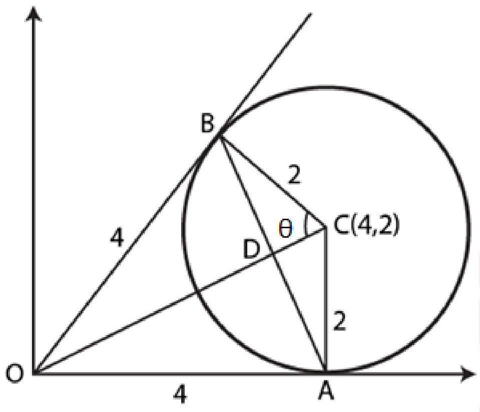
Question 1: Let the tangents drawn from the origin to the circle $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the point A and B. The $(AB)^2$ is equal to

(a) 32/5 (b) 64/5 (c) 52/5 (d) 56/5

Answer: (b)

Solution:
$x^2 + y^2 - 8x - 4y + 16 = 0$
Rearranging above equation, we get
$(x - 4)^2 + (y - 2)^2 = 4$
Centre = (4, 2) and
Radius = 2



OA = OB = 4
In triangle, OBC,
$\tan \theta = 4/2 = 2$
and $\sin\theta = 2/\sqrt{5}$
In triangle, BDC
$\sin\theta = BD/2 \Rightarrow BD = 4/\sqrt{5}$
Length of chord of contact = AB = $8/\sqrt{5}$

Question 2: The circle passing through (1, -2) and touching the axis of x at (3, 0) also passes through the point:

(a) (–5, 2) (b) (2, –5) (c) (5, –2) (d) (–2, 5)

Answer: (c)

Solution: The equation of circle passing through the point (a,b) and having radius r is $(x - a)^2 + (y - b)^2 = r^2$

Since given circle touches the x-axis at (3, 0) and passes through the point (1, -2).
Find radius of the circle using distance formula,
So, $(1 - 3)^2 + (r + 2)^2 = r^2$
$4 + r^2 + 4 + 4r - r^2 = 0$
$\Rightarrow r = 2$
So, circle is $(x - 3)^2 + (y + 2)^2 = 4$
Point (5, -2) satisfy the equation.

Question 3: Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to :

(a) $\frac{\sqrt{3}}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Answer: (d)

Solution:
C be the circle with centre at (1, 1) and radius = 1.
T is the circle centred at (0, y), passing through origin and touching the circle C externally.
Let r_1 and r_2 be the radius of two circles.
$CT = r_1 + r_2 = \sqrt{[(1+(1-y)^2]} = 1 + y$
Where "y" be the radius of circle T.
Squaring both sides, we have
$1 + 1 + y^2 - 2y = 1 + y^2 + 2y$
$\Rightarrow y = \frac{1}{4}$

Question 4: The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on,
(a) A parabola
(b) An ellipse which is not a circle
(c) A circle
(d) A hyperbola
Answer: (a)

Solution:
The general equation of the circle with center (h, k) and radius r is, $(x - h)^2 + (y - k)^2 = r^2$ ...(1)
$r_1 = \sqrt{[g^2+f^2-c]} = \sqrt{[16+16+4]} = \sqrt{36} = 6$
Where r_1 is the radius of given circle.
Since circle (1) touch the given circle.
$(h - 4)^2 + (k - 4)^2 = (6 + k)^2$

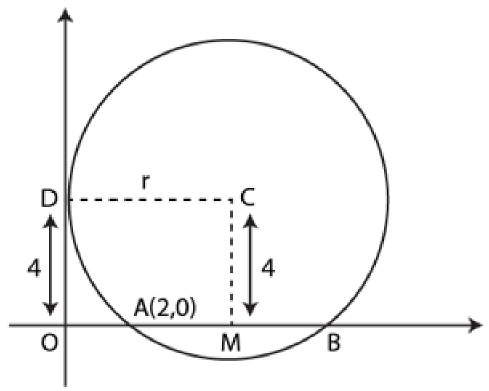
Solving above equation, we have
$(h - 4)^2 = 20k + 20$
Replace (h, k) by (x, y)
$(x - 4)^2 = 4(5y + 5)$
Which is locus of parabola.

Question 5: If a circle "C" passing through any point (4, 0) touches the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ externally at a point (1,-1), then what will be the radius of the circle.

Solution:
We know, equation of family of circles: $(x - 1)^2 + (y + 1)^2 + \lambda(x^2 + y^2 + 4x - 6y - 12) = 0$ ...(1)
(4, 0) satisfy the equation as it passes through it.
$\Rightarrow 9 + 1 + \lambda(16 + 16 - 12) = 0$
$\Rightarrow \lambda = -1/2$
Therefore, (1)$\Rightarrow x^2 + y^2 - 2x + 2y + 2 - 1/2\,(x^2 + y^2 + 4x - 6y - 12) = 0$
$\Rightarrow x^2 + y^2 - 8x + 10y + 16 = 0$
Now, Radius, $r = \sqrt{[16+25-16]} = \sqrt{25} = 5$

Question 6: A circle touches the y-axis at the point (0, 4) and passes through point (2,0). which of the following lines is not a tangent to the circle?
(a) $4x - 3y + 17 = 0$
(b) $3x + 4y - 6 = 0$
(c) $4x + 3y - 8 = 0$
(d) $3x - 4y - 24 = 0$
Answer: (c)

Solution:



From figure,
$OD^2 = OA \times OB$
$\Rightarrow 16 = 2 \times OB$
$\Rightarrow OB = 8$
So, AB = 6
AM = 3 and CM = 4

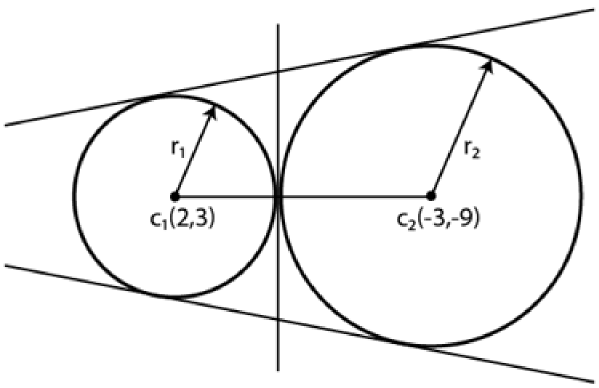
=> CA = 5
Therefore, OM = 5
Now, we have center = (5, 4) and radius = 5
Check for all the given options.
4x + 3y - 8 = 0 is not a tangent.
so, $[20+12-8]/\sqrt{[3^2+4^2]} = 24/5$

Question 7: The number of common tangents to the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is :

(a) 3 (b) 4 (c) 1 (d) 2

Answer: (a)

Solution:

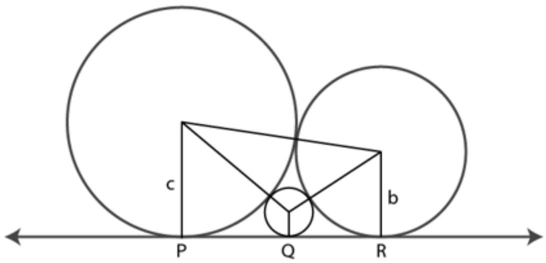


From diagram,
$C_1C_2 = 14$
(Using distance formula)
Now, $r_1 = \sqrt{[4+9+12]} = 5$ and $r_2 = \sqrt{[9+81-26]} = 8$
$C_1 C_2 = r_1 + r_2$ is case of external touching. So, number of common tangents can be drawn = 3.

Question 8: Three circles of radii, a, b, c (a<b<c) touch each other externally and have x−axis as a common tangent then
(a) a, b, c are in A.P.
(b) $1/\sqrt{b} = 1/\sqrt{a} + 1/\sqrt{c}$
(c) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.
(d) $1/\sqrt{a} = 1/\sqrt{b} + 1/\sqrt{c}$

Answer: (d)

Solution:



From figure,
PQ + QR = PR

$$\sqrt{(c+a)^2-(c-a)^2}+\sqrt{(b+a)^2-(b-a)^2}=\sqrt{(b+c)^2-(c-b)^2}$$

$$\sqrt{4ac}+\sqrt{4ab}=\sqrt{4bc}$$

Dividing each side with √(4abc)
=> 1/√b + 1/√c = 1/√a

Question 9: If the circle $x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$ touches the axis of x, then "a" equals : -
(a) ± 4 (b) ±3 (c) 0 (d) ±2

Answer: (a)

Solution:

Circle $x^2 + y^2 - 6x - 8y + (25 - a^2) = 0$ touches the axis of x.
On rearranging given equation, we have
$(x^2 - 6x + 9) + (y^2 - 8y + 16) = a^2$
$\Rightarrow (x - 3)^2 + (y - 4)^2 = a^2$ ...(1)
We know the general equation of circle passing through (a, b) having radius "r" is: $(x - a)^2 + (y - b)^2 = r^2$
Comparing with (1), we get
a = 3, b = 4 and r = ±a
Since, circle touches the axis of x, then r = b = 4
Therefore, Radius = r = ± 4

Question 10: If the curves $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0, (k > 0)$ touch each other at a point, then the largest value of k is __________.

Solution: Two circles touch each other if $C_1C_2 = |r_1 \pm r_2|$
$\sqrt{k} + 1 = 5$ or $|\sqrt{k} - 1| = 5$
This implies, k = 16 or 36
Maximum value of k is 36

Question 11: Let the orthocentre and centroid of a triangle be A(-3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:
(a) $2\sqrt{10}$ (b) $3\sqrt{(5/2)}$ (c) $3\sqrt{5}/2$ (d) $\sqrt{10}$

Answer:(b)

Solution: Let C be any point having coordinates (h, k).
Using section formula, we have
$[2h+(-3)]/3 = 3$, $[2k+5]/3 = 3$
=> h = 6 and k = 2
So, point C is (6, 2).
Here, Point A is (-3, 5)
=> Diameter = AC = $\sqrt{(9^2+3^2)} = 3\sqrt{10}$
Therefore, radius = diameter/2 = $3\sqrt{10}/2 = 3\sqrt{(5/2)}$ units

Question 12: If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is
(a) 79 (b) 83 (c) 95 (d) 105

Answer: (c)

Solution:
Given equation is $x^2 = y - 6$
Or $y = x^2 + 6$
Find slope of tangent at (1, 7)
=>Let m = dy/dx = 2x
=>m = 2x 1 = 2
Equation of tangent at the point (1,7) is
$(y - 7) = 2(x - 1)$
=>$y - 7 = 2x - 2$
=> $2x - y + 5 = 0$
Given equation of circle can be rearranged as:
$x^2 + y^2 + 16x + 12y + c = 0 \Rightarrow (x + 8)^2 + (y + 6)^2 + c - 64 - 36 = 0$
=>$(x + 8)^2 + (y + 6)^2 = 100 - c$
If tangent touches the circle, then radius is :
Distance of (-8, -6) from $2x - y + 6 = 0$

So, d = $\left|\frac{2(-8)-(-6)}{\sqrt{4+1}}\right| = \left|\sqrt{5}\right|$

Radius = $\sqrt{(100-c)} = \sqrt{5} \Rightarrow c = 95$