Question 1: Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140$ m, then m is equal to

(a) 68 (b) 34 (c) 33 (d) 66

Answer: (b)

Solution:
We know, nth term of A.P. is $a_n = a + (n - 1) d$

$a_9 + a_{43} = 66$

Therefore, a + 8d + a + 42d = 66

Or a + 25d = 33 ……(1)

Now, $\sum_{k=0}^{12} a_{4k+1} = 416$

Therefore, 13a + 312d = 416

Or a + 24d = 32 …(2)

Solving (1) and (2), we get

a = 8 and d = 1

So,

$\sum_{k=1}^{17} a_k^2 = 8^2 + 9^2 + \ldots + 24^2$

$= (1^2 + 2^2 + \ldots + 24^2) - (1^2 + 2^2 + \ldots + 7^2)$

Using sum of squares of n natural numbers formula, we have

= [24x25x49]/6 – [7x8x15]/6

= 4750 = 140 x 34

Thus, answer is 34.

Question 2: Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is −1/2 , then the greatest number amongst them is

(a) 16 (b) 27 (c) 7 (d) 21/2

Answer: (a)
Solution:
Let 5 numbers be a − 2d, a − d, a, a + d, a + 2d
so, Sum of numbers = 5a = 25 or a = 5
Product of Numbers = (a − 2d)(a − d)a(a + d)(a + 2d) = 2520 (25 − 4d2)(25 − d2) = 504
=> $4d^4 - 125d^2 + 121 = 0$
=> $4d^4 - 4d^2 - 121d^2 + 121 = 0$
=> $d^2 = 1$ or $d^2 = 121/4$
=> d = ± 11/2
For d = 11/2, a + 2d is the greatest term, a + 2d = 5 + 11 = 16

Question 3: Let f: R -> R be such that for all x ∈ R, $(2^{1+x} + 2^{1-x})$, f(x) and $(3^x + 3^{-x})$ are in A.P., then the minimum value of f(x) is :

(a) 0 (b) 4 (c) 3 (d) 2

Answer: (c)

Solution:
Given: $(2^{1+x} + 2^{1-x})$, f(x) and $(3^x + 3^{-x})$ are in A.P.
Therefore,

$$f(x) = \frac{3^x + 3^{-x} + 2^{1+x} + 2^{1-x}}{2} = \frac{(3^x + 3^{-x})}{2} + \frac{2^{1+x} + 2^{1-x}}{2}$$

Applying, A.M. ≥ G.M. inequality,

$$\frac{(3^x + 3^{-x})}{2} \geq \sqrt{3^x . 3^{-x}}$$

$$\Rightarrow \frac{(3^x+3^{-x})}{2} \geq 1 \quad \dots (1)$$

By A.M. ≥ G.M. inequality,

$$\frac{2^{1+x} + 2^{1-x}}{2} \geq \sqrt{2^{1+x} . 2^{1-x}}$$

$$\Rightarrow \frac{2^{1+x} + 2^{1-x}}{2} \geq 2 \quad \dots (2)$$

Add (1) and (2)
f(x) ≥ 1 + 2 = 3
Thus, minimum value of f(x) is 3. Answer!

Question 4: If the 10th term of an A.P. is 1/20 and its 20th term is 1/10 , then the sum of its first 200 terms is:
(a) 201/4 (b) 100 (c) 50 (d) 201/2

Answer: (d)

Solution:
10th term of an A.P. is 1/20 and its 20th term is 1/10.
So, $a_{10} = 1/20$ and $a_{20} = 1/10$
Now, $a_{20} - a_{10} = 10$ d
or d = 1/200 and a = 1/200
Now,
Sum of first 200 terms:
$S_{200} = 200/2\ [2(1/100) + 199(1/200)]$
= 201/2

Question 5: If three positive numbers a, b and c are in A.P. such that abc = 8, then the minimum possible value of b is :

(a) 2 (b) $4^{1/3}$ (c) 8 (d) 4

Answer: (a)

Solution:
For a set of positive numbers, the AM ≥ GM.
Given: abc = 8
Since the numbers are in AP, their arithmetic mean is b.
The geometric mean of the three numbers:
(a+b+c)/3 = b
=> $b \geq (abc)^{1/3}$
Therefore, the minimum possible value of b is obtained as b ≥ 2 .

Question 6: Let $a_1, a_2, a_3, \ldots\ldots a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, ...,11
If $[a_1^2 + a_2^2 + \ldots. + a_{11}^2]/11 = 90$ then the value of $[a_1 + a_2 + \ldots. + a_{11}]/11$

Solution:
First term $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4, ...,11
And $[a_1^2 + a_2^2 + \ldots. + a_{11}^2]/11 = 90$
$a_{k-1} = [a_k + a_{k-2}]/2$
Now,
Let the numbers are, $a_6 + 5d$, $a_6 + 4d$, …… a_6, …….., $a_6 - 5d$

$11a_6^2 + 110d^2 = 990$
$a_6 = 15 - 5d$
$a_6^2 + 10d^2 = 90$
Now, $(15 - 5d)^2 + 10d^2 = 90$
$\Rightarrow d = 3, 9/7$
For $d = 3 \Rightarrow a_2 = 12$
For $d = 9/7 \Rightarrow a_2 = 13.7$ (not possible, as $a_2 < 27/2$)
Thus, $[a_1 + a_2 + ... + a_{11}]/11 = (11/2) \times [30-10(3)]/11 = 0$

Question 7: In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression is

(a) $[1-\sqrt{5}]/2$ (b) $\sqrt{5}/2$ (c) $[\sqrt{5} - 1]/2$ (d) $\sqrt{5}$

Answer: (c)

Solution:
Let a geometric progression consisting of positive terms, a, ar, ar^2
Then, $a = ar + ar^2$
$\Rightarrow r^2 + r - 1 = 0$
Solving equation using quadratic formula, we get
$r = [\sqrt{5} - 1]/2$

Question 8: A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs.40 more than the saving of immediately previous month. After how many months his total savings from the start of service will be Rs. 11040 after:
(a) 18 months
(b) 19 months
(c) 20 months
(d) 21 months
Answer: (d)

Solution:
From the question, saving are 200, 200, 200, 240, 280,........ upto n terms.

But 200, 240, 280,........(n-2) terms are in AP.

Using sum of n terms of an AP, we have

$400 + (n-2)/2\ [2\times200 + (n - 2-2)40] = 11040$

$\Rightarrow (n-2)[200 + 20n - 60] = 10640$

=> 20(n + 7)(n - 2) = 10640

=> $n^2 + 5n - 546 = 0$

=> (n + 26)(n - 21) = 0

-ve value is not possible.

n = 21

Therefore, total time = 21 months

Question 9: If 100 times the 100th term of an AP with non-zero common difference equal to the 50 times its 50th term, then the 150th term of this AP is
(a) -150
(b) 150 times its 50th term
(c) 0
(d) 150

Answer: (c)

Solution:
Given 100 times the 100th term of an AP = 50 times its 50th term.
$100 \times T_{100} = 50 \times T_{50}$
=> 100(a + 99d) = 50(a + 49d)
=> 2a + 198d = a + 49d
=> a + 149d = 0
=> $T_{150} = 0$

Question 10: If the nth term of an AP be (2n-1), then find the sum of its first n terms.
Solution:
Let $a_n = (2n-1)$
$a_1 = 2 \times 1 - 1 = 1$
$a_2 = 4 - 1 = 3$
Now, $d = a_2 - a_1 = 3 - 1 = 2$
Sum of first n terms = (n/2) [2 + 2n - 2]
= n^2

Question 11: The sum of the infinity of the series $1 + 2/3 + 6/3^2 + 10/3^3 + 14/3^4 + \ldots$ is
(a) 2 (b) 3 (c) 4 (d) 5
Answer: (b)

Solution:

Let $S = 1 + 2/3 + 6/3^2 + 10/3^3 + 14/3^4 + \ldots$ ...(1)

$(1/3)S = 1/3 + 2/3^2 + 6/3^3 + 10/3^4 + 14/3^5 + \ldots$ ....(2)

Diving (1) and (2), we get

$(2/3)S = 4/3 + 4/3^2(1 + 1/3 + 1/3^2 + \ldots)$ ...(3)

We know, $1 + 1/3 + 1/3^2 + \ldots = 1/[1-1/3] = 3/2$

$(2/3)S = 4/3 + 4/3^2 \times 3/2 = 6/3$

$\Rightarrow S = 3$

Question 12: The pth , qth and rth terms of an A.P are a, b,c respectively. Show that $(q - r)a + (r - p)b + (p - q)c = 0$

Solution:

Let a, a + d, a+ 2d, ..are in A.P.

pth term = $a + (p-1)d = a$

qth term = $a + (q - 1)d = b$ and

rth term = $a + (r - 1)d = c$

L.H.S. = $(q - r)a + (r - p)b + (p - q)c$

$= (q - r)(a + (p-1)d) + (r - p)(a + (q - 1)d) + (p - q)(a + (r - 1)d)$

Solving above equation, we have

$a(q - r) + b(r - p) + c(p - q)$

$= a[(q - r) + (r - p) + (p - q)] + d[(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)]$

$= a(0) + d(0)$

$= 0$

Hence Proved.