**Question 1:** If y = mx + 4 is a tangent to both the parabolas, $y^2 = 4a$ and $x^2 = 2by$, then b is equal to

(a) -64 (b) 128 (c) -128 (d) -32

**Answer: (c)**

**Solution:**

Any tangent to parabola $y^2 = 4x$ is y = mx + a/m

On comparing with y = mx + 4, we have

1/m = 4 => m = 1/4

Now, equation of tangent y = x/4 + 4 to $x^2 = 2by$

=> $x^2 = 2b(x/4 + 4)$

or $2x^2 - bx - 16b = 0$

Find the value of b:

Discriminant, D = 0

so, $b^2 + 128\,b = 0$

=> b(b + 128) = 0

b = 0 or b = -128

=> b = 0 (not possible) so, b = -128. Answer!

**Question 2:** Find the shortest distance between the line x - y = 1 and the curve $x = y^2$.

**Solution:**

$x = y^2$

Differentiate above equation w.r.t. x to find the slope

1 = 2y dy/dx

=> dy/dx = 1/2y

$1/2y = 1 \Rightarrow y = 1/2$

and $x = (1/2)^2 = 1/4$

(1/4, 1/2) is point on a parabola.

Therefore, the shortest distance between line and curve is

$$= \frac{\left|\frac{1}{4}-\frac{1}{2}+1\right|}{\sqrt{1+1}}$$

$= 3/4\sqrt{2}$ or $3\sqrt{2}/8$

**Question 3:** Two common tangents to the circle $x^2 + y^2 = 2a^2$ and parabola $y^2 = 8ax$ are

(a) $x = \pm (y + 2a)$

(b) $y = \pm (x + 2a)$

(c) $x = \pm(y + a)$

(d) $y = \pm(x + a)$

**Answer: (d)**

**Solution:** Let $y = mx + 2a/m$ be equation of tangent to any parabola $y^2 = 8ax$

If above tangent is tangent to the circle, $x^2 + y^2 = 2a^2$, then

$$\sqrt{2}a = \pm\frac{2a}{m\sqrt{m^2+1}}$$

$\Rightarrow m^2(1 + m^2) = 2$

$\Rightarrow (m^2 + 2)(m^2 - 1) = 0$

$\Rightarrow m = \pm 1$

So, required equation of tangent is $y = \pm(x + 2a)$

**Question 4:** The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(a) vertex is (2a/3, 0)

(B) directrix is x = 0

(C) latus rectum is 2a/3

(D) focus is (a, 0)

Answer: (A) and (D)

**Solution:**

Let P($at^2$, 2at) be any point on the parabola $y^2 = 4ax$.

Find the equation of tangent and normal:

Equation of tangent: y = x/t + at or yt = x + $at^2$ and

Equation of normal: y = -tx + 2at + $at^2$

Let centroid of triangle PTN is S(h, k)

Therefore,

$$h = \frac{at^2 + (-at^2) + 2a + at}{3}$$

and, k = 2at/3

=> 3h = 2a + $9k^2/4a$

=> $9k^2 = 4a(3h - 2a)$

Latus rectum = 4a/3

Locus of centroid is $y^2 = 4a/3(x - 2a/3)$ and vertex (2a/3, 0)

Directrix: x - 2a/3 = -a/3 => x = a/3

So, the focus is (a/3 + 2a/3, 0) = > (a, 0)

**Question 5:** Let P be the point (1, 0) and Q a point on the locus $y^2 = 8x$. The locus of mid point of PQ is

(a) $y^2 – 4x + 2 = 0$

(b) $y^2 + 4x - 2 = 0$

(c) $x^2 + 4y + 2 = 0$

(d) $x^2 – 4y + 2 = 0$

**Answer: (a)**

**Solution:**

Given point is P = (1, 0) and let another point Q = (h, k)

Such that, $k^2 = 8h$

Let (x, y) be the midpoint of PQ

So, $x = (h + 1)/2$ and $y = (k+0)/2$

=> $x = 2m – 1$ and $y = 2n$

Now, $(2y)^2 = 8(2x – 1)$

=> $y^2 = 4x – 2$

=> $y^2 - 4x + 2 = 0$, which is locus of mid point PQ.

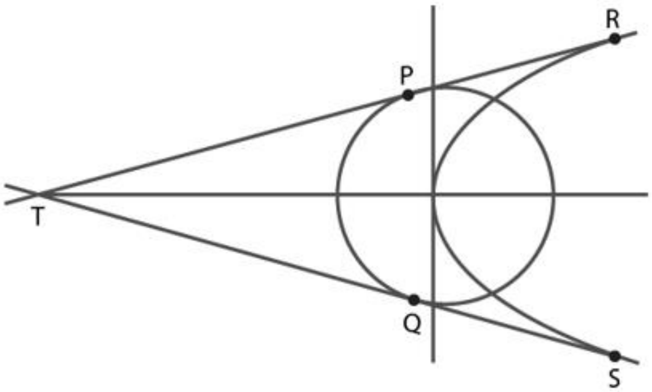
**Question 6:** The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the point P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is

(A) 3 (B) 6 (C) 9 (D) 15

**Answer: (d)**

**Solution:**

Circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ (Given)




Equation of tangent to parabola, y = mx + a/m

In this case a = 2

So, y = mx + 2/m

Now,

$$\frac{|0-0+2/m|}{\sqrt{1+m^2}} = \sqrt{2}$$

=> $2 = m^2 (1 + m^2)$

=> m = ± 1

Now equation of TP: -x + y = 2

So, P(-1, 1) and Q(-1, -1) and R (2,4) and S(2, -4)

Now, the area of the quadrilateral PQRS = 1/2 x (PQ + RS) × (Distance between PQ and RS)

= 1/2 x (2 + 8) × (1 + 2)

= 1/2 x 10 × 3

= 15 sq. units.

**Question 7:** The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$ then

(a) $t_2 = -t_1 - 2/t_1$

(b) $t_2 = -t_1 + 2/t$

(c) $t_2 = t_1 - 2/t_1$

(d) $t_2 = t_1 + 2/t_1$

**Answer: (a)**

**Solution:**

Equation of the normal to a parabola $y^2 = 4bx$ at point $(bt_1^2, 2bt_1)$ is

$y = -t_1 x + 2bt_1 + bt_1^3$

We per the question, it passes through the point $(bt_2^2, 2bt_2)$, then

$2bt_2 = -t_1bt_2^2 + 2bt_1 + bt_1^3$

$2t_2 - 2t_1 = -t_1 (t_2^2 - t_1^2) = -t_1(t_2 - t_1) (t_2 + t_1)$

$\Rightarrow 2 = -t_1(t_2 + t_1)$

$\Rightarrow (t_2 + t_1) = -2/t_1$

Or $t_2 = -t_1 - 2/t_1$

**Question 8:** A parabola has the origin as its focus and the line x = 2 as the directrix. Then the vertex of the parabola is at

(a) (0, 2)   (b) (1, 0)   (c) (0, 1)   (d) (2, 0)

**Answer: (b)**

**Solution:**

Vertex is mid point of origin (0, 0) and focus (2,0)

Vertex of the parabola:

$$= \left(\frac{0+2}{2}, \frac{0+0}{2}\right) = (1, 0)$$

**Question 9:** If $a \neq 0$ and the line $2bx + 3cy + 4d = 0$ passes through the points of intersection of the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ then

(a) $d^2 + (2b + 3c)^2 = 0$

(b) $d^2 + (3b + 2c)^2 = 0$

(c) $d^2 + (2b - 3c)^2 = 0$

(d) $d^2 + (3b - 2c)^2 = 0$

**Solution:**

Here (0, 0) and (4a, 4a) are the points of intersection of given parabolas.

=> Equation of line passing through these points is $y = x$.

On comparing $y = x$ with given line $2bx + 3cy + 4d = 0$, we get

$d = 0$ and $2b + 3c = 0$

=> $(2b + 3c)^2 + d^2 = 0$

**Question 10:** The locus of the vertices of the family of parabolas

$y = \frac{a^3x^3}{3} + \frac{a^2x}{2} - 2a$ is

(a) $xy = 105/64$ (b) $xy = 3/4$ (c) $xy = 35/16$ (d) $xy = 64/105$

**Answer: (a)**

**Solution:**

The family of parabolas

$$y = \frac{a^3x^3}{3} + \frac{a^2x}{2} - 2a$$

Let (x, y) be the vertex of the parabola.

x = $\frac{-a^2/2}{2a^3/3} = \frac{-3}{4a}$ and

y = $\frac{-(\frac{a^4}{4}+4\times\frac{a^3}{3}\times 2a)}{4\times\frac{4a^3}{3}}$

= $\frac{-(\frac{1}{4}+\frac{8}{3})\times a^4}{\frac{4a^3}{3}}$

= -35/12 x a/4 x 3

= ( -35/16) a

Because, vertex of y = $ax^2 + bx + c$ is

$(\frac{-b}{2a}, \frac{-(b^2-4ac)}{4a})$

Now, xy = (-3/4a ) ( -35/16)a = 105/64