Question 1: The equation of $2x^2 + 3y^2 - 8x - 18y + 35 = k$ represents a ________.

Solution:

Given equation, $2x^2 + 3y^2 - 8x - 18y + 35 - k = 0$

Compare with $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$, we get

$a = 2, b = 3, h = 0, g = -4, f = -9, c = 35 - k$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 6(35 - k) + 0 - 162 - 48 - 0$

$\Delta = 210 - 6k - 210 = -6k$;

$\Delta = 0$, if $k = 0$

So, that given equation is a point if $k = 0$.

Question 2: The locus of the midpoint of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with the directrix ___________.

Solution:

$\alpha = [at^2 + a] / 2, \beta = [2at + 0] / 2$

$\Rightarrow 2\alpha = at^2 + a, at = \beta$

$2\alpha = a[\beta^2 / a^2] + a$ or

$2a\alpha = \beta^2 / a^2$

The locus is $y^2 = 4a / 2 (x - [a / 2])$

$= 4b (x - b), (b = a / 2)$

Directrix is $(x - b) + b = 0$ or $x = 0$.

Question 3: On the parabola $y = x^2$, the point least distance from the straight line $y = 2x - 4$ is

___________.

Solution:

Given, parabola $y = x^2$ .....(i)

Straight line $y = 2x - 4$ .....(ii)

From (i) and (ii),

$x^2 - 2x + 4 = 0$

Let $f(x) = x^2 - 2x + 4$,

$f'(x) = 2x - 2$.

For least distance, $f'(x) = 0$

$\Rightarrow 2x - 2 = 0$

$x = 1$

From $y = x^2$, $y = 1$

So, the point least distant from the line is (1, 1).

Question 4: The line $x - 1 = 0$ is the directrix of the parabola, $y^2 - kx + 8 = 0$. Then, one of the values of k is ________.

Solution:

The parabola is $y^2 = 4 * [k / 4] (x - [8 / k])$.

Putting $y = Y$, $x - 8k = X$, the equation is $Y^2 = 4 * [k / 4] * X$

The directrix is $X + k / 4 = 0$,i.e. $x - 8 / k + k / 4 = 0$.

But $x - 1 = 0$ is the directrix.

So, $[8 / k] - k / 4 = 1$

$\Rightarrow k = -8, 4$

Question 5: The centre of the circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is ____________.

Solution:

Tangent to the parabola $y = x^2$ at (2, 4) is $[1 / 2] (y + 4) = x * 2$ or

$4x - y - 4 = 0$

It is also a tangent to the circle so that the centre lies on the normal through (2, 4) whose equation is $x + 4y = \lambda$, where $2 + 16 = \lambda$.

Therefore, $x + 4y = 18$ is the normal on which lies (h, k).

$h + 4k = 18$ ..... (i)

Again, distance of centre (h, k) from (2, 4) and (0, 1) on the circle are equal.

Hence, $(h - 2)^2 + (k - 4)^2 = h^2 + (k - 1)^2$

So, $4h + 6k = 19$ .....(ii)

Solving (i) and (ii), we get the centre = (−16 / 5, 53 / 10)

Question 6: Find the equation of the axis of the given hyperbola $x^2/3 - y^2/2 = 1$ which is equally inclined to the axes.

Solution:

$x^2/3 - y^2/2 = 1$

Equation of tangent is equally inclined to the axis i.e., tan θ = 1 = m.

Equation of tangent y = mx + $\sqrt{a^2m^2 - b^2}$

Given equation is $x^2/3 - y^2/2 = 1$ is an equation of hyperbola which is of form $[x^2/a^2] - [y^2/b^2] = 1$.

Now, on comparing $a^2 = 3$, $b^2 = 2$

y = 1 * x + $\sqrt{3 \times (1)^2 - 2}$

y = x + 1

Question 7: If $4x^2 + py^2 = 45$ and $x^2 - 4y^2 = 5$ cut orthogonally, then the value of p is ______.

Solution:

Slope of 1st curve $(dy / dx)_I = -4x / py$

Slope of 2nd curve $(dy / dx)_{II} = x / 4y$

For orthogonal intersection (−4x / py) (x / 4y) = −1

$x^2 = py^2$

On solving equations of given curves x = 3, y = 1

p (1) = $(3)^2$ = 9

p = 9

Question 8: If the foci of the ellipse $x^2 / 16 + y^2 / b^2 = 1$ and the hyperbola $x^2 / 144 - y^2 / 8 = 1 / 25$ coincide, then the value of b^2 is _______.

Solution:

Hyperbola is $x^2 / 144 - y^2 / 8 = 1/25$

a = $\sqrt{\frac{144}{25}}$, b = $\sqrt{\frac{81}{25}}$ and

$e_1 = \sqrt{1 + \left(\frac{81}{144}\right)} = 5/4$

Therefore, foci = (ae_1, 0) = ([12 / 5] * [5 / 4], 0) = (3, 0)

Therefore, focus of ellipse = (4e, 0) i.e. (3, 0)

Hence, $b^2 = 16 (1 - [9 / 16]) = 7$

Question 9: Let E be the ellipse $x^2 / 9 + y^2 / 4 = 1$ and C be the circle $x^2 + y^2 = 9$. Let P and Q be the points (1, 2) and (2, 1), respectively. Then

A) Q lies inside C but outside E

B) Q lies outside both C and E

C) P lies inside both C and E

D) P lies inside C but outside E

Solution:

The given ellipse is $[x^2 / 9] + [y^2 / 4] = 1$. The value of the expression $[x^2 / 9] + [y^2 / 4] - 1$ is positive for x = 1, y = 2 and negative for x = 2, y = 1. Therefore, P lies outside E and Q lies inside E. The value of the expression $x^2 + y^2 - 9$ is negative for both the points P and Q. Therefore, P and Q both lie inside C. Hence, P lies inside C but outside E.

Question 10: The equation of the director circle of the hyperbola $[x^2 / 16] - [y^2 / 4] = 1$ is given by _______.

Solution:

Equation of the director circle of hyperbola is $x^2 + y^2 = a^2 - b^2$. Here $a^2 = 16$, $b^2 = 4$

Therefore, $x^2 + y^2 = 12$ is the required director circle.

Question 11: If m_1 and m_2 are the slopes of the tangents to the hyperbola $x^2 / 25 - y^2 / 16 = 1$ which pass through the point (6, 2), then find the relation between the sum and product of the slopes.

Solution:

The line through (6, 2) is $y - 2 = m (x - 6)$

$y = mx + 2 - 6m$

Now from condition of tangency, $(2 - 6m)^2 = 25m^2 - 16$

$36m^2 + 4 - 24m - 25m^2 + 16 = 0$

$11m^2 - 24m + 20 = 0$

Obviously, its roots are m_1 and m_2, therefore $m_1 + m_2 = 24 / 11$ and $m_1 m_2 = 20 / 11$.

Question 12: The eccentricity of the curve represented by the equation $x^2 + 2y^2 - 2x + 3y + 2 = 0$ is __________.

Solution:

Equation $x^2 + 2y^2 - 2x + 3y + 2 = 0$ can be written as $(x - 1)^2 / 2 + (y + 3 / 4)^2 = 1 / 16$

$[(x - 1)^2] / (1/8) + [(y + 3 / 4)^2] / (1/16) = 1$, which is an ellipse with $a^2 = 1 / 8$ and $b^2 = 1 / 16$

Therefore, $1 / 16 = 1 / 8 (1 - e^2)$

$e^2 = 1 - 1 / 2$

$e = 1 / \sqrt{2}$

Question 13: The foci of the ellipse $25 (x + 1)^2 + 9 (y + 2)^2 = 225$ are at ___________.

Solution:

$25 (x + 1)^2 + 9 (y + 2)^2 = 225$

Or $25(x + 1)^2 /225 + 9(y + 2)^2/225 = 1$

$a = \sqrt{[225 / 25]} = 15 / 5$, $b = \sqrt{[225/9]} = 15 / 3$

$e = \sqrt{[1 - 9 / 25]} = 4 / 5$

Focus = $(-1, -2 \pm [15 / 3] * [4 / 5])$

$= (-1, -2 \pm 4)$

$= (1, 2); (1, 6)$

Question 14: The locus of a variable point whose distance from (2, 0) is 2/3 times its distance from the line x = −9 / 2, is __________.

Solution:

Let point P (x_1, y_1)

So, $\sqrt{(x_1+2)^2+y_1^2}$ = 2/3 $(x_1 + 9 / 2)$

$(x_1 + 2)^2 + y^2{}_1 = 4 / 9\ (x_1 + 9 / 2)^2$

$9\ [x_1^2 + y^2{}_1 + 4x1 + 4] = 4(x_1^2 + 81 / 4 + 9x_1)$

$5x_1^2 + 9y^2{}_1 = 45$

$x_1^2 / 9 + y^2{}_1 / 5 = 1$,

Locus of (x_1, y_1) is $x^2 / 9 + y^2 / 5 = 1$, which is the equation of an ellipse.

Question 15: The equation of the ellipse whose latus rectum is 8 and whose eccentricity is 1/√2, referred to the principal axes of coordinates, is __________.

Solution:

$2b^2 / a = 8$, $e = 1/\sqrt{2}$

$a^2 = 64$, $b^2 = 32$

Hence, the required equation of ellipse is $x^2/64 + y^2/32 = 1$.