

ICSE Class 10 Maths Question Paper Solution 2015

Question 1

- (a) A shopkeeper bought an article for ₹3,450. He marks the price of the article 16% above the cost price. The rate of sales tax charged on the article is 10%. Find the:
- marked price of the article.
 - price paid by a customer who buys the article. [3]
- (b) Solve the following inequation and write the solution set:
 $13x - 5 < 15x + 4 < 7x + 12, x \in \mathbb{R}$
 Represent the solution on a real number line. [3]
- (c) Without using trigonometric tables evaluate:
 $\frac{\sin 65^\circ}{\cos 25^\circ} + \frac{\cos 32^\circ}{\sin 58^\circ} - \sin 28^\circ \cdot \sec 62^\circ + \operatorname{cosec}^2 30^\circ$ [4]

Examiners' Comments

- (a) Some candidates calculated 10% sales tax on the cost price instead of marked price that led to a number of calculation errors. They also expressed the final customer's price as ₹4402.2 instead of ₹4402.20.
- (b) Common errors were made in transposing x terms on one side and constants on the other. Some candidates tried to work out both inequalities simultaneously and hence made errors, both with signs and transposition of terms. Solution to the inequality was not written in the set form as stated in the question. It is necessary to put arrows on both sides of the number line with at least one extra element on each side so as to indicate the continuity of the infinite real number line.
- (c) Candidates adopted incorrect methods of conversion of trigonometric ratio of complementary angles, e.g.

| Incorrect Method | Correct Method |
|---|---|
| $\sin 65^\circ = \cos(90^\circ - 25^\circ)$ | $\sin 65^\circ = \cos(90^\circ - 65^\circ)$ |
| $\sin 65^\circ = \sin(90^\circ - 65^\circ)$ | $\sin 65^\circ = \sin(90^\circ - 25^\circ)$ |

Candidates were unable to write the value of $\operatorname{cosec} 30^\circ$ as 2 and missed out on a number of essential steps that led to incorrect answer.

Suggestions for teachers

Students must be advised to read the question carefully for correct identification of data given and the result that is to be found. Further all decimal answers related to money must be written to two decimal places e.g., ₹4402.20

It is advisable to solve the inequation by taking the two inequalities separately, e.g., $13x - 5 < 15x + 4 < 7x + 12$ to be taken as $13x - 5 < 15x + 4$ and $7x + 12$.

Students need to understand the concept that $-3x < 9$ is $3x > -9$ and not $3x < -9$.

Students must always write the solution in the appropriate set form.

It is necessary to have rigorous practice of complementary angles of trigonometric ratios and the common errors must be highlighted. Emphasis must be given on the concepts of values of special trigonometric angles, e.g., $0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° . Examine assignments to ensure essential steps are being followed, e.g. $\sin 65^\circ$ must not be directly written as $\cos 25^\circ$

MARKING SCHEME

Question 1

(a) C.P. of the article is Rs.3,450/-

$$(i) \text{ M.P. of the article} = 3450 + \frac{16}{100} \times 3450 + 552 = \text{`} 4002$$

$$(ii) \text{ S.T.} = \frac{10}{100} \times 4002 = \text{`} 400.20$$

$$\therefore \text{Price paid by the customer} = 4002 + 400.20 = \text{`} 4402.20$$

(b) $13x - 5 < 15x + 4 < 7x + 12, \quad x \in R$

$$13x - 5 < 15x + 4$$

$$15x + 4 < 7x + 12$$

$$13x - 15x < 4 + 5$$

$$15x - 7x < 12 - 4$$

$$-2x < 9$$

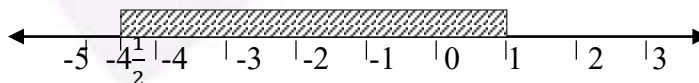
$$8x < 8$$

$$\therefore x > -4\frac{1}{2}$$

$$\therefore x < 1$$

Any one correctly transposed

$$\text{Solution } \left\{ x: -4\frac{1}{2} < x < 1, x \in R \right\}$$



(c) $\frac{\sin 65^\circ}{\cos 25^\circ} + \frac{\cos 32^\circ}{\sin 58^\circ} - \sin 28^\circ \sec 62^\circ + \operatorname{cosec}^2 30^\circ$

$$\frac{\cos(90 - 65)}{\cos 25} + \frac{\sin(90 - 32)}{\sin 58} - \cos(90 - 28) \times \sec 62 + (2)^2$$

Any one complementary angle correct

$$= \frac{\cos 25}{\cos 25} + \frac{\sin 58}{\sin 58} - \cos 62 \times \frac{1}{\cos 62} + 4$$

$$1 + 1 - 1 + 4 = 5$$

Question 2

- (a) If $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$, find x and y when $A^2 = B$. [3]
- (b) The present population of a town is 2,00,000. Its population increases by 10% in the first year and 15% in the second year. Find the population of the town at the end of the two years. [3]
- (c) Three vertices of a parallelogram ABCD taken in order are A (3, 6), B (5, 10) and C (3, 2) find:
- the coordinates of the fourth vertex D.
 - length of diagonal BD.
 - equation of side AB of the parallelogram ABCD. [4]

Examiners' Comments

- (a) Some candidates took entries of matrix A^2 by squaring the elements of A instead of finding the product $A \times A$. Thus got $A^2 = \begin{bmatrix} 9 & x^2 \\ 0 & 1 \end{bmatrix}$ instead of $\begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}$. Some added the elements and then equated the corresponding elements of $\begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$ hence got the incorrect values of x and y . e.g. $9 + 4x = 9 + 16$
- (b) Some candidates lacked basic clarity about the concepts of compound interest. Due to incorrect concept they took the principal of Second year as ₹2,00,000 instead of ₹2,20,000 which is the amount at the end of first year.
- (c) Candidates adopted steps that led to cumbersome calculations. The numerical problem could easily be solved by using midpoint theorem or by finding slopes, e.g., $\frac{5+x}{2} = \frac{3+3}{2}$ (equating the x – coordinates of midpoint of AC and BD) or by equating slopes of AB & DC, e.g. $\frac{2-y}{3-x} = \frac{6-10}{-5+3}$.

Suggestions for teachers

- Multiplication of matrices must be made clear and to understand that A^2 is the matrix product $A \times A$ and is not a matrix whose elements are the square of the elements of A .
- Basic concepts of Compound Interest calculation needs to be made clear. Students must be able to differentiate between Simple Interest and Compound Interest.
- Concepts on Coordinate Geometry need to be explained in different methods of solving a particular numerical problem. This would assist students avoid the cumbersome approaches of solving a numerical problem.

If worked out by equating two distances AD and BC then working becomes tedious and answer to the sum is incorrect, e.g., $\sqrt{(3-x)^2 + (y-6)^2} = \sqrt{(5-3)^2 + (10-2)^2}$

Marking Scheme

Question 2

(a) $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$

$$A^2 = B$$

$$\begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$$

$$\begin{bmatrix} 9 + 0 & 3x + x \\ 0 + 0 & 0 + 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$$

$$\therefore \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$$

$$\therefore 4x = 16 \Rightarrow x = 4$$

$$1 = -y \Rightarrow y = -1$$

Equating both correctly

(b) Present population 2,00,000, rate of increase 10% and 15%

Method 1:

Population after 2 years

$$= 200000 \left(1 + \frac{10}{100}\right) \left(1 + \frac{15}{100}\right)$$

$$= 200000 \times \frac{11}{10} \times \frac{23}{20} = 253000$$

Simplifying

Method 2:

Population at the end of 1st year

$$= 200000 + \frac{10}{100} \times 200000$$

$$= 220000$$

Population at the end of 2nd year

$$= 220000 + \frac{15}{100} \times 220000$$

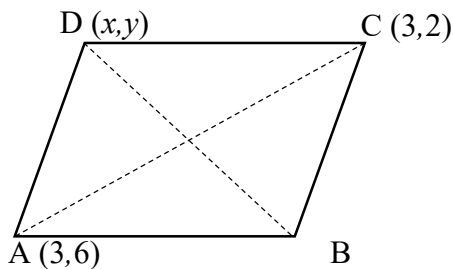
$$= 220000 + 33000 = 253000$$

(c) (i) Mid point of AC = Mid point of BD

$$\therefore \frac{3+3}{2} = \frac{5+x}{2} \text{ and } \frac{6+2}{2} = \frac{10+y}{2}$$

$$\therefore x = 1 \qquad y = -2$$

$$\therefore D(1, -2)$$



(ii) Length of BD :

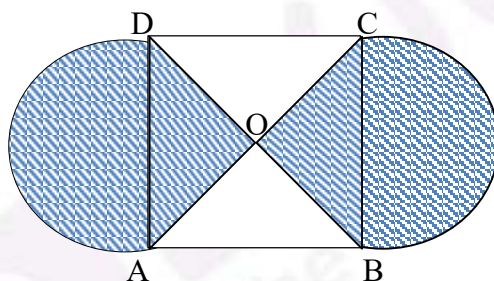
$$\sqrt{(10+2)^2 + (5-1)^2} = \sqrt{12^2 + 4^2} = \sqrt{160} = 4\sqrt{10} \text{ units}$$

(iii) Slope of AB = $\frac{10-6}{5-3} = \frac{4}{2} = 2$

$$\therefore \text{Eqn of AB : } y - 6 = 2(x - 3) \text{ or } y - 6 = 2x - 6$$

Question 3

(a) In the given figure, ABCD is a square of side 21 cm. AC and BD are two diagonals of the square. Two semi circles are drawn with AD and BC as diameters. Find the area of the shaded region. (Take $\pi = \frac{22}{7}$) [3]



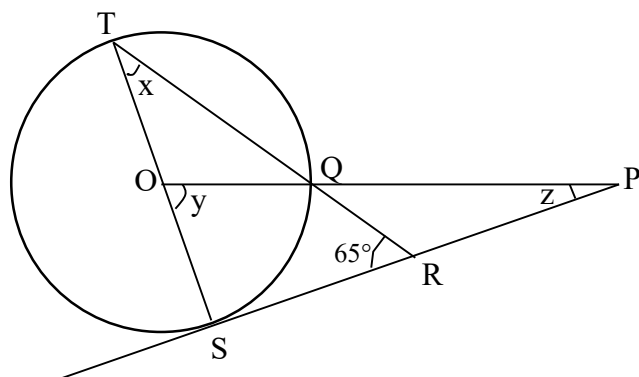
(b) The marks obtained by 30 students in a class assessment of 5 marks is given below:

| | | | | | | |
|-----------------|---|---|---|----|---|---|
| Marks | 0 | 1 | 2 | 3 | 4 | 5 |
| No. of Students | 1 | 3 | 6 | 10 | 5 | 5 |

Calculate the mean, median and mode of the above distribution.

[3]

- (c) In the figure given below, O is the centre of the circle and SP is a tangent. If $\angle SRT = 65^\circ$, find the value of x, y and z.



[4]

Examiners' Comments

- (a) Candidates committed calculation errors made as they did not follow the appropriate approach, e.g. (i) taking $\pi = 3.14$ instead of $\frac{22}{7}$ as instructed in the question. This led to lengthy working; (ii) Finding the length of diagonal to get the area of the triangles where as they could have just found it by using the fact that the area of the two triangles $= \frac{1}{2} \text{Area of the Square}$. Some made mistakes by taking the radius as 21 instead of $21/2$.
- (b) Some candidates solved the numerical problem correctly however they incorrectly mentioned mean for median and vice versa. Candidates made errors in calculating fx or finding cumulative frequency c and some calculated fc instead of fx . A few candidates went on to find median graphically for the given non-grouped distribution and arrived at incorrect answers.
- (c) Most candidates failed to cite reasons while solving numerical problems. Some candidates were not versed with the circle properties, hence could not identify the fact $\angle y = 2\angle x$ or $\angle PST = 90^\circ$

Suggestions for teachers

- Students need to be trained on the correct approach of solving Mensuration numerical problems with the value of π taken as $22/7$ if given.
- If students had identified that, the area of the two triangles is equal to half the area of the square they need not find the length of the diagonal.
- Insist on instructions to be followed as given in a question. It is essential to draw graphs only when it is specified in the question. Students must be made to understand the meaning of $\sum f$ some students took $\sum f$ as 6 instead of 30.
- Emphasise on the following while teaching: (i) identifying angles correctly. $\angle TRS$ cannot be written as $\angle R$. (ii) Essential working must be shown, (iii) reasons must be clearly stated.

MARKING SCHEME

Question 3

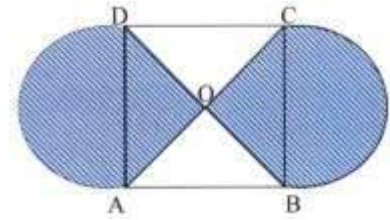
- (a) ABCD is a square of side 21 cm.

$$\therefore \text{Area of square} = 21^2 = 441$$

\therefore Area of the shaded part = area of the two triangular part + Area of the two semi circles.

$$\therefore \text{Area of shaded part} = \frac{1}{2} \times 441 + \pi r^2 = \frac{1}{2} \times 441 + \frac{22}{7} \times \left(\frac{21}{2}\right)^2$$

$$= \frac{441}{2} + \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{441}{2} + \frac{693}{2} = \frac{1134}{2} = 567 \text{ sq units}$$



| (b) | Marks (x) | Number of students (f) | fx | cf |
|-----|---------------|----------------------------|-----------|------|
| | 0 | 1 | 0 | 1 |
| | 1 | 3 | 3 | 4 |
| | 2 | 6 | 12 | 10 |
| | 3 | 10 | 30 | 20 |
| | 4 | 5 | 20 | 25 |
| | 5 | 5 | 25 | 30 |
| | | <u>30</u> | <u>90</u> | |

$$\therefore \text{Mean} = \frac{\sum fx}{\sum f} = \frac{90}{30} = 3$$

$$\text{Median} = 3$$

$$\text{Mode} = 3$$

- (c) In ΔRST , $\angle S = 90^\circ$ (\therefore ST is a diameter)

$\therefore x = 180^\circ - (90^\circ + 65^\circ)$; (angles of a triangle upto 180°)

$$= 25^\circ$$

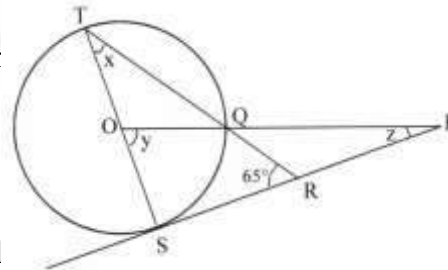
$$\angle y = 2\angle x = 2 \times 25^\circ = 50^\circ$$

(angle of the centre is double the angle at the circumference)

In the remaining circumference

$$x = 180^\circ - (90^\circ + 50^\circ) = 40^\circ \quad (\text{With at least one reason})$$

(angles of a triangle otherwise adds upto 180°)



Question 4

- (a) Katrina opened a recurring deposit account with a Nationalised Bank for a period of 2 years. If the bank pays interest at the rate of 6% per annum and the monthly instalment is ₹1,000, find the:

- (i) interest earned in 2 years.
(ii) matured value. [3]
- (b) Find the value of 'K' for which $x = 3$ is a solution of the quadratic equation,
 $(K + 2)x^2 - Kx + 6 = 0$.
Thus find the other root of the equation. [3]
- (c) Construct a regular hexagon of side 5 cm. Construct a circle circumscribing the hexagon. All traces of construction must be clearly shown. [4]

Examiners' Comments

- (a) Errors of candidates were mostly based on formula, application of the formula or calculation error, e.g., (i) n is taken as '2' instead of 24 months; (ii) r is taken as 6 instead of $\frac{6}{12}$ as interest is being calculated per month; (iii) First part of question is to find the interest which is equal to ₹1500,
Some candidates found the final answer as matured value = ₹25,500 which is the second part of the question hence led to incorrect answers.
- (b) Candidates were able to find $k = -4$. Some used the direct method of replacing x by 3 and did it easily. On the other hand there were candidates who used the formula method of solving x and equating to 3 to find k . This led to more working and thereby incorrect answers.
A few candidates were unable to find the other factor which was simply to solve the quadratic equation.
- (c) It is necessary to use a ruler and compass and show all traces while doing any geometrical construction. According to the question it is necessary to construct the hexagon and locate the circumcentre by construction and finally to draw the circle. Drawing a circle of 5cm radius and then constructing the hexagon by cutting arcs is incorrect according to the given conditions.

Suggestions for teachers

- Advise students to read the question carefully so as to identify correctly what is the given data and what is required to be found.
- A thorough clarity on concepts to ensure students are confident of attempting the question. They should be able to identify a quadratic equation and hence solve to find the roots, e.g., after substituting k in the equation it becomes the quadratic $x^2 - 2x - 3 = 0$, hence $(x-3)(x+1) = 0$. \therefore other root is -1.
- The question must be read carefully before an attempt is made to answer.

MARKING SCHEME

Question 4

- (a) Instalment = `1000, number of months = $12 \times 2 = 24$, Rate = 6%
- (i) $\therefore \text{Interest} = \frac{PRT}{100} = \frac{1000 \times 6 \times 24(24+1)}{100 \times 12 \times 2} = `1500$
- (ii) Matured value = Total amount deposited + Interest
 $= 1000 \times 24 + 1500 = `25,500$

(b) $(K + 2)x^2 - Kx + 6 = 0$, $x = 3$ is a root of the equation

$$\therefore (K + 2)3^2 - K \times 3 + 6 = 0$$

$$9K + 18 - 3K + 6 = 0$$

$$\therefore 6K = -24 \text{ or } K = -4$$

\therefore The equation is $-2x^2 + 4x + 6 = 0$

$$x^2 - 2x - 3 = 0$$

$$\therefore (x - 3)(x + 1) = 0$$

$$\therefore x = 3, x = -1$$

Hence the second root is -1

(c) One side and one 120°

Hexagon

Bisection of one side

Two sides bisection to locate centre and circumcircle

Question 5

(a) Use a graph paper for this question taking 1 cm = 1 unit along both the x and y axis:

(i) Plot the points A(0, 5), B(2, 5), C(5, 2), D(5, -2), E(2, -5) and F(0, -5).

(ii) Reflect the points B, C, D and E on the y-axis and name them respectively as B', C', D' and E'.

(iii) Write the coordinates of B', C', D' and E'.

(iv) Name the figure formed by B C D E E' D'C'B'.

(v) Name a line of symmetry for the figure formed.

[5]

(b) Virat opened a Savings Bank account in a bank on 16th April 2010. His pass book shows the following entries:

| Date | Particulars | Withdrawal (₹) | Deposit (₹) | Balance (₹) |
|------------------------|-------------|----------------|-------------|-------------|
| April 16, 2010 | By cash | - | 2500 | 2500 |
| April 28 th | By cheque | - | 3000 | 5500 |
| May 9 th | To cheque | 850 | - | 4650 |
| May 15 th | By cash | - | 1600 | 6250 |
| May 24 th | To cash | 1000 | - | 5250 |
| June 4 th | To cash | 500 | - | 4750 |
| June 30 th | By cheque | - | 2400 | 7150 |
| July 3 rd | By cash | - | 1800 | 8950 |

Calculate the interest Virat earned at the end of 31st July, 2010 at 4% per annum interest. What sum of money will he receive if he closes the account on 1st August, 2010? [5]

Examiners' Comments

- (a) Candidates did not use the given scale i.e. $1\text{ cm} = 1\text{ unit}$ along x and y axis. Further the following errors were observed: (i) incorrectly plotted points A and F ; (ii) Some candidates marked the positive and negative parts of both x and y axis incorrectly; (iii) Some did not join the points in the proper order and hence were unable to name the figure as octagon; (iv) some did not draw or name the line of symmetry.
- (b) Many candidates made errors by taking the April balance as ₹2500 instead of '0' as the account was opened on 16th April 2010. Errors were made in calculating the minimum balance and also in finding the amount received on 1st August 2010. Candidates worked out ₹(18350 + 61.17) instead of ₹(8950 + 61.17), Some candidates took $t = 4$ instead of 1.

Suggestions for teachers

- It is necessary to see that students read their question paper and identify the given data. If a scale is given for a question on graph the same must be used. All coordinates must be noted. Students must be instructed to form the figure by joining the points and also draw the line of symmetry and name it.
- Concepts of finding interest using the formula: $\text{Interest} = \frac{(P \times R \times t)}{100 \times 12}$ needs to be made clear. Taking $t = 1$ is important as interest is calculated monthly and must be divided by 12 as rate given is 4% per annum and not per month. Students must be made aware that the answer related to money must be written correctly to 2 decimal places. So interest earned is ₹61.17 and not ₹61.166 or ₹61.2 or ₹61.16

Marking Scheme

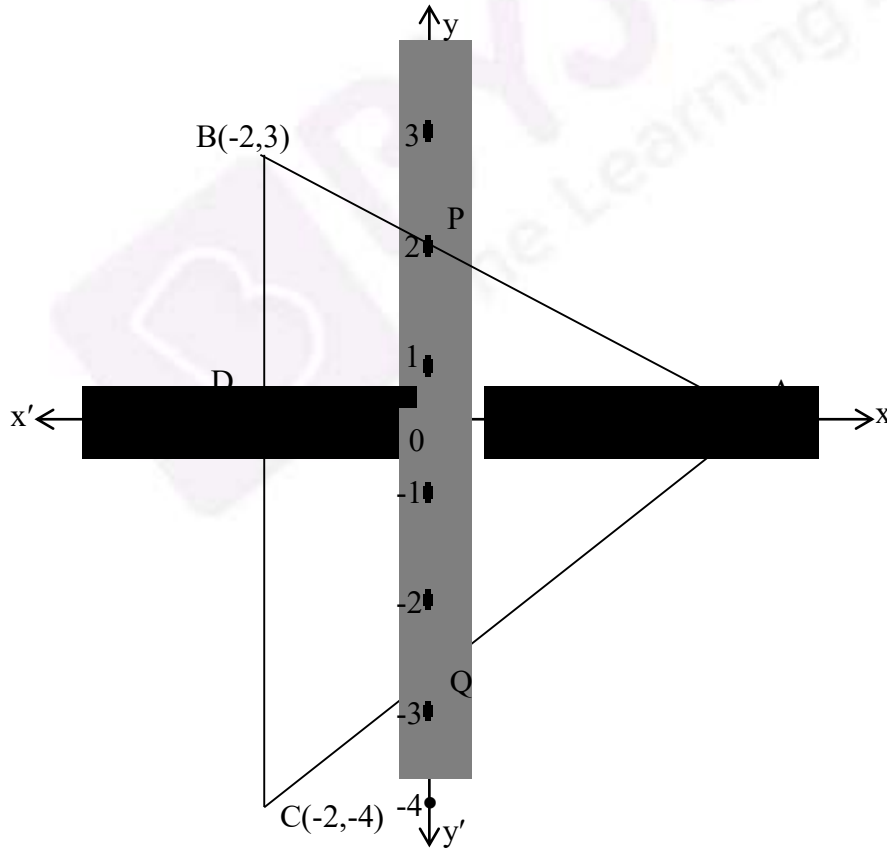
Question 5.

- (a) (i) Plotting 3 points correctly
(ii) Plotting 3 images correctly
(iii) Writing all coordinates correctly $B'(-2,5)$, $C'(-5,2)$, $D'(-5,-2)$, $E'(-2,-5)$
(iv) Octagon
(v) y -axis or x -axis or any other correct line of symmetry $B_1 \times 5 =$

| (b) | Month | Minimum Balance in ₹ |
|-----|--|-------------------------|
| | April | 0 |
| | May | 4650 |
| | June | 4750 |
| | July | <u>8950</u> |
| | | <u>18350</u> |
| | $\therefore \text{Interest} = \frac{PRT}{100} = \frac{18350 \times 4 \times 1}{100 \times 12} = \text{`}61.17$ | |
| | Amount received on 1 st August = 8950 + 61.17 | |
| | = `9011.17 = `9011 | |

Question 6

- (a) If a, b, c are in continued proportion, prove that
 $(a + b + c) (a - b + c) = a^2 + b^2 + c^2$. [3]
- (b) In the given figure ABC is a triangle and BC is parallel to the y-axis. AB and AC intersect the y-axis at P and Q respectively.



- (i) Write the coordinates of A.
 (ii) Find the length of AB and AC.
 (iii) Find the ratio in which Q divides AC.
 (iv) Find the equation of the line AC.

[4]

- (c) Calculate the mean of the following distribution:

| Class Interval | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|----------------|------|-------|-------|-------|-------|-------|
| Frequency | 8 | 5 | 12 | 35 | 24 | 16 |

[3]

Examiners' Comments

- (a) Some candidates failed to apply the property of continued proportion, i.e., $b^2 = ac$ or $\frac{a}{b} = \frac{b}{c} = k$ and hence failed to solve the numerical problem. In some cases, candidates used the property correctly but were unable to simplify and obtain the correct answer.
- (b) Some candidates wrote the coordinates of A as $(0,4)$ instead of $(4,0)$ and hence incorrectly attempted the remaining part of the question. There was an incorrect application of the Distance formula, Section formula and the slope. Hence varied incorrect answers were obtained by candidates.
- (c) Most candidates solved the numerical correctly however calculation errors were noticed. Some candidates made errors in finding the class mark and in finding $\sum fx, \sum f$.

Suggestions for teachers

- All properties of ratio and proportion must be given equal importance and a thorough drilling so as to help in handling all types of problems.
- Explain the concept that points on the x -axis and y -axis must be represented as $(a,0)$ and $(0,b)$ respectively. It is necessary for a thorough explanation and revision of basic formulae and concepts. Students need to understand that if the question is to find the ratio in which Q divides AC it implies that we are to find $AQ:QC$ and not $CQ:QC$.
- Students must be made to understand all three methods of obtaining Mean and explain how to find the class mark. In using the shortcut or step deviation method students must choose the assumed mean of the class mark for convenience of working.

MARKING SCHEME

Question 6

- (a) a,b,c are in continued proportion

$$\therefore \frac{a}{b} = \frac{b}{c} \text{ or } b^2 = ac$$

$$\text{LHS} = (a + b + c)(a - b + c) = a^2 + c^2 + 2ac - b^2$$

$$= a^2 + c^2 + 2b^2 - b^2$$

$$= a^2 + b^2 + c^2 = \text{R.H.S. proved}$$

- (b) (i) $A \rightarrow (4,0) \rightarrow$
- (ii) $AB = \sqrt{(4+2)^2 + (0-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$
 $AC = \sqrt{(4+2)^2 + (0+4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ any one correct
- (iii) $\frac{m}{n} = \frac{x-x_1}{x_2-x} = \frac{0-4}{-2-0} = \frac{2}{1}$ i.e. $m:n = 2:1$
- (iv) Slope of $AC = \frac{0+4}{4+2} = \frac{4}{6} = \frac{2}{3} \therefore$ eqn. $y - 0 = \frac{2}{3}(x - 4)$

Or $2x - 3y = 8$

| (c) | Class Interval | Frequency | X | d=X-A | fd |
|-----|----------------|------------------|----|-------|-------------------|
| | 0 - 10 | 8 | 5 | -30 | -240 |
| | 10 - 20 | 5 | 15 | -20 | -100 |
| | 20 - 30 | 12 | 25 | -10 | -120 |
| | 30 - 40 | 35 | 35 | 0 | 0 |
| | 40 - 50 | 24 | 45 | 10 | 240 |
| | 50 - 60 | 16 | 55 | 20 | 320 |
| | | $\Sigma f = 100$ | | | $\Sigma fd = 100$ |

$$\text{Mean} = A + \frac{\Sigma fd}{\Sigma f} = 35 + \frac{100}{100} = 36$$

Question 7

- (a) Two solid spheres of radii 2 cm and 4 cm are melted and recast into a cone of height 8 cm. Find the radius of the cone so formed. [3]
- (b) Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leaves the same remainder when divided by $x+3$. [3]
- (c) Prove that $\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} = \cos \theta + \sin \theta$ [4]

Examiners' Comments

- (a) Candidates made mistakes in writing the formula of the cone and sphere. Some made calculation errors mostly because of using the lengthy method of finding out each volume separately. Workings would be easier if the volumes are equated as $\frac{4}{3}\pi 2^3 + \frac{4}{3}\pi 4^3 = \frac{1}{3}\pi r^2 \times 8$. Hence certain terms common throughout the expression cancel off and a simplified expression is obtained.
- (b) Most candidates solved the sum correctly. Some candidates failed to equate $f(-3)$ and $g(-3)$ to solve for 'a' instead wrote $f(-3) = 0, g(-3) = 0$ and tried finding 'a'. Some wrote $f(3) = g(3)$ and thus obtained an incorrect answer.

Suggestions for teachers

- To avoid long calculations there is a need to guide students of using shorter methods of equating and hence solving of numerical problems.
- More of application based problems are necessary for students to practice.
- Simple tips is necessary for avoiding calculation errors and to save time.

- (c) The common drawback noticed was of not making the denominator same before taking the LCM. Other errors detected were in the calculation and formula.

For example $\frac{\sin^2\theta}{\sin\theta-\cos\theta} + \frac{\cos^2\theta}{\cos\theta-\sin\theta}$

$\therefore \frac{\sin^2\theta}{\sin\theta-\cos\theta} - \frac{\cos^2\theta}{\sin\theta-\cos\theta} \therefore LCM \text{ is } \sin\theta - \cos\theta, \text{ not } (\sin\theta - \cos\theta)(\cos\theta - \sin\theta)$

MARKING SCHEME

Question 7

- (a) Sum of the volumes of the two sphere = volume of the cone

$$\frac{4}{3}\pi \times 2^3 + \frac{4}{3}\pi 4^3 = \frac{1}{3}\pi r^2 \times 8$$

Equating $4 \times 8 + 4 \times 64 = 8r^2 \quad \therefore r^2 = \frac{288}{8} = 36$

$\therefore r = 6\text{cm}$

- (b) $f(x) = ax^3 + 3x^2 - 9 \quad g(x) = 2x^3 + 4x + a$

Remainder on dividing by $x + 3$ is

$$f(-3) = a(-3)^3 + 3 \times (-3)^2 - 9 \quad g(-3) = 2 \times (-3)^3 + 4 \times (-3) + a$$

$$= -27a + 27 - 9 \quad \text{OR} \quad = -54 - 12 + a$$

$$\therefore -27a + 18 = -66 + a$$

- (c) $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} = \cos\theta + \sin\theta$

$$\text{L.H.S} = \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} = \frac{\sin^2\theta}{\sin\theta-\cos\theta} + \frac{\cos^2\theta}{\cos\theta-\sin\theta}$$

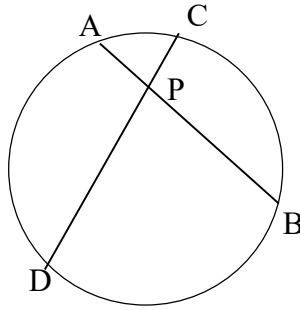
$$= \frac{\sin^2\theta}{\sin\theta-\cos\theta} - \frac{\cos^2\theta}{\sin\theta-\cos\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta} = \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{\sin\theta - \cos\theta} = \sin\theta + \cos\theta = \text{R.H.S.}$$

Question 8

- (a) AB and CD are two chords of a circle intersecting at P. Prove that

$$AP \times PB = CP \times PD$$



[3]

- (b) A bag contains 5 white balls, 6 red balls and 9 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is:

- a green ball
 - a white or a red ball
 - is neither a green ball nor a white ball.
- [3]

- (c) Rohit invested ₹ 9,600 on ₹ 100 shares at ₹ 20 premium paying 8% dividend. Rohit sold the shares when the price rose to ₹ 160. He invested the proceeds (excluding dividend) in 10% ₹ 50 shares at ₹ 40. Find the:

- original number of shares.
 - sale proceeds.
 - new number of shares.
 - change in the two dividends.
- [4]

Examiners' Comments

- (a) In this geometrical proof it was necessary to prove two triangles similar. For this candidates needed to join AD and CD or AC and DB. Hence drawing of diagram was essential. Thus some failed to prove the result. The problem though a circle theorem is actually a direct application of similar triangles, i.e. $\triangle APD \sim \triangle CPB$. Some candidates proved the result but did not give reasons.
- (b) Candidates failed to write the favourable outcome and the total instead they directly wrote the required probability. Further, answers were not given in the simplest form, e.g., $\frac{6}{20}$ needed to be written as $\frac{3}{10}$.

Suggestions for teachers

- Emphasise on the following points while solving geometry problems :
- The diagram must always be drawn and labelled carefully;
- (ii) all reasons supporting the result must be given;
- (iii) $\angle APC$ must not be written as $\angle P$ as there are four angles at that point. Hence supervision is required with regards to naming angles.

- (c) Candidates did not seem to be versed with the concept that ₹100 share at ₹20 premium implies that the MV is ₹120. As a result the number of shares was incorrect. Some worked out as $₹9600 \div 100$ instead of $₹9600 \div 120$. Some candidates misunderstood the concept of the proceeds, hence committed errors in finding the new number of shares and dividend.

- Revise problems on shares and dividends extensively so that students are familiar with various terms.
- It is necessary to teach students the three basic points of solving a probability sum; (i) identify the total number of outcomes and the number of outcomes favorable for the event; (ii) Finding probability of event E by using $P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total number of outcomes}}$; (iii) Writing of the final answer in the simplest form.

MARKING SCHEME

Question 8

- (a) In the two triangles APD and CPB

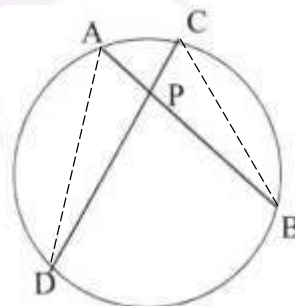
$$\angle A = \angle C \quad (\text{angles in the same segment})$$

$$\angle D = \angle B$$

$$\angle APD = \angle CPB$$

$$\therefore \Delta APD \sim \Delta CPB \quad (\text{AAA})$$

$$\text{Hence } \frac{AP}{CP} = \frac{PD}{PB} \quad \text{or } AP \times PB = CP \times PD$$



- (b) 5 white balls, 6 red balls and 9 green balls

- (i) There are 9 green balls and $5 + 6 + 9 = 20$ balls altogether

$$\therefore P(\text{a green ball}) = \frac{9}{20}$$

- (ii) There are 5 white balls and 6 red balls and their sum is 11

$$\text{Total number of balls} = 20$$

$$\therefore P(\text{a white or red ball}) = \frac{11}{20}$$

- (iii) Neither green or white means probability of a red ball. There are 6 red balls

$$\text{Total number of balls} = 20$$

$$\therefore P(\text{neither green nor white}) = \frac{6}{20} = \frac{3}{10}$$

(c) Total investment = ₹ 9600 N.V. = ₹100 M.V. = $100 + 20 = ₹120$, rate of dividend = 8%.

(i) \therefore original number of shares = $\frac{9600}{120} = 80$

(ii) Sale proceeds = $80 \times 160 = ₹12800$

(iii) The new number of shares = $12800 \div 40 = 320$

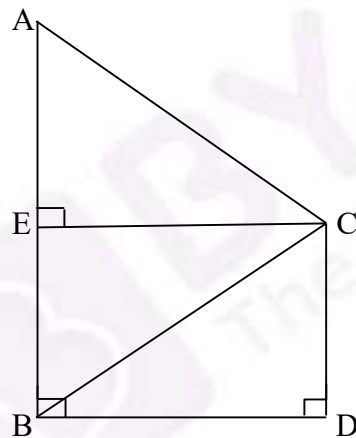
(iv) Dividend from the original shares = $80 \times 100 \times \frac{8}{100} = ₹640$

Dividend from the new shares = $320 \times 50 \times \frac{10}{100} = ₹1600$

\therefore Change in the two dividends = $₹1600 - ₹640 = ₹960$

Question 9

- (a) The horizontal distance between two towers is 120m. The angle of elevation of the top and angle of depression of the bottom of the first tower as observed from the second tower is 30° and 24° respectively.



Find the height of the two towers. Give your answer correct to 3 significant figures. [4]

- (b) The weight of 50 workers is given below:

| Weight in Kg | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 | 100-110 | 110-120 |
|---------------|-------|-------|-------|-------|--------|---------|---------|
| No of Workers | 4 | 7 | 11 | 14 | 6 | 5 | 3 |

Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis and 2cm = 5 workers along the other axis. Use a graph to estimate the following:

- (i) the upper and lower quartiles.
- (ii) if weighing 95 Kg and above is considered overweight find the number of workers who are overweight. [6]

Examiners' Comments

- (a) The following errors were observed among different candidates (i) Incorrect value of $\tan 24^\circ$ or was unable to find its value at all; (ii) Many did not write their answer correct to 3 significant figures as given in the question; (iii) Some candidates rounded off $\tan 24^\circ$ as 0.445 or 0.44 and $\sqrt{3}$ as 1.73 or 1.7 initially and hence wrote an inaccurate answer.
- (b) A few common errors made by candidates in solving this sum are as follows: (i) scale not according to that given in question; (ii) Some made mistakes in finding the cumulative frequency. It is necessary to tally the last *cf i.e.* (50) with $\sum f = 50$ to avoid errors; (iii) Some used a ruler to join the plotted points instead of drawing a free hand curve. A *kink* was not shown between interval 0 and 50 since the next marking is 60; (iv) while using the graph to find median quartiles etc, the perpendicular lines on *x* and *y*-axis must be drawn; (v) Some candidates interchanged the *x*-axis and *y*-axis.

Suggestions for teachers

- Advise students to (a) read trigonometric tables; (b) read the question carefully and to specially note whether an approximation of answer is necessary; (c) round off their answers according to the question only at the final step of working so as to get the right answer.
- It must be noted that the ogive is a cumulative frequency curve and the plotted points must be joined freehand and not with a ruler.
- It is necessary to use the given scale and mark the axes accordingly. With intent drilling and supervision such errors may be avoided.

MARKING SCHEME

Question 9

$$\begin{aligned} \text{(a)} \quad \tan 24^\circ &= \frac{CD}{120} \text{ or } CD = 120 \tan 24 \\ &= 120 \times 0.4452 \\ &= 53.424 \\ &= 53.4\text{m} \end{aligned}$$

$$\tan 30^\circ = \frac{AE}{EC} = \frac{AE}{120}$$

$$\therefore AE = 120 \times \tan 30 = 120 \times \frac{1}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40 \times 1.732 = 69.28$$

$$\begin{aligned} \therefore \text{Tower } AB &= AE + EB = AE + CD = 69.28 + 53.4 = 122.68 \\ &= 123\text{m} \end{aligned}$$

$$\text{Tower } CD = 53.4\text{m}$$

(A)

$$\tan 24^\circ = \frac{CD}{120} \text{ or } CD = 120 \tan 24$$

$$= 120 \times 0.4452$$

$$= 53.424$$

$$= 53.4\text{m}$$

$$\tan 30^\circ = \frac{AE}{EC} = \frac{AE}{120}$$

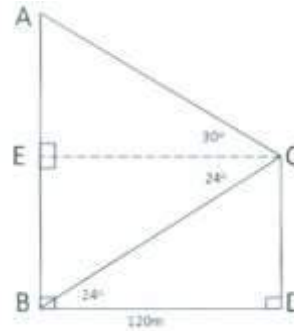
$$\therefore AE = 120 \times \tan 30 = 120 \times \frac{1}{\sqrt{3}} = \frac{120\sqrt{3}}{3}$$

$$= 40 \times 1.73 = 69.28$$

$$\therefore \text{Tower } AB = AE + EB = AE + CD = 69.28 + 53.4 = 122.68$$

$$= 123\text{m}$$

$$\text{Tower } CD = 53.4\text{m} \quad (\text{A}_1)$$



| (b) | Weight in Kg | Number of persons | C.f. |
|-----|--------------|-------------------|------|
| | 50 – 60 | 4 | 4 |
| | 60 – 70 | 7 | 11 |
| | 70 – 80 | 11 | 22 |
| | 80 – 90 | 14 | 36 |
| | 90 – 100 | 6 | 42 |
| | 100 – 110 | 5 | 47 |
| | 110 – 120 | 3 | 50 |

S-curve plotted with respect to upper boundaries and C.f.

$$(i) \quad Q_1 \text{ position} = \frac{1}{4} \times 50 = 12.5 \quad \therefore Q_1 = 71 \text{ kg } (\pm 1)$$

$$Q_3 \text{ position} = \frac{3}{4} \times 50 = 37.5 \quad \therefore Q_3 = 93 \text{ kg } (\pm 1)$$

Correct axis and perpendiculars dropped for result

$$(ii) \quad \text{Number of persons who are overweight is equal to}$$

$$50 - 38.5 = 11.5 (\pm 1) = 11 \text{ or } 12 \text{ approximately}$$

Question 10

(a) A wholesaler buys a TV from the manufacturer for ` 25,000. He marks the price of the TV 20% above his cost price and sells it to a retailer at a 10% discount on the marked price. If the rate of VAT is 8% , find the:

- (i) marked price.
- (ii) retailer's cost price inclusive of tax.
- (iii) VAT paid by the wholesaler.

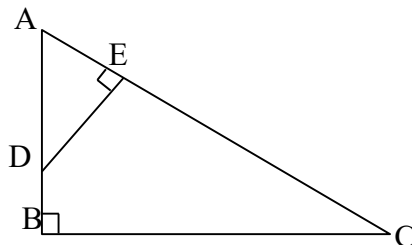
[3]

(b) If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

Find $AB - 5C$.

[3]

- (c) ABC is a right angled triangle with $\angle ABC = 90^\circ$. D is any point on AB and DE is perpendicular to AC . Prove that:-



- (i) $\triangle ADE \sim \triangle ACB$.
 (ii) If $AC = 13$ cm, $BC = 5$ cm and $AE = 4$ cm. Find DE and AD .
 (iii) Find, area of $\triangle ADE$: area of quadrilateral $BCED$.

[4]

Examiners' Comments

- (a) Some candidates calculated 10% discount on ₹25000 instead of the MP which is $\frac{120}{100} \times 25000 = ₹30000$. Hence the VAT found was also incorrect. A number of calculation errors were also identified. The wholesaler's VAT was incorrect due to lack of understanding.
- (b) Some candidates lost marks due to incorrect matrix multiplication. Some made sign errors in finding $-5C$ where $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$
- (c) Few candidates failed to prove $\triangle ADE \sim \triangle ACB$. Some were unable to write the proportional sides $\frac{DE}{BC} = \frac{AD}{AC} = \frac{AE}{AB}$, hence could not get the correct values of DE and AD . Some candidates took the ratio of area $\frac{\Delta ADE}{\Delta ABC} = \frac{AE}{AB}$ instead of $\frac{AE^2}{AB^2}$ hence the area of $\triangle ADE$ and quadrilateral $BCED$ was incorrect.

Suggestions for teachers

- Students must be advised to read the question carefully and try finding answers to each subpart by working out the sum one step at a time. This helps in avoiding errors.
- Adequate practice is necessary for various matrix operations.
- It is essential to explain similar triangles and explain that corresponding proportional sides are sides opposite to the corresponding equal angles. It is necessary to explain how the ratio of the areas of similar triangles is proportional to the square of the corresponding

MARKING SCHEME

Question 10

- (a) (i) Wholesaler's price is `25000
$$\text{M.P.} = 25000 + \frac{20}{100} \times 25000 = `30,000$$
- (ii) Retailer's Price = $30,000 - \frac{10}{100} \times 30,000 = `27,000$
$$\therefore \text{Price inclusive of tax} = 27,000 + \frac{8}{100} \times 27000 = ` (27000 + 2160)$$

$$= `29,160$$
- (iii) VAT paid by wholesaler
$$= (27000 - 25000) \times \frac{8}{100} = `160 \quad (\text{or by taking difference of two taxes})$$
- (b) $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$
$$AB - 5C = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 35 & 6 + 21 \\ 0 + 20 & 4 + 12 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

Question 11

- (a) Sum of two natural numbers is 8 and the difference of their reciprocal is $\frac{2}{15}$.
Find the numbers. [3]
- (b) Given $\frac{x^3 + 12x}{6x^2 + 8} = \frac{y^3 + 27y}{9y^2 + 27}$. Using componendo and dividendo find x: y. [3]
- (c) Construct a triangle ABC with AB = 5.5. cm, AC = 6 cm and $\angle BAC = 105^\circ$. Hence:
(i) Construct the locus of points equidistant from BA and BC.
(ii) Construct the locus of points equidistant from B and C.
(iii) Mark the point which satisfies the above two loci as P. Measure and write the length of PC. [4]

Examiners' Comments

- (a) Some candidates could not frame the equation. Some framed it correctly but failed to solve it as they were unable to factorize $2x^2 + 7x - 60 = 0$.
- (b) Candidates were unable to apply the property of componendo and dividendo and hence solved the sum incorrectly. The correct application is $\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2-8} = \frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$; simplify and get the result. Many failed to write the answer in the simplest form, e.g., $\frac{2x}{4} = \frac{2y}{6}$ is $\frac{x}{2} = \frac{y}{3} \therefore x:y = 2:3$. Some were unable to identify $x^3 + 12x + 6x^2 + 8 = (x + 2)^3$.
- (c) In the construction of 105° ruler and compass was not used. The length of AB and AC was incorrect and must be drawn carefully. Some candidates bisected their required line and angle incorrectly and hence were unable to construct the right figure. Correct the answer to the nearest whole number.

Suggestions for teachers

- Emphasis on solving quadratic equation application based problems are necessary.
- Adequate practice of all properties of ratio and proportion is essential. The basic formula for $(a \pm b)^2, (a \pm b)^3$ must be recapitulated periodically.
- Instruct students to read the question carefully so that they do not miss any part of the question. The basic two concepts that (i) point equidistant from two arms of an angle lies on the bisector of the angle (ii) a point equidistant from two fixed points lies on the perpendicular bisector of the line segment joining the two fixed points must be explained thoroughly.

MARKING SCHEME

Question 11

- a) Let the number be x and $8 - x$

$$\frac{1}{8-x} - \frac{1}{x} = \frac{2}{15}$$

$$\therefore \frac{x-(8-x)}{x(8-x)} = \frac{2}{15}$$

$$\therefore (2x - 8) \times 15 = 2(8x - x^2)$$

$$\therefore 30x - 120 = 16x - 2x^2$$

$$2x^2 + 14x - 120 = 0$$

OR $x^2 + 7x - 60 = 0$

$$(x + 12)(x - 5) = 0$$

$$\therefore x = 5 \quad (x = -12 \text{ not possible})$$

\therefore The number are 3 and 5

b) $\frac{x^3+12x}{6x^2+8} = \frac{y^3+27y}{9y^2+27}$

By componendo & dividendo

$$\frac{x^3+12x+6x^2+8}{x^3+12x-6x^2+8} = \frac{y^3+27y+9y^2+27}{y^3+27y-9y^2-27}$$

$$\frac{(x+2)^3}{(x-2)^3} = \frac{(y+3)^3}{(y-3)^3} \quad \therefore \frac{x+2}{x-2} = \frac{y+3}{y-3}$$

$$\therefore \frac{x+2+x-2}{x+2-x+2} = \frac{y+3+y-3}{y+3-y+3}$$

$$\text{Hence } \frac{2x}{4} = \frac{2y}{6}$$

$$\therefore x : y = 2 : 3$$

c) $\angle BAC = 105^\circ$

ΔABC

Bisector $\angle B$

Bisector of BC or

$$PC = 5\text{cm}$$

Topics/Concepts Found Difficult

- Value Added Tax (VAT)
- Compound Interest inverse problems.
- Trigonometry
- Similarity
- Rounding off final result e.g. significant figures.
- Theorems on properties of circle.
- Properties of proportion.
- Constructing a circle about a constructed Hexagon.
- Calculation of mean
- Coordinate geometry, Section formula and identifying points on x or y axis.
- Quadratic equation problem
- Inequation: writing solution and representation on number line

Suggestions for Candidates

- Reading time must be utilized to make the right choice of questions and make oneself familiar with all given data
- More practice must be done on rounding off of digits
- Use graph paper for questions based on graphs
- Use of log table to find square root of numbers
- Avoid skipping steps. All necessary steps must be clearly shown
- Working for matrix multiplication is essential
- Rounding off of decimals
- Adopt methods where lesser calculation is necessary to get final result
- Necessary sample space must be written for probability problems.
- Steps of working is necessary in conversion of trigonometric ratios of complementary angles.
- Reasons must be provided for all geometry problems.

