

Total No. of Questions - 24

Total No. of Printed Pages - 4

Regd.
No.

--	--	--	--	--	--	--	--	--	--

Part - III

MATHEMATICS, Paper - II (A)

(Algebra and Probability)

(English Version)

Time : 3 Hours

Max. Marks : 75

Note : This question paper consists of three Sections A, B and C.

SECTION A

10 × 2 = 20

I. Very Short Answer Type Questions.

- i) Attempt all questions.
- ii) Each question carries two marks.

1. If the $\text{Arg}(z_1)$ and $\text{Arg}(z_2)$ are $\frac{\pi}{5}$ and $\frac{\pi}{3}$ respectively, then find $(\text{Arg } z_1 + \text{Arg } z_2)$.
2. If $(\sqrt{3} + i)^{100} = 2^{99}(a + ib)$, then show that $a^2 + b^2 = 4$.
3. If A, B, C are angles of a triangle such that $x = \text{cis } A, y = \text{cis } B, z = \text{cis } C$, then find the value of xyz .
4. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ in terms of a, b, c .

5. If α , β and 1 are roots of $x^3 - 2x^2 - 5x + 6 = 0$, then find α and β .
6. If ${}^nP_r = 5040$ and ${}^nC_r = 210$, then find n and r .
7. Find the number of positive divisors of 1080.
8. If ${}^{22}C_r$ is the largest binomial coefficient in the expansion of $(1+x)^{22}$ then find the value of ${}^{13}C_r$.
9. Find the mean deviation about the median for the following data.
13, 17, 16, 11, 13, 10, 16, 11, 18, 12, 17
10. The mean and variance of a binomial distribution are 4 and 3 respectively. Fix the distribution and find $P(X \geq 1)$.

SECTION B

$5 \times 4 = 20$

II. Short Answer Type Questions.

- i) Attempt **any five** questions.
- ii) Each question carries **four** marks.

11. If $x + iy = \frac{1}{1 + \cos \theta + i \sin \theta}$ then, show that $4x^2 - 1 = 0$.
12. Prove that $\frac{1}{3x+1} + \frac{1}{x+1} - \frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4, if x is real.
13. If the letters of the word EAMCET are permuted in all possible ways and if the words thus formed are arranged in the dictionary order, find the rank of the word EAMCET.

14. Simplify : ${}^{34}C_5 + \sum_{r=0}^4 {}^{38-r}C_4$

15. Resolve $\frac{x^2 - 3}{(x + 2)(x^2 + 1)}$ into partial fractions.

16. The probability for a contractor to get a road contract is $\frac{2}{3}$ and to get a building contract is $\frac{5}{9}$. The probability to get at least one contract is $\frac{4}{5}$. Find the probability that he gets both the contracts.

17. A speaks truth in 75% of the cases and B in 80% cases. What is the probability that their statements about an incident do not match?

SECTION C

5 × 7 = 35

III. Long Answer Type Questions.

- i) Attempt any five questions.
- ii) Each question carries seven marks.

18. If $\cos \alpha + \cos \beta + \cos \gamma = 0 = \sin \alpha + \sin \beta + \sin \gamma$, then

prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \frac{3}{2} = \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$.

19. Solve the equation $2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0$.

20. If P and Q are the sum of odd terms and the sum of even terms respectively, in the expansion of $(x + a)^n$ then prove that

(i) $P^2 - Q^2 = (x^2 - a^2)^n$ (ii) $4PQ = (x + a)^{2n} - (x - a)^{2n}$

21. Find the sum to infinite terms of the series

$$\frac{7}{5} \left(1 + \frac{1}{10^2} + \frac{1.3}{1.2} \cdot \frac{1}{10^4} + \frac{1.3.5}{1.2.3} \cdot \frac{1}{10^6} + \dots \right)$$

22. Find the mean deviation about the mean for the following data :

Marks obtained	0–10	10–20	20–30	30–40	40–50
Number of students	5	8	15	16	6

23. Three boxes numbered I, II, III contains the balls as follows :

	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

One box is randomly selected and a ball is drawn from it. If the ball is red, then find the probability that it is from box II.

24. A random variable X has the following probability distribution.

$X = x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) The mean and (iii) $P(0 < X < 5)$