Question 1: Consider the system of equations \( x + y + z = 1, \ 2x + 3y + 2z = 1, \ 2x + 3y + (a^2 - 1)z = a + 1 \) then

(a) System has a unique solution for \( |a| = \sqrt{3} \)
(b) System is inconsistence for \( |a| = \sqrt{3} \)
(c) System is inconsistence for \( a = 4 \)
(d) System is inconsistence for \( a = 3 \)

Answer: (b)

Solution:
Given system of linear equations:
\[
\begin{align*}
x + y + z &= 1 \quad \text{...(1)} \\
2x + 3y + 2z &= 1 \quad \text{....(2)} \\
2x + 3y + (a^2 - 1)z &= a + 1 \quad \text{.....(3)}
\end{align*}
\]
Consider \( a^2 - 1 = 2 \)
then LHS of (2) and (3) are same but RHS are not.
Hence \( a^2 = 3 \Rightarrow |a| = \sqrt{3} \)
For \( |a| = \sqrt{3} \), system is inconsistence.
Option (b) is correct.

Question 2: If the system of linear equations
\[
\begin{align*}
2x + 2ay + az &= 0 \\
2x + 3by + bz &= 0 \quad \text{and} \\
2x + 4cy + cz &= 0,
\end{align*}
\]
where \( a, b, c \in \mathbb{R} \) are non-zero and distinct; has non-zero solution, then

(a) \( a + b + c = 0 \)
(b) \( 1/a, 1/b, 1/c \) are in A.P.
(c) \( a, b, c \) are in A.P.
(d) \( a, b, c \) are in G.P.

Answer: (b)

Solution:
Given system of linear equations
\[
\begin{align*}
2x + 2ay + az &= 0 \\
2x + 3by + bz &= 0 \quad \text{and} \\
2x + 4cy + cz &= 0,
\end{align*}
\]
Now,

\[
\begin{vmatrix}
2 & 2a & a \\
2 & 3b & b \\
2 & 4c & c \\
\end{vmatrix} = 0
\]

\[R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1\]

\[
\begin{vmatrix}
2 & 2a & a \\
0 & 3b-2a & b-a \\
0 & 4c-2a & c-a \\
\end{vmatrix} = 0
\]

=> (3b − 2a)(c − a) − (4c − 2a)(b − a) = 0

=> 3bc − 2ac − 3ab + 2a^2 − [4bc − 4ac − 2ab + 2a^2] = 0

=> −bc + 2ac − ab = 0

=> ab + bc = 2ac

=> 1/c + 1/a = 2/b

Which shows that \(1/a, 1/b, 1/c\) are in A.P.

**Question 3:** If system of linear equations

\[x + y + z = 6\]

\[x + 2y + 3z = 10\] and

\[3x + 2y + \lambda z = \mu\]

has more than two solutions, then \(\mu - \lambda^2\) is equal to ________.

**Solution:**
The system of equations has more than 2 solutions.

Find for \(D = D_3 = 0\)
Question 4: For which of the following ordered pairs \((\mu, \delta)\), the system of linear equations
\[
\begin{align*}
x + 2y + 3z &= 1 \\
3x + 4y + 5z &= \mu \\
4x + 4y + 4z &= \delta
\end{align*}
\]
is inconsistent?
(a) (4, 6)  (b) (3, 4)  (c) (1, 0)  (d) (4, 3)

Answer: (d)

Solution:

\[
\begin{align*}
\begin{vmatrix}
1 & 1 & 1 \\
1 & 2 & 3 \\
3 & 2 & \lambda
\end{vmatrix} &= 0 \\
\Rightarrow 2\lambda - 6 - \lambda + 9 + 2 - 6 &= 0 \\
\Rightarrow \lambda &= 1 \\
\begin{vmatrix}
1 & 1 & 6 \\
1 & 2 & 10 \\
3 & 2 & \mu
\end{vmatrix} &= 0 \\
\Rightarrow 2\mu - 20 - \mu + 30 - 24 &= 0 \\
\Rightarrow \mu &= 14 \\
\end{align*}
\]
So, \(\mu - \lambda^2 = 13\)

For inconsistent system, one of \(D_x, D_y, D_z\) should not be equal to 0.
Now,

\[ D_x = \begin{vmatrix} \mu & 4 & 5 \\ 1 & 2 & 3 \\ \delta & 4 & 4 \end{vmatrix} \]

\[ D_y = \begin{vmatrix} 3 & \mu & 5 \\ 1 & 1 & 3 \\ 4 & \delta & 4 \end{vmatrix} \]

\[ D_z = \begin{vmatrix} 3 & 4 & \mu \\ 1 & 2 & 1 \\ 4 & 4 & \delta \end{vmatrix} \]

For inconsistent system, \(2\mu \neq \delta + 2\)
Therefore, the system will be inconsistent for \(\mu = 4, \delta = 3\).

**Question 5:** The system of linear equations

\[
\begin{align*}
\lambda x + 2y + 2z &= 5 \\
2\lambda x + 3y + 5z &= 8 \\
4x + \lambda y + 6z &= 10
\end{align*}
\]

has:

(a) no solution when \(\lambda = 2\)
(b) infinitely many solutions when \(\lambda = 2\)
(c) no solution when \(\lambda = 8\)
(d) a unique solution when \(\lambda = -8\)

**Answer:** (a)

**Solution:**

\[
D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix} = 18\lambda - 5\lambda^2 - 24\lambda + 40 + 4\lambda^2 - 24
\]

\[ \Rightarrow D = -\lambda^2 - 6\lambda + 16 \]

Now, \(D = 0\)

\[ \Rightarrow \lambda^2 + 6\lambda - 16 = 0 \]
\[ \Rightarrow \lambda = -8 \text{ or } 2 \]

For \(\lambda = 2\)

\[
D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix} = 40 + 4 - 28 \neq 0
\]

Therefore, Equations have no solution for \(\lambda = 2\).
Question 6: The following system of linear equations
7x + 6y − 2z = 0,
3x + 4y + 2z = 0
x − 2y − 6z = 0, has
(a) infinitely many solutions, (x, y, z) satisfying y = 2z
(b) infinitely many solutions, (x, y, z) satisfying x = 2z
(c) no solution
(d) only the trivial solution

Answer: (b)

Solution:
Given system of linear equations
7x + 6y − 2z = 0,
3x + 4y + 2z = 0
x − 2y − 6z = 0,

As the system of equations are Homogeneous
=> The system is consistent.

\[
\begin{vmatrix}
7 & 6 & -2 \\
3 & 4 & 2 \\
1 & -2 & -6
\end{vmatrix} = 0
\]

=> Infinite solutions exist (both trivial and non-trivial solutions)

When, y = 2z
Let’s take y = 2 and z = 1
When (x, 2, 1) is substituted in the system of equations
=> 7x + 10 = 0,
3x + 10 = 0 and
x − 10 = 0 (which is not possible)

Therefore, y = 2z
=> Infinitely many solutions not exist.

For x = 2z, let’s take x = 2, z = 1, y = y

Substitute (2, y, 1) in system of equations
=> y = −2
So, for each pair of (x, z), we get a value of y.

Therefore, for x = 2z infinitely many solutions exist.
Question 7: If the system of linear equations
\[ x + ky + 3z = 0 \]
\[ 3x + ky - 2z = 0 \]
\[ 2x + 4y - 3z = 0 \]
has a non zero solution \((x, y, z)\) then \(xz/y^2\) is equal to

(a) -10   (b) 10   (c) -30   (d) 30

Answer: (b)

Solution:
Given system of linear equations
\[ x + ky + 3z = 0 \]
\[ 3x + ky - 2z = 0 \]
\[ 2x + 4y - 3z = 0 \]
System has non zero solution, so
\[ D = 0 \]
\[
\begin{vmatrix}
1 & k & 3 \\
3 & k & -2 \\
2 & 4 & -3 \\
\end{vmatrix} = 0
\]
1(-3k + 8) - k(-9 + 4) + 3(12 - 2k) = 0
Solving above equation, we have \(-4k = -44\)
or \(k = 11\)
\[ x + 11y + 3z = 0 \quad ...(i) \]
\[ 3x + 11y - 2z = 0 \quad ...(ii) \]
\[ 2x + 4y - 3z = 0 \quad ....(iii) \]
Solving (i) and (iii)
\[ x = -5y \]
Using \(x = -5y\) in (iii), \(-10y + 4y - 3z = 0\)
\(-6y - 3z = 0\)
or \(z = -2y\)
Now, \(xz/y^2 = (-5y)(-2y)/y^2 = 10\)

Question 8: The number of real values of \(\lambda\) for which the system of linear equations
\[ 2x + 4y - \lambda z = 0 \]
\[ 4x + \lambda y + 2z = 0 \]
\[ \lambda x + 2y + 2z = 0 \]
has infinitely many solutions, is :
(a) 0   (b) 1   (c) 2   (d) 3
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Answer: (b)

Solution:
For infinitely many solutions, \( D = 0 \), \( D_x = 0 \), \( D_y = 0 \) and \( D_z = 0 \)

\[
\begin{vmatrix}
2 & 4 & -\lambda \\
4 & \lambda & 2 \\
\lambda & 2 & 2 \\
\end{vmatrix}
\]

\[
= 2(2\lambda - 4) - 4(8 - 2\lambda) - \lambda(8 - \lambda^2)
\]

\[
= \lambda^3 - 4\lambda - 40
\]

If \( D = 0 \), then

\[
\lambda^3 - 4\lambda - 40 = 0
\]

Again, \( D_x = 0 \)

\[
\begin{vmatrix}
0 & 4 & -\lambda \\
0 & \lambda & 2 \\
0 & 2 & 2 \\
\end{vmatrix} = 0
\]

Similarly, \( D_y = 0 \) and \( D_z = 0 \)

The equation, \( \lambda^3 + 4\lambda - 40 \) has only one solution.
Since \( \lambda(-\infty) = -\infty \) and \( \lambda(\infty) = \infty \)

Here \( \alpha \) is only one solution, so \( \lambda(\alpha) = 0 \)
[Using intermediate value property]
Now, differentiating \( \lambda^3 + 4\lambda - 40 \) w.r.t. \( \lambda \) we get

\[
3\lambda^2 + 4 > 0
\]

The equation can not have \( y, m, \) so

\[
\lambda(m) = 0 \text{ and } \lambda(y) = 0
\]

Thus, the number of real values of \( \lambda \) is 1.

Question 9: If \( x = a, \ y = b, \ z = c \) is a solution of the system of linear equations

\[
x + 8y + 7z = 0 \\
9x + 2y + 3z = 0 \\
x + y + z = 0
\]

such that the point \( (a, b, c) \) lies on the plane \( x + 2y + z = 6 \), then \( 2a + b + c \) equals :

(a) -1 \quad (b) 0 \quad (c) 1 \quad (d) 2

Answer: (c)
Solution:
Given system of linear equations
\[x + 8y + 7z = 0 \quad \text{(i)}
\]
\[9x + 2y + 3z = 0 \quad \text{(ii)}
\]
\[x + y + z = 0 \quad \text{(iii)}
\]
Operate: (ii) \(-3 \times (\text{iii})\)
\[6x - y = 0 \quad \text{or} \quad y = 6x \quad \text{(iv)}
\]
Using (iv) in (i)
\[x + 8(6x) + 7z = 0\]
\[z = -7x \quad \text{(v)}
\]
Since \(x = a, y = b, z = c\) (Given)
\[b = 6a \quad \text{and} \quad c = -7a\]
Also, \((a, b, c)\) lies on the plane \(x + 2y + z = 6\).
Therefore, \(a + 2b + c = 6 \quad \text{(vi)}\)
Putting the values of \(b\) and \(c\) in (vi),
\[a + 2(6a) - 7a = 6\]
\[\Rightarrow a = 1\]
Also, we get \(b = 6\) and \(c = -7\)
Now, \(2a + b + c = 2(1) + 6 - 7 = 1\)

Question 10: It \(S\) is the set of distinct values of ‘\(b\)’ for which the following system of linear equations \(x + y + z = 1\)
\(x + ay + z = 1\)
\(ax + by + z = 0\) has no solution, then \(S\) is:

(a) an empty set
(b) an infinite set
(c) a finite set containing two or more elements
(d) a singleton

Answer: (d)

Solution:
We get first two planes co-incident for \( a = 1 \).

\[
\begin{align*}
x + y + z &= 1 \\
x + y + z &= 1 \\
x + by + z &= 0
\end{align*}
\]

If \( b = 1 \), the system will be inconsistent and hence no solution.

If \( b \neq 1 \), the system will produce infinite solutions.

Hence, for no solution, \( S \) has to be a singleton set \( \{1\} \).