Question 1: The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the point

(a) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$

(b) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$

(c) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$

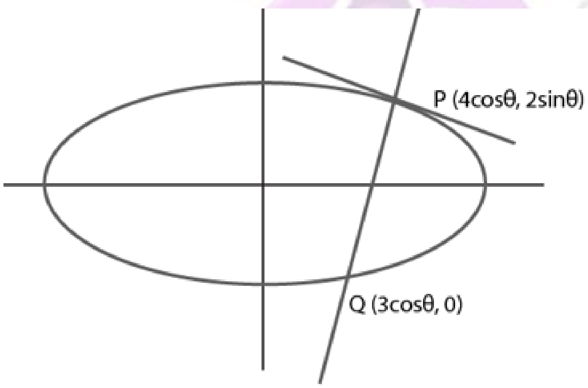
(d) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Answer: (c)

Solution:

Equation of ellipse $x^2 + 4y^2 = 16$ or

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$



Form figure,

$4x \sec\theta - 2y \operatorname{cosec}\theta = 12$ and $x = 3\cos\theta$

Coordinates of point Q: $(3\cos\theta, 0)$

Also, 2h = 7 cos θ and 2k = 2 sin θ

=> $\frac{4x^2}{49} + \frac{y^2}{1} = 1$

Latus rectum = x = 2√3

and y = ± 1/7

Therefore, required point is (± 2√3, ±1/7)

Question 2: The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn in inscribed in another ellipse that passes through the point (4, 0). Find the equation of the ellipse.

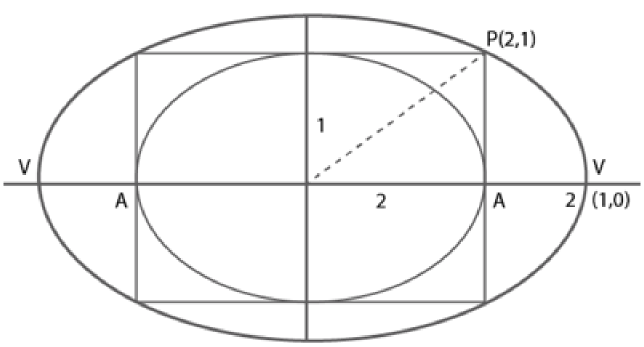
Solution:

Given equation of ellipse : $x^2 + 4y^2 = 4$

=> $x^2/4 + y^2/1 = 1$

Here a = 2 and b = 1

We know the general equation of ellipse is $x^2/a^2 + y^2/b^2 = 1$



$4/16 + 1/b^2$

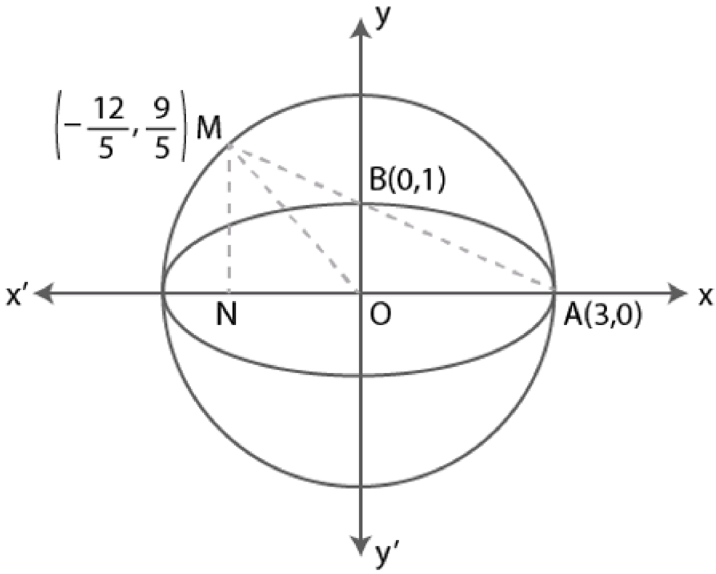
=> $b^2 = 4/3$

Therefore, $x^2/16 + y^2/(4/3) = 1$

=> $x^2/16 + 3y^2/4 = 1$

=> $x^2 + 12 y^2 = 16$

Question 3: The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets its auxiliary circle at the point M. Find the area of the triangle with vertices at A, M and the origin O.



Solution:
Equation of AM: y = -1/3(x + 3)

or x + 3y + 3 = 0, Slope of line = -1/3

So, y - 1 = -1/3(x - 0) => 3y + x - 3 = 0

=> Equation of AM is 3y + x - 3 = 0, which is chord of auxiliary circle, $x^2 + 9y^2 = 9$.

Coordinates of point A(3,0) and B(0, 1).

Distance of AM from the origin = |(0+0-3)/(√(9+1)| = 3/√10
The equation of an auxiliary circle of an ellipse is $x^2 + y^2 = 9$

From figure, line AM cuts auxiliary circle at M.

By solving these equations, we have M(-12/5, 9/5) co-ordinates of point M.

Area of triangle AOM = 1/2 |0(0 – 9/5) + 3(9/5 – 0) – 12/5(0-0)|
= 27/10

Question 4: If 3x + 4y = 12√2 is a tangent to the ellipse $x^2/a^2 + y^2/9 = 1$, for some a ∈ R then the distance between the foci of the ellipse is:
(a) 2√5 (b) 2√7 (c) 2√2 (d) 4

Answer: (b)
Solution:
Here 3x + 4y = 12√2 is the tangent to ellipse.
=> y = -3x/4 + 3√2 ..(1)

We know, the equation of tangent to ellipse $x^2/a^2 + y^2/b^2 = 1$ is $y = mx + \sqrt{(a^2m^2 + b^2)}$

Here $b^2 = 9$

So, equation of tangent to given ellipse is $y = mx + \sqrt{(a^2m^2 + 9)}$

Now, using (1), we have
m = -3/4 and $\sqrt{(a^2m^2 + 9)} = 3\sqrt{2}$

=> $(a^2m^2 + 9) = 18$

=> $(a^2(-3/4)^2 + 9) = 18$

=> $a^2 \times 9/16 = 9$

=> a = 4

Now, $e^2 = 1 – b^2/a^2 = 1 - 9/16 = 7/16$

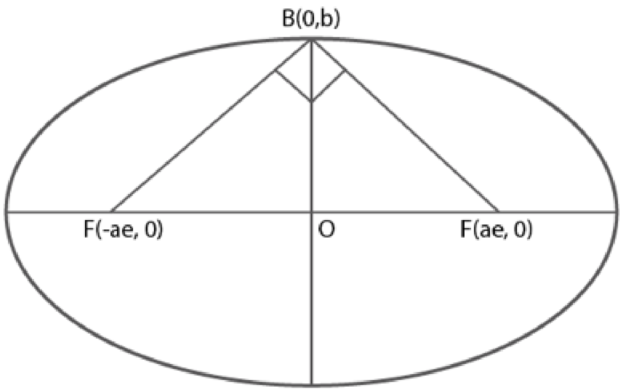
Or e = √7/4

The distance between foci is 2ae = 2 x 4 x √7/4 = 2√7

Question 5: An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

(a) 1/√2 (b) 1/2 (c) 1/4 (d) 1/√3

Solution:



The angle FBF' is a right angle.

Therefore,

$$(\sqrt{a^2e^2+b^2})^2 + (\sqrt{a^2e^2+b^2})^2 = (2ae)^2$$

=> $2(a^2e^2 + b^2) = 4a^2e^2$

=> $e^2 = b^2/a^2$

We know, $e^2 = 1 - b^2/a^2 = 1 - e^2$

=> $2e^2 = 1$

or e = 1/√2

Question 6: Find the equation of the ellipse whose axes are the axes of coordinates and which passes through the point (-3, 1) and has eccentricity √(2/5)

Solution:

Given, eccentricity = √(2/5)

$e^2 = 2/5$

we know, $e^2 = 1 - b^2/a^2$

=> $b^2/a^2 = 1 – 2/5 = 3/5$

=> $x^2/5k + y^2/3k = 1$, this equation passes through the point (-3, 1)

=> (9/5) + (1/3) = k

Or k = 32/15
Therefore, required equation of ellipse is $3x^2/32 + 5y^2/32 = 1$

Question 7: The eccentricity of an ellipse, with its centre at the origin is 1/2. If one of the directrices is x = 4, then what will be the equation of the ellipse.

Solution:
We know, equation of directrix is x = a/e = 4
=> a = 2
Now, $b^2 = a^2 (1 - e^2)$
=> $b^2 = 3$
Therefore, the equation of ellipse is $3x^2 + 4y^2 = 12$

Question 8: Tangents are drawn from the point P(3, 4) to the ellipse $x^2/9 + y^2/4 = 1$ touching the ellipse at points A and B
The coordinates of A and B are
(a) (3, 0) and (0, 2)
(b) (-8/5, 2√26/15) and (-9/8, 8/5)
(c) (-8/5, 2√16/15) and (0, 2)
(d) (3,0) and (-9/5, 8/5)

Answer: (d)

Solution:
Given equation of ellipse is $x^2/9 + y^2/4 = 1$ ...(1)
The equation of chord is
$xx_1/a^2 + yy_1/b^2 = 1$
3x/9 + 4y/4 = 1
=> x + 3y - 3 = 0
=> x = 3(-y + 1)

Now,
(1) =>
$\frac{9(-y+1)^2}{9} + \frac{y^2}{4} = 1$
=> $y^2 - 2y + 1 + y^2/4 = 1$
=> $5y^2 - 2y = 0$
=> $y(5y/4 - 2) = 0$
=> $y = 0$ or $y = 8/5$
So, $x = 3 - 3y$
At $y = 0$ => $x = 3$
At $y = 8/5$ => $x = -9/5$
Therefore, the required coordinates are (3, 0) and (-9/5, 8/5)

Question 9: In an ellipse, the distance between its foci is 6 and minor axis is 8. Then its eccentricity is
(a) 3/5 (b) ½ (c) 4/5 (d) 1/√5

Answer: (a)

Solution:
In an ellipse, the distance between its foci is 6 and minor axis is 8

The equation of ellipse,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \ a^2 > b^2$

and $b^2/a^2 = 1 - e^2$ ...(1)

Therefore, $ae/b = 3/4$

=> $e^2 = 9/16 \times b^2/a^2$

Using (1), we have

$e = 3/5$

Question 10: If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is

(a) $2\sqrt{3}$ (b) $\sqrt{3}$ (c) $3/\sqrt{2}$ (d) $3\sqrt{2}$

Answer: (d)
Solution:
Let the equation of ellipse be $x^2/a^2 + y^2/b^2 = 1$ $(a > b)$

Now, $2ae = 6$ and $2a/e = 12$

$\Rightarrow ae = 3$ and $a/e = 6$

$\Rightarrow a^2 = 18$

$\Rightarrow a^2e^2 = c^2 = a^2 - b^2 = 9$

$\Rightarrow b^2 = 9$

Length of latus rectum $= 2b^2/a$

$= 18/\sqrt{18}$

$= 3\sqrt{2}$