ICSE Class 10 Maths Important Questions and Solutions

Question 1
Ben deposits a certain sum of money each month in a recurring deposit account of a bank. If the rate of interest is 8% per annum and Ben gets Rs. 8,088 from the bank after 3 years, find the value of his monthly instalment.

Solution:
Let P be the monthly instalment.
Given,
Rate of interest = r = 8%
Time = n = 36 (i.e. 3 years)
Maturity value at the end of 3 years = Rs. 8088
SI = P × [n(n + 1)/ (2 × 12)] × (r/100)
= P × [36(36 + 1)/ 24] × (8/100)
= P × (3 × 37/ 2) × (2/25)
= P × (3 × 37)/25
= 4.44P
Total amount at the time of maturity = (P × n) + SI
8088 = 36P + 4.44P
8088 = 40.44P
P = 8088/ 40.44
P = 200
Therefore, the monthly instalment is Rs. 200.

Question 2
A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is:
(i) a green ball
(ii) a white or a red ball
(iii) neither a green ball nor a white ball

Solution:
Given,
A bag contains 5 white balls, 6 red balls and 9 green balls.
Total number of outcomes = n(S) = 5 + 6 + 9 = 20
(i) Let A be the event of getting a green ball.
Number of outcomes favourable to A = n(A) = 9
P(A) = n(A)/ n(S)
= 9/20
(ii) Let B be the event of getting a white or a red ball.
Number of outcomes favourable to B = n(B) = 11
i.e. 5(white) + 6(red) = 11 balls
P(B) = n(B)/ n(S)
= 11/20
(iii) Let C be the event of getting neither a green ball nor a white ball.
Number of outcomes favourable to C = n(C) = 6

i.e. Total number of balls - (green + white balls) = 20 - (9 + 5) = 6
P(C) = n(C)/ n(S)
= 6/20
= 3/10

Question 3
Find the minimum length in cm and correct to the nearest whole number of a thin metal sheet required to make a hollow and closed cylindrical box of diameter 20 cm and height 35 cm. The width of the metal sheet is 1 m. Also, find the cost of the sheet at the rate of Rs. 56 per m. Find the area of the metal sheet required if 10% of it is wasted in cutting, overlapping etc.

Solution:
Given,
Diameter of hollow and close cylinder = 20 cm
Radius of the cylinder = r = 20/2 = 10 cm
Height of cylinder = h = 35 cm
Total surface area of the cylinder = $2\pi r(r + h)$
= 2 × (22/7) × 10 × (10 + 35)
= (44/7) × 10 × 45
= 2828.57 cm^2
= 0.282 m^2
Width of the sheet = 1 m
Cost of sheet per m = Rs. 56
Total cost of the sheet = 0.282 × Rs. 56 = Rs. 15.8
Length of the sheet = Area of sheet/ Width of the sheet
= 0.282/ 1 = 0.282 m = 28.2 cm
Area of the metal sheet required if 10% of it is wasted in cutting, overlapping etc.,
= 2828.57 + 10% of 2828.57
= 2828.37 + 282.8
= 3111.57 cm^2

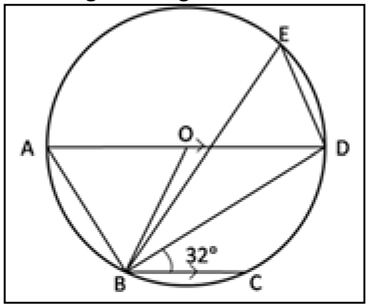
Question 4
Geetha repays her total loan of 1,18,000 by paying instalments every month. If the instalment for the first month is 1,000 and it increases by 100 every month, what amount will she pay as the 30th instalment of the loan? What amount of the loan has she to still pay after the 30th instalment?

Solution:

Total loan amount = Rs. 1,18,000
First instalment = Rs. 1000
Second instalment = Rs. 1000 + Rs. 100 = Rs. 1100
Third instalment = Rs. 1100 + Rs. 100 = Rs. 1200
This is an AP with a = 1000 and d = 100
nth term of an Ap:
an = a + (n - 1)d
a_{30} = 1000 + (30 - 1) × 100
= 1000 + 29 × 100
= 1000 + 2900
= 3900
Therefore, the amount paid in the 30th instalment is Rs. 3900.

Sum of first n term of an AP:
$S_n = n/2\ [a + a_n]$
$S_{30} = (30/2) \times (1000 + 3900)$
$= 15 \times 4900$
$= 73500$
Therefore, the amount paid in 30 instalments is Rs. 73,500
Loan amount to be paid after 30th instalment = Total loan amount - Amount paid in 30 instalments
= Rs. 1,18,000 - Rs. 73,500
= Rs. 44,500

Question 5
In the given figure, AD is a diameter. O is the centre of the circle. AD is parallel to BC and ∠CBD = 32°.



Find:
(i) ∠OBD
(ii) ∠AOB
(iii) ∠BED

Solution:
Given,
AD is a diameter of the circle with centre O.
AD is parallel to BC and ∠CBD = 32°.
From the given figure,
AD || BC
(i) ∠ODB = ∠CBD (alternate interior angles)
OB = OD (radii of the same circle)
∠OBD = ∠ODB = 32°
(ii) AD is the diameter.
We know that the angle in a semicircle is 90°.
∠ABD = 90°
And
∠ABO = ∠ABD - ∠OBD
= 90° - 32°
= 58°
∠ABO = 58°
Also, the angles opposite to the equal sides are equal.
∠OAB = ∠OBA = 58° (OA = OB)
In triangle AOB,
∠OAB + ∠AOB + ∠OBA = 180
58° + ∠AOB + 58° = 180°
∠AOB = 180° - 58° - 58°
∠AOB = 64°

(iii) Angles in the same segment are equal.
∠BED = ∠BAD = 58°

Question 6
Prove that:

$$\frac{1}{(\sec\theta - \tan\theta)} - \frac{1}{\cos\theta} = \frac{1}{\cos\theta} - \frac{1}{(\sec\theta + \tan\theta)}$$

Solution:
LHS = [1/ (sec θ - tan θ)] - (1/cos θ)
Using the identity $\sec^2 A - \tan^2 A = 1$,
= [($\sec^2\theta - \tan^2\theta$) / (sec θ - tan θ)] - sec θ
= [(sec θ + tan θ) (sec θ - tan θ) / (sec θ - tan θ)] - sec θ
= sec θ + tan θ - sec θ
= tan θ….(i)
RHS = (1/cos θ) - [1/ (sec θ + tan θ)]
= sec θ - [($\sec^2\theta - \tan^2\theta$) / (sec θ + tan θ)]
= sec θ - [(sec θ + tan θ) (sec θ - tan θ) / (sec θ + tan θ)]
= sec θ - sec θ + tan θ
= tan θ….(ii)
From (i) and (ii),
LHS = RHS
Hence proved.

Question 7
Prove by factor theorem that
(i) (x - 2) is a factor of $2x^3 - x^2 - 7x + 2$
(ii) (2x + 1) is a factor of $4x^3 + 12x^2 + 7x +1$
(iii) (3x - 2) is a factor of $18x^3 - 3x^2 + 6x - 8$

Solution:
(i) Let $p(x) = 2x^3 - x^2 - 7x + 2$
For checking (x - 2) is a factor of p(x), substitute x = 2 in p(x).
$p(2) = 2(2)^3 - (2)^2 - 7(2) + 2$
= 2(8) - 4 - 14 + 2
= 16 - 18 + 2
= 0
Therefore, (x - 2) is a factor of $2x^3 - x^2 - 7x + 2$.
(ii) Let $p(x) = 4x^3 + 12x^2 + 7x +1$
For checking (2x + 1) is a factor of p(x), substitute x = -1/2 in p(x).
$p(-½) = 4(-½)^3 + 12(-½)^2 + 7(-½) + 1$
= 4(-⅛) + 12(¼) - (7/2) + 1
= (-½) + 3 - (7/2) + 1
= (-8/2) + 4
= -4 + 4
= 0
Therefore, (2x + 1) is a factor of $4x^3 + 12x^2 + 7x +1$.
(iii) Let $p(x) = 18x^3 - 3x^2 + 6x - 8$
For checking (3x - 2) is a factor of p(x), substitute x = 2/3 in p(x).

$p(⅔) = 18(⅔)^3 - 3(⅔)^2 + 6(⅔) - 12$
$= 18(8/27) - 3(4/9) + 4 - 8$
$= (16/3) - (4/3) - 4$
$= (12/3) - 4$
$= 4 - 4$
$= 0$

Question 8
Draw an Ogive for the following distribution:

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	6	10	18	22	20	15	6	3

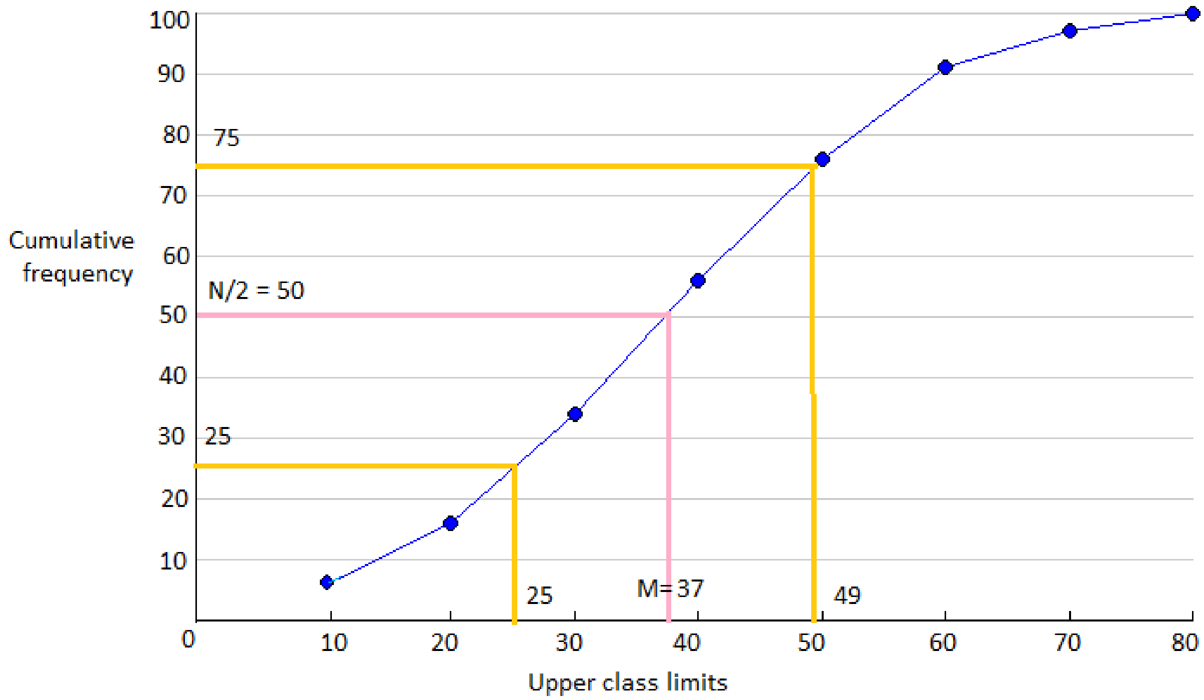
Find the median and interquartile range from the obtained ogive.

Solution:
Cumulative frequency distribution table:

Class	Frequency	Cumulative frequency
0 - 10	6	6
10 - 20	10	16
20 - 30	18	34
30 - 40	22	56
40 - 50	20	76
50 - 60	15	91
60 - 70	6	97
70 - 80	3	100

$N/2 = 100/ 2$

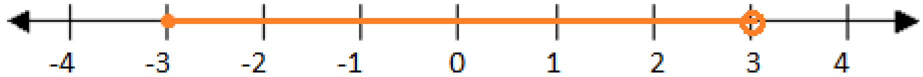
Ogive:



Median = 37
Lower quartile = (N/4)th observation = (100/4) = 25th observation = 25
Upper quartile = (3N/4)th observation = (300/4) = 75th observation = 49
Interquartile range = Upper quartile - Lower quartile
= 49 - 25
= 24

Question 9
Solve the following inequation.
$-2\ 2/3 \leq x + (1/3) < 3 + (1/3), x \in R$
Represent the solution set on the number line.

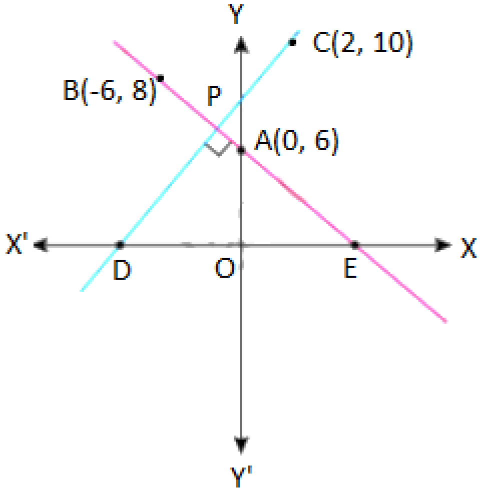
Solution:
$-2\ 2/3 \leq x + (1/3) < 3 + (1/3)$
$-(8/3) \leq x + (1/3) < (10/3)$
$(-8/3) - (1/3) \leq x < (10/3) - (1/3)$
$-9/3 \leq x < 9/3$
$-3 \leq x < 3$



The open circle at 3 indicates that 3 is not included in the solution set.

Question 10
In the given figure. line AB meets the y-axis at point A. The line through C(2, 10) and D intersects line AB at a right angle at point P. Find:
(i) equation of line AB
(ii) equation of line CD

(iii) coordinates of points E and D



Solution:

From the given figure,

A = (0, 6)

B = (-6, 8)

C = (2, 10)

(i) Slope of line AB = (8 - 6)/ (-6 - 0) = -2/6 = -1/3

i.e. m = -1/3

y-intercept of the line AB = 6

Equation of the line in slope intercept form is y = mx + c

y = (-1/3)x + 6

y = (-x + 18)/3

3y = -x + 18

x + 3y - 18 = 0

(ii) Given,

AB and CD interest at right angles.

Slope of AB × Slope of CD = -1

(-1/3) × Slope of CD = -1

Slope of CD = -1 × (-3/1) = 3

Equation of line with slope 3 and passing through the point C(2, 10) is:

y - 10 = 3(x - 2)

y - 10 = 3x - 6

3x - 6 - y + 10 = 0

3x - y + 4 = 0

Therefore, the equation of line CD is 3x - y + 4 = 0.

(iii) Point E lies on the line AB and on the x - axis.

The y-coordinate of E is 0.

Let the coordinates of E be (x, 0).

Substituting E(x, 0) in the equation of line AB.

x + 3(0) - 18 = 0

x = 18

Thus, E = (18, 0)

Similarly,

y-coordinate of D is 0.

Substituting D in the equation of line CD.
$3x - 0 + 4 = 0$
$3x = -4$
$x = -4/3$
Therefore, the coordinates of D are (-4/3, 0).

Question 11
Find a GP for which the sum of the first two terms is -4 and the fifth term is 4 times the third term.

Solution:
Let a be the first term and r be the common ratio of a GP.
nth term of a GP = $a_n = ar^{n-1}$
Given,
$a_1 + a_2 = -4$
$a + ar = -4$
$a(1 + r) = -4$….(i)
And
$a^5 = 4a^3$
$ar^{(5-1)} = 4 \times ar^{(3-1)}$
$r^4 = 4r^2$
$r^2 = 4$
$r = \pm 2$
When r = 2,
$a(1 + 2) = -4$ [From (i)]
$3a = -4$
$a = -4/3$
$ar = (-4/3) \times 2 = -8/3$
$ar^2 = (-4/3) \times (2)^2 = -16/3$
When r = -2,
$a(1 - 2) = -4$
$-a = -4$
$a = 4$
$ar = 4(-2) = -8$
$ar^2 = 4(-2)^2 = 4 \times 4 = 16$
Therefore, the required GP is -4/3, -8/3, -16/3,.... Or 4, -8, 16,....

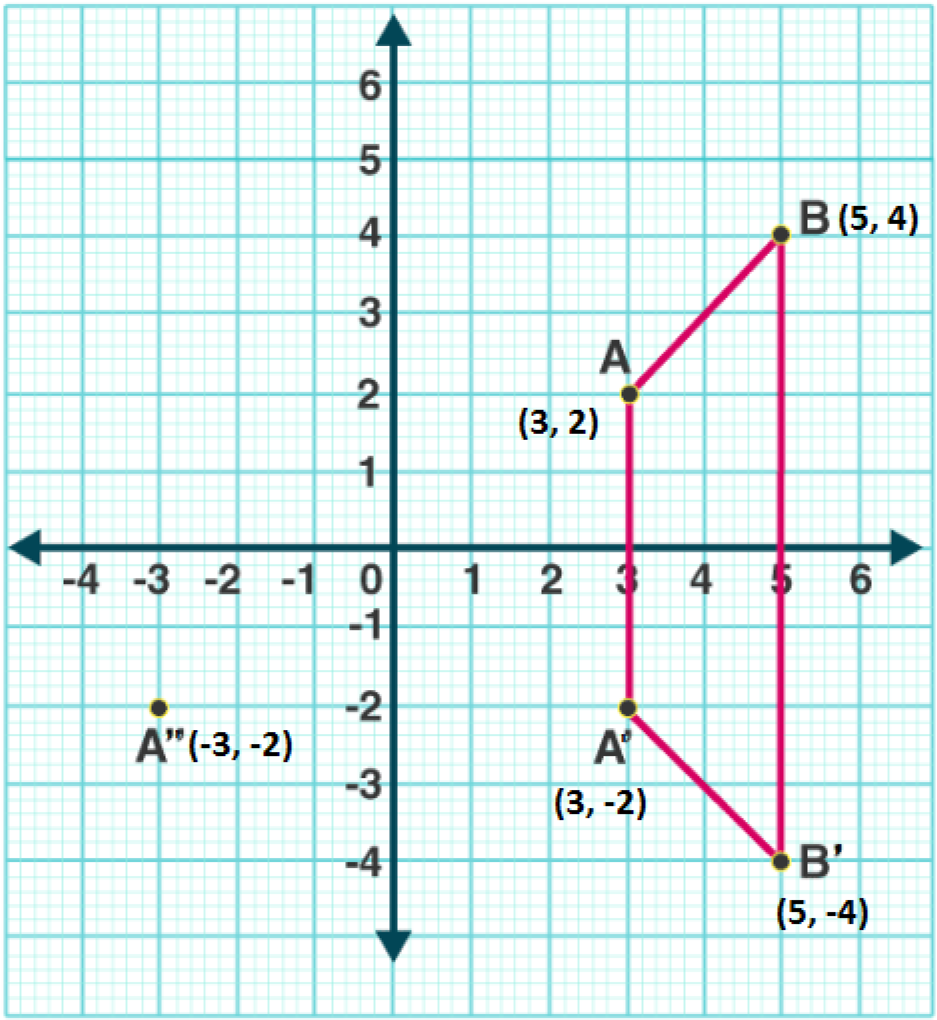
Question 12
A man invests a certain sum on buying 15% Rs. 100 shares at 20% premium. Find:
(i) His income from one share
(ii) The number of shares bought to have an income from the dividend as Rs. 6480
(iii) The sum invested.

Solution:
(i) Dividend on one share = 15% of Rs. 100
= (15/100) × Rs. 100
= Rs. 15
Therefore, the income from one share is Rs. 15.
(ii) The number of shares bought to have an income from the dividend as Rs. 6480
= Annual income/ Dividend on one share
= 6480/ 15

= Rs. 432
(iii) Given that the man bought shares of Rs 100 at 20% premium, the market value of one share
= (120/100) × Rs. 100
= Rs. 120
Total investment = Number of shares x Market value of one share
= 432 × Rs. 120
= Rs. 51,840

Question 13
Attempt this question on a graph paper.
(i) Plot A (3, 2) and B (5, 4) on a graph paper. Take 2 cm = 1 unit on both axes.
(ii) Reflect A and B in the x-axis to A' and B', respectively. Plot these points also on the same graph paper.
(iii) Write down:
(a) the geometrical name of the figure ABB'A'
(b) the measure of angle ABB'
(c) the image of A" of A when A is reflected in the origin
(d) the single transformation that maps A' to A".

Solution:
Given,
A (3, 2) and B (5, 4)
A' and B' are the reflections of A and B respectively in the x-axis.
A" is the image of A when A is reflected in the origin.
(i)



(ii) Coordinates of A'(3, -2) and the coordinates of B'(5, -4).
(iii)
(a) ABB'A' is an isosceles trapezium.
(b) The measure of angle ABB' is 45°.
(c) Coordinates of A" = (-3, -2)
(d) Single transformation that maps A' to A" is the reflection in the y-axis.

Question 14
A school has 630 students. The ratio of the number of boys to the number of girls is 3 : 2. This ratio changes to 7 : 5 after the admission of 90 new students. Find the number of newly admitted boys.

Solution:
Given,
Total number of students = 630
The ratio of the number of boys to the number of girls is 3 : 2.
Let 3x be the number of boys and 2x be the number of girls.
⇒ 3x + 2x = 630
5x = 630
x = 630/5

x = 126
Number of boys = 3x = 3 × 126 = 378
Number of girls = 2x = 2 × 126 = 252
After admission of 90 new students, the ratio of boys to girls is 7 : 5. (given)
Total number of students = 630 + 90 = 720
Let 7x be the number of boys and 5x be the number of girls.
⇒ 7x + 5x = 720
12x = 720
x = 720/12
x = 60
The number of boys = 7x = 7 × 60 = 420
The number of girls = 5x = 5 × 60 = 300
Therefore, the number of newly admitted boys = 420 - 378 = 42

Question 15
Prove:

$$\boxed{\frac{1+\sin A}{\cos A}+\frac{\cos A}{1+\sin A}=2\sec A}$$

Solution:
LHS = [(1 + sin A)/ cos A] + [cos A/ (1 + sin A)]
= [(1 + sin A)^2 + cos^{2}A] / [(1 + sin A) cos A]
= [1 + sin^{2}A + 2 sin A + cos^{2}A] / [(1 + sin A) cos A]
= (1 + 1 + 2 sin A) / [(1 + sin A) cos A]
= (2 + 2 sin A)/ [(1 + sin A) cos A]
= [2(1 + sin A)] / [(1 + sin A) cos A]
= 2/cos A
= 2 sec A
= RHS
Hence proved.

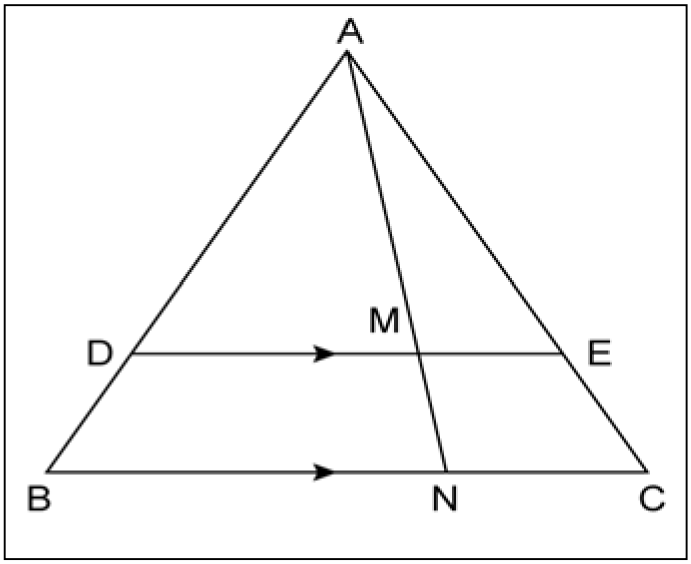
Question 16
Solve for x using the quadratic formula. Write your answer correct to the two significant figures. (x - 1)^2 - 3x + 4 = 0

Solution:
Given,
(x - 1)^2 - 3x + 4 = 0
x^2 + 1 - 2x - 3x + 4 = 0
x^2 - 5x + 5 = 0
Comparing with the standard form ax^2 + bx + c = 0,
a = 1, b = -5, c = 5
Using quadratic formula,
x = [-b ± √(b^2 - 4ac)] / 2a
= [-(-5) ± √{(-5)^2 - 4(1)(5)}] / 2(1)
= [5 ± √(25 - 20)]/ 2
= (5 ± √5)/2
= (5 ± 2.24)/2
x = (5 + 2.24)/2, x = (5 - 2.24)/2

x = 7.24/2, x = 2.76/2
x = 3.62, x = 1.38

Question 17

In the given figure, DE ∥ BC, AE = 15 cm, EC = 9 cm, NC = 6 cm and BN = 24 cm. Find the lengths of ME and DM.



Solution:

Given,
DE ∥ BC, AE = 15 cm, EC = 9 cm, NC = 6 cm and BN = 24 cm.
From the given figure,
ME || NC
In Δ AME and Δ ANC,
∠AME = ∠ANC
∠MAE = ∠NAC (common angle)
By AA similarity criterion,
ΔAME ~ ΔANC
We know that the corresponding sides of similar triangles are proportional.
ME/NC = AE/AC
ME/ 6 = 15/ 24
ME = 3.75 cm
In Δ ADM and Δ ABN,
∠ADM = ∠ABN (DE || BC so, DM || BN)
∠DAM = ∠BAN (common angle)
By AA similarity criterion,
ΔADM ~ ΔABN
In Δ ADE and Δ ABC,
∠ADE = ∠ABC (since DE || BC so, ME || NC)
∠AED = ∠ACB
By AA similarity criterion,
ΔADE ~ ΔABC
ΔADE ~ ΔABC (proved above)
⇒ AD/AB = AE/AC = 15/24 …. (i)

$\Delta ADM \sim \Delta ABN$ (proved above)
$\Rightarrow DM/BN = AD/AB = 15/24$ [From (i)]
$DM/24 = 15/24$
$DM = 15$ cm
Therefore, ME = 3.75 cm and DM = 15 cm.

Question 18
One pipe can fill a cistern in 3 hours less than the other. The two pipes together can fill the cistern in 6 hours 40 minutes. Find the time that each pipe will take to fill the cistern.

Solution:
Let one pipe fill the cistern in x hours and the other fills the cistern in (x - 3) hours.
Given,
The two pipes together can fill the cistern in 6 hours 40 minutes.
i.e. 6 2/3 hours = 20/3 hours
$\Rightarrow (1/x) + [1/(x - 3)] = 3/20$
$\Rightarrow (x - 3 + x)/[x(x - 3)] = 3/20$
$\Rightarrow (2x - 3)/(x^2 - 3x) = 3/20$
$\Rightarrow 20(2x - 3) = 3(x^2 - 3x)$
$\Rightarrow 40x - 60 = 3x^2 - 9x$
$\Rightarrow 3x^2 - 9x - 40x + 60 = 0$
$\Rightarrow 3x^2 - 49x + 60 = 0$
$\Rightarrow 3x^2 - 45x - 4x + 60 = 0$
$\Rightarrow 3x(x - 15) - 4(x - 15) = 0$
$\Rightarrow (3x - 4)(x - 15) = 0$
$\Rightarrow 3x - 4 = 0, x - 15 = 0$
$\Rightarrow x = 4/3, x = 15$
Now,
$x - 3 = (4/3) - 3 = -5/4$ (not possible)
Therefore, $x = 15$
$x - 3 = 15 - 3 = 12$
Hence, one pipe can fill the cistern in 15 hours and other in 12 hours.

Question 19
A circus tent is in the shape of a cylinder surmounted by a conical top of the same diameter. If their common diameter is 56 m, the height of the cylindrical part is 6 m and the total height of the tent above the ground is 27 m. Find the area of the canvas used in making the tent.

Solution:
Given,
Total height of the tent = 27 m
Diameter of the circular bases = 56 m
Radius of cylinder = Radius of conical top = r = 56/2 = 28 m
Height of the cylinder = h = 6 m
Height of the conical part = H = 27 - h = 27 - 6 = 21 m
Slant height of the conical part = $\sqrt{(r^2 + h^2)}$
$l = \sqrt{[(28)^2 + (21)^2]}$
$= \sqrt{(784 + 441)}$
$= \sqrt{1225}$
= 35 m

Area of the canvas used = CSA of cylinder + CSA of cone
$= 2\pi rh + \pi rl$
$= \pi r(2h + l)$
$= (22/7) \times 28 \times (2 \times 6 + 35)$
$= 22 \times 4 \times (12 + 35)$
$= 88 \times 47$
$= 4136 \text{ m}^2$
Therefore, the area of the canvas used in making the tent is 4136 m^2.

Question 20
Ankit deposits Rs 2,250 per month in a recurring deposit account for a period of 3 years. At the time of maturity, he will get Rs 90,990.
(i) Find the rate of simple interest per annum
(ii) Find the total interest earned by Ankit.

Solution:
Given,
Monthly instalment = P = Rs. 2250
Time = n = 36 months (i.e. 3 years)
Amount at the time of maturity = Rs. 90,990
Let r be the rate of interest.
Total amount deposited = Rs. 2250 × 36 = Rs. 81,000
Interest = Amount at the time of maturity - Total amount deposited
= Rs. 90,990 - Rs. 81,000
= Rs. 9990
$SI = P \times [n(n + 1)/ (2 \times 12)] \times (r/100)$
$9990 = 2250 \times (36 \times 37/ 24) \times (r/100)$
$9990 = 2250 \times (3 \times 37) \times (r/200)$
$r = (9990 \times 200)/ (2250 \times 3 \times 37)$
$r = 8$
Therefore, the rate of interest is 8% and the total interest earned by Ankit is Rs. 9990.

Question 21
Ms. Kumar has an account in ICICI Bank. Entries of his passbook has given below:

Date , 2008	Particulars	Withdrawals (in Rs.)	Deposits (in Rs.)	Balance (in Rs.)
Jan 3	B/F	_	_	2642
Jan 16	To self	640	_	2002
March 4	By cash	_	850	2852
April 10	To self	1130	_	1722
April 25	By cheque	_	650	2372
June 15	By cash	577	_	1795

Calculate the total amount received by Kumar if he closes his account on August 1, 2008 and the rate of interest is 5% per annum.

Solution:
Qualifying amount on monthly basis:
Jan = 2002
Feb = 2002
March = 2852
April = 1722
May = 2372
June = 1795
July = 1795
Total = Rs. 14540
Let principal amount for one month = Rs. 14540
Rate of interest = 5%
Interest = $(14540 \times 1 \times 5)/ (12 \times 100)$
= Rs. 60.58
The total amount received by Kumar if he closes his account on August 1, 2008
= Balance at the end of July + Interest earned from Jan to July
= Rs. 1795 + Rs. 60.58
= Rs. 1855.58