a cone = $\pi r l$

Here,

r = Radius of the circular base

l = Slant height

6. "The median of observations -2, 5, 3, -1, 4, 6 is 3.5". Is it correct? Justify your answer.

Solution:

Given observations are:

-2, 5, 3, -1, 4, 6

Ascending order:

-2, -1, 3, 4, 5, 6

Number of observations = $n = 6$

Median = $\frac{1}{2} [(n/2)\text{th} + \{(n/2) + 1\}\text{th}]$ observations

$= \frac{1}{2} [3\text{rd} + 4\text{th observations}]$

$= \frac{1}{2} [3 + 4]$

$= 7/2$

$= 3.5$

Therefore, the median is 3.5.

7. If $\cos \theta = 1/\sqrt{2}$, then find the value of $4 + \cot \theta$.

Solution:

Given,

$\cos \theta = 1/\sqrt{2}$

$\cos \theta = \cos 45^\circ$

$\Rightarrow \theta = 45^\circ$

$4 + \cot \theta = 4 + \cot 45^\circ$

$= 4 + 1$

$= 5$

SECTION - II

8. The diameter of a solid sphere is 6 cm. It is melted and recast into a solid cylinder of height 4 cm. Find the radius of the cylinder.

Solution:

Given,

Diameter of sphere = 6 cm

Radius of sphere = $r = 6/2 = 3$ cm

Height of cylinder = $h = 4$ cm

Let R be the radius of the cylinder.

Given that, the sphere is melted and recast into a cylinder.

Volume of sphere = Volume of cylinder

$$\left(\frac{4}{3}\right)\pi r^3 = \pi R^2 h$$

$$\left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 3 \times 3 \times 3 = \left(\frac{22}{7}\right) \times R^2 \times 4$$

$$R^2 = 36/4$$

$$R^2 = 9$$

$$R = 3 \text{ cm}$$

Hence, the radius of the cylinder is 3 cm.

9. Write the formula of mode for grouped data and explain each term in it.

Solution:

$$\text{Mode for grouped data} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

Here,

l = Lower limit of the modal class

f_1 = Frequency of the modal class

f_0 = Frequency of the class preceding the modal class

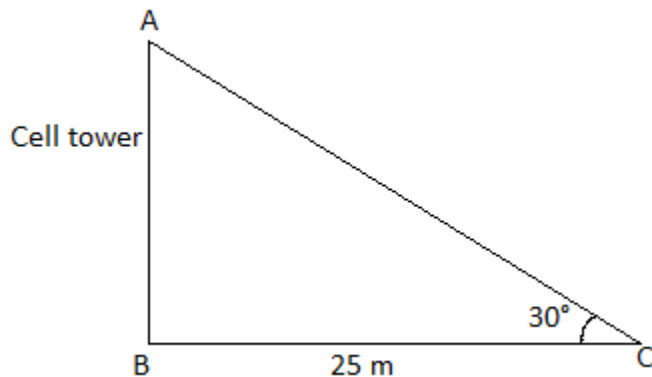
f_2 = Frequency of the class succeeding the modal class

h = class size (or class height)

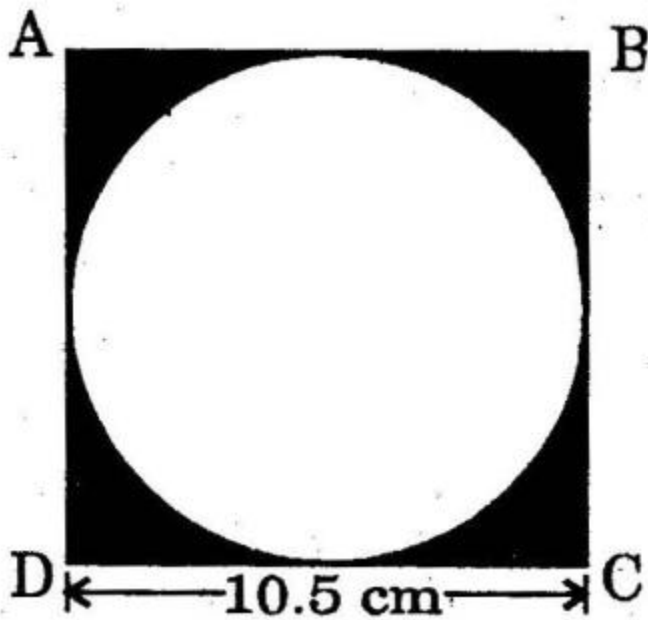
10. A person 25 mts away from a cell tower observes the top of the cell tower at an angle of elevation 30° . Draw the suitable diagram for this situation.

Solution:

Let AB be the cell tower and C be the point of observation.



11. Find the area of the shaded region in the given figure. ABCD is a square with side 10.5 cm.



Solution:

Given,

Side of square = $a = 10.5$ cm

Diameter of circle = 10.5 cm

Radius of circle = $r = 10.5/2 = 5.25$ cm

Area of the shaded region = Area of square - Area of circle

$$= a^2 - \pi r^2$$

$$= (10.5)^2 - (22/7) \times 5.25 \times 5.25$$

$$= 110.25 - 86.625$$

$$= 23.625 \text{ cm}^2$$

12. One card is selected from a well shuffled deck of 52 cards. Find the probability of getting a red card with a prime number.

Solution:

Total number of outcomes = $n(S) = 52$

Let E be the event of getting a red card with a prime number.

Prime numbers on red card (on diamonds and hearts) = 2, 3, 5, 7

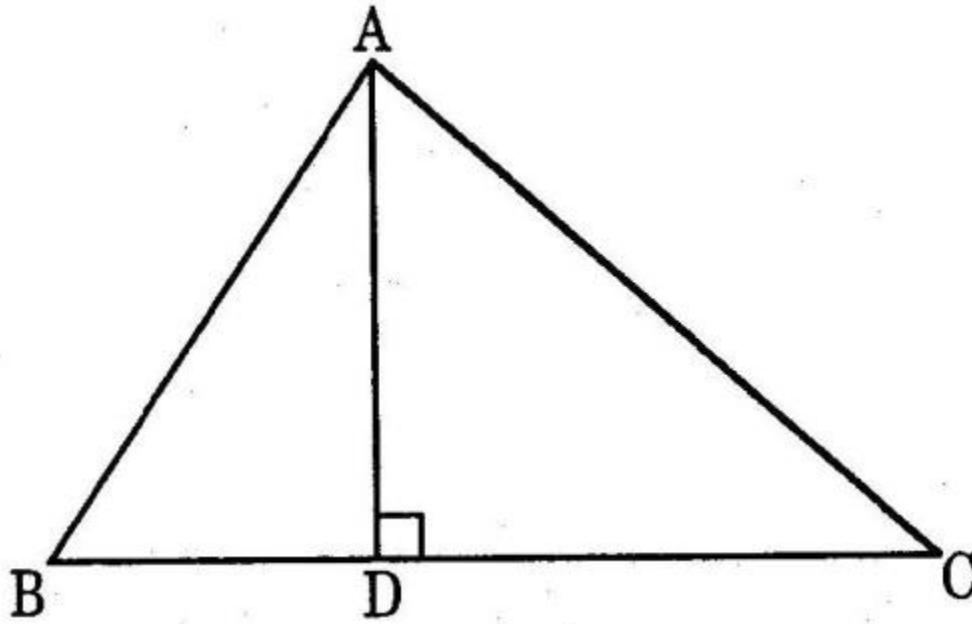
$$n(E) = 8$$

$$P(E) = n(E)/n(S)$$

$$= 8/52$$

$$= 2/13$$

13. In a ΔABC , $AD \perp BC$ and $AD^2 = BD \times CD$, prove that ΔABC is a right angled triangle.



Solution:

Given,

$$AD \perp BC$$

In $\triangle ADB$ and $\triangle ADC$,

By Pythagoras theorem,

$$AB^2 = AD^2 + BD^2 \dots (i)$$

$$AC^2 = AD^2 + DC^2 \dots (ii)$$

Adding (i) and (ii),

$$AB^2 + AC^2 = 2AD^2 + BD^2 + DC^2$$

$$= 2BD \cdot CD + BD^2 + CD^2 \text{ [given } AD^2 = BD \cdot CD \text{]}$$

$$= (BD + CD)^2$$

$$= BC^2$$

In triangle ABC, $AB^2 + AC^2 = BC^2$

Hence, the triangle ABC is a right triangle.

SECTION - III

14. The length of cuboid is 12 cm, breadth and height are equal in measurements and its volume is 432 cm^3 . The cuboid is cut into 2 cubes. Find the lateral surface area of each cube.

Solution:

Given,

$$\text{Length of cuboid} = l = 12 \text{ cm}$$

Let x be the breadth and height of cuboid.

$$\text{Volume of cuboid} = 432 \text{ cm}^3 \text{ (given)}$$

$$lbh = 432$$

$$12(x)(x) = 432$$

$$x^2 = 432/12$$

$$x^2 = 36$$

$$x = 6$$

Breadth = Height = 6 cm

Given that, the cuboid is cut into 2 cubes along its length.

Side of each cube = $12/2 = 6$ cm

Lateral surface of cube = $4(\text{side})^2$

$= 4 \times 6 \times 6$

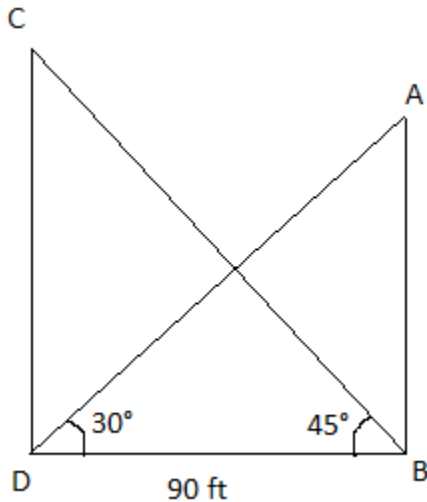
$= 144 \text{ cm}^2$

OR

Two poles are standing opposite to each other on either side of the road which is 90 feet wide. The angle of elevation from bottom of first pole to top of the second pole is 45° , the angle of elevation from bottom of second pole to top of first pole is 30° . Find the heights of poles. (use $\sqrt{3} = 1.732$)

Solution:

Let AB be the first pole and CD be the second pole.



In right triangle BDC,

$\tan 45^\circ = CD/BD$

$1 = CD/90$

$CD = 90 \text{ ft}$

In right triangle ABD,

$\tan 30^\circ = AB/BD$

$1/\sqrt{3} = AB/90$

$AB = 90/\sqrt{3}$

$AB = (90/\sqrt{3}) \times (\sqrt{3}/\sqrt{3})$

$AB = 90\sqrt{3}/3$

$AB = 30\sqrt{3}$

$AB = 30 \times 1.732$

$AB = 51.96 \text{ ft}$

Hence, the heights of the two poles are 90 ft and 51.96 ft.

15. A bag contains some square cards. A prime number between 1 and 100 has been written on each card. Find the probability of getting a card that the sum of the digits of the prime number written on it is 8.

Solution:

Number of prime numbers between 1 and 100 = 25

Total number of outcomes = 25

i.e. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Prime numbers in which the sum of their digits is 8 are 17, 53, 71.

Number of favourable outcomes = 3

$P(\text{getting a card that the sum of the digits of the prime number written on it is 8}) = \frac{3}{25}$

OR

The daily wages of 80 workers of a factory.

Daily wages (Rs)	500 - 600	600 - 700	700 - 800	800 - 900	900 - 1000
Number of workers	12	17	28	14	9

Find the mean daily wages of the workers of the factory by using an appropriate method.

Solution:

Assumed mean method:

CI	x_i	Number of workers (f_i)	$d_i = x_i - A$	$f_i d_i$
500 - 600	550	12	-200	-2400
600 - 700	650	17	-100	-1700
700 - 800	750 = A	28	0	0
800 - 900	850	14	100	1400
900 - 1000	950	9	200	1800
		$\sum f_i = 80$		$\sum f_i d_i = -900$

$$\text{Mean} = A + \left(\frac{\sum f_i d_i}{\sum f} \right)$$

$$= 750 + \left(\frac{-900}{80} \right)$$

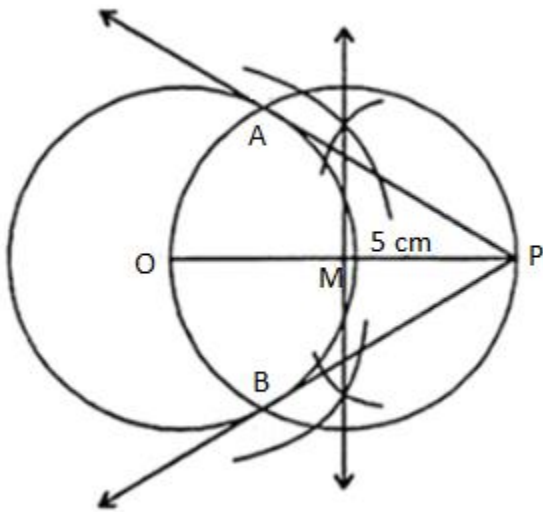
$$= 750 - \left(\frac{90}{8} \right)$$

$$= 750 - 11.25$$

$$= 735.75$$

16. Draw a circle of diameter 6 cm from a point 5 cm away from its centre. Construct the pair of tangents to the circle and measure their length.

Solution:



Hence, PA and PB are the required tangents to the circle.
 $PA = PB = 4 \text{ cm}$ (by measurement)

OR

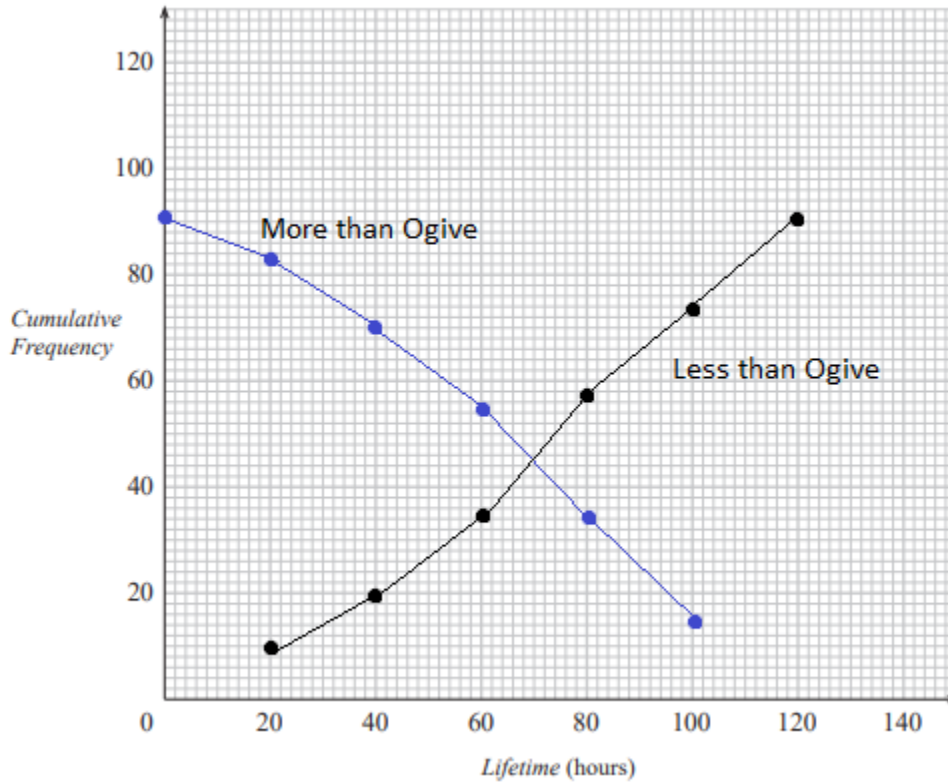
The following data gives the information on the observed life span (in hours) of 90 electrical components.

Life span (in hours)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	8	12	15	23	18	14

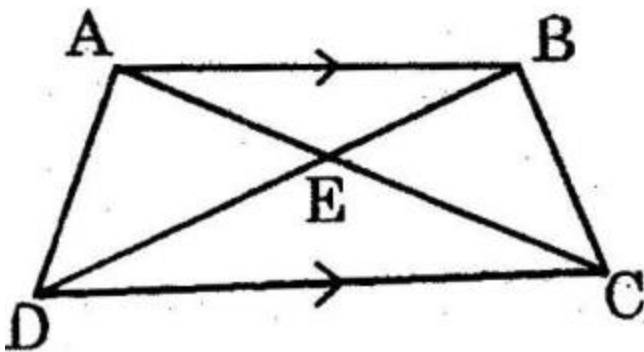
Draw both Ogives for the above data.

Solution:

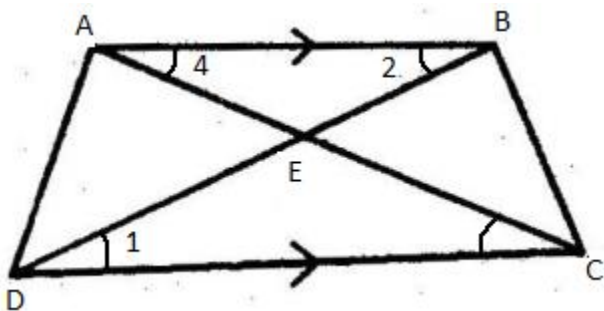
CI	Less than cumulative frequency	CI	More than cumulative frequency
Less than 20	8	More than 0	90
Less than 40	20	More than 20	82
Less than 60	35	More than 40	70
Less than 80	58	More than 60	55
Less than 100	76	More than 80	32
Less than 120	90	More than 100	14



17. ABCD is a trapezium with $AB \parallel DC$, the diagonals AC and BD are intersecting at E. If $\triangle AED$ is similar to $\triangle BEC$, then prove that $AD = BC$.



Solution:



In $\triangle EDC$ and $\triangle EBA$,
 $\angle 1 = \angle 2$ (alternate angles)
 $\angle 3 = \angle 4$ (alternate angles)
and $\angle CED = \angle AEB$ (vertically opposite angles)
 $\triangle EDC \sim \triangle EBA$
 $\Rightarrow ED/EB = EC/EA$
 $\Rightarrow ED/EC = EB/EA$ (i)
Given that, $\triangle AED \sim \triangle BEC$
 $ED/EC = EA/EB = AD/BC$ (ii)
From (i) and (ii),
 $EB/EA = EA/EB$
 $\Rightarrow EB^2 = EA^2$
 $\Rightarrow EB = EA$
Substituting $EB = EA$ in (ii),
 $EA/EA = AD/BC$
 $\Rightarrow AD/BC = 1$
 $\Rightarrow AD = BC$
Hence proved.

OR

Prove that $(1 + \tan^2\theta) + [1 + (1/\tan^2\theta)] = 1/(\sin^2\theta - \sin^4\theta)$

Solution:

We know that,
 $\sec^2\theta - \tan^2\theta = 1$
 $1 + \tan^2\theta = \sec^2\theta$
 $1 + \tan^2\theta = 1/\cos^2\theta$
Similarly,
 $1 + \cot^2\theta = \operatorname{cosec}^2\theta = 1/\sin^2\theta$
 $\text{LHS} = (1 + \tan^2\theta) + [1 + (1/\tan^2\theta)]$
 $= (1 + \tan^2\theta)(1 + \cot^2\theta)$
 $= 1/(\sin^2\theta \cos^2\theta)$
 $= 1/\sin^2\theta(1 - \sin^2\theta)$
 $= 1/(\sin^2\theta - \sin^4\theta)$
 $= \text{RHS}$
Hence proved.