**Question 1:** Consider the system of equations x + y + z = 1, 2x + 3y + 2z = 1, 2x + 3y + $(a^2 - 1)$z = a + 1 then

(a) System has a unique solution for |a| = √3

(b) System is inconsistence for |a| = √3

(c) System is inconsistence for a = 4

(d) System is inconsistence for a = 3

**Answer: (b)**

**Solution:**

Given system of linear equations:

x + y + z = 1 ....(1)

2x + 3y + 2z = 1 ....(2)

2x + 3y + $(a^2 - 1)$z = a + 1 .....(3)

Consider $a^2 - 1 = 2$

then LHS of (2) and (3) are same but RHS are not.

Hence $a^2 = 3$ => |a| = √3

For |a| = √3, system is inconsistence.

Option (b) is correct.

**Question 2:** If the system of linear equations

2x + 2ay + az = 0

2x + 3by + bz = 0 and

2x + 4cy + cz = 0,

where a, b, c ∈ R are non-zero and distinct; has non-zero solution, then

(a) a + b + c = 0

(b) 1/a, 1/b, 1/c are in A.P.

(c) a, b, c are in A.P.

(d) a, b, c are in G.P.

**Answer: (b)**

**Solution:**

Given system of linear equations

2x + 2ay + az = 0

2x + 3by + bz = 0 and

2x + 4cy + cz = 0,

Now,

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$R_2 \to R_2 - R_1 \; and \; R_3 \to R_3 - R_1$$

$$\begin{vmatrix} 2 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

=> (3b − 2a)(c − a) − (4c − 2a)(b − a) = 0

=> $3bc - 2ac - 3ab + 2a^2 - [4bc - 4ac - 2ab + 2a^2] = 0$

=> −bc + 2ac − ab = 0

=> ab + bc = 2ac

=> 1/c + 1/a = 2/b

Which shows that 1/a, 1/b, 1/c are in A.P.

**Question 3:** If system of linear equations
x + y + z = 6

x + 2y + 3z = 10 and

3x + 2y + λz = μ

has more than two solutions, then $\mu - \lambda^2$ is equal to ________.

**Solution:**
The system of equations has more than 2 solutions.

Find for $D = D_3 = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 2\lambda - 6 - \lambda + 9 + 2 - 6 = 0$$

$$\Rightarrow \lambda = 1$$

$$\begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix} = 0$$

$$\Rightarrow 2\mu - 20 - \mu + 30 - 24 = 0$$

$$\Rightarrow \mu = 14$$

So, $\mu - \lambda^2 = 13$

**Question 4:** For which of the following ordered pairs (μ, δ), the system of linear equations
x + 2y + 3z = 1

3x + 4y + 5z = μ

4x + 4y + 4z = δ

is inconsistent?

(a) (4, 6) (b) (3, 4) (c) (1, 0) (d) (4, 3)

**Answer: (d)**

**Solution:**

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{vmatrix}$$

$$R_3 \to R_3 - 2R_1 + 2R_2$$

$$D = \begin{vmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

For inconsistent system, one of $D_x$, $D_y$, $D_z$ should not be equal to 0.

Now,

$$D_x = \begin{vmatrix} \mu & 4 & 5 \\ 1 & 2 & 3 \\ \delta & 4 & 4 \end{vmatrix}$$

$$D_y = \begin{vmatrix} 3 & \mu & 5 \\ 1 & 1 & 3 \\ 4 & \delta & 4 \end{vmatrix}$$

*and*

$$D_z = \begin{vmatrix} 3 & 4 & \mu \\ 1 & 2 & 1 \\ 4 & 4 & \delta \end{vmatrix}$$

For inconsistent system, $2\mu \neq \delta + 2$

Therefore, the system will be inconsistent for $\mu = 4$, $\delta = 3$.

**Question 5:** The system of linear equations

$\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$ has:

(a) no solution when $\lambda = 2$

(b) infinitely many solutions when $\lambda = 2$

(c) no solution when $\lambda = 8$

(d) a unique solution when $\lambda = -8$

**Answer: (a)**

**Solution:**

$$D = \begin{bmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{bmatrix} = 18\lambda - 5\lambda^2 - 24\lambda + 40 + 4\lambda^2 - 24$$

$$\Longrightarrow D = -\lambda^2 - 6\lambda + 16$$

$\text{Now, } D = 0$

$$\Rightarrow \lambda^2 + 6\lambda - 16 = 0$$

$$\Rightarrow \lambda = -8 \text{ or } 2$$

$\text{For } \lambda = 2$

$$D_1 = \begin{bmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{bmatrix} = 40 + 4 - 28 \neq 0$$

Therefore, Equations have no solution for $\lambda = 2$.

**Question 6:** The following system of linear equations
$7x + 6y - 2z = 0$ ,
$3x + 4y + 2z = 0$
$x - 2y - 6z = 0$, has
(a) infinitely many solutions, (x, y, z) satisfying y = 2z
(b) infinitely many solutions, (x, y, z) satisfying x = 2z
(c) no solution
(d) only the trivial solution

**Answer: (b)**

**Solution:**
Given system of linear equations
$7x + 6y - 2z = 0$ ,
$3x + 4y + 2z = 0$
$x - 2y - 6z = 0$,

As the system of equations are Homogeneous
=> The system is consistent.

$$\Rightarrow \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix} = 0$$

=> Infinite solutions exist (both trivial and non-trivial solutions)

When, y = 2z
Let's take y = 2 and z = 1
When (x, 2, 1)is substituted in the system of equations
=> 7x + 10 = 0,
3x + 10 = 0 and
x − 10 = 0 (which is not possible)

Therefore, y = 2z
=> Infinitely many solutions not exist.

For x = 2z, let's take x = 2, z = 1, y = y

Substitute (2, y, 1) in system of equations
=> y = −2
So, for each pair of (x, z), we get a value of y.

Therefore, for x = 2z infinitely many solutions exist.

**Question 7:** If the system of linear equations

x + ky + 3z = 0

3x + ky - 2z = 0 and

2x + 4y - 3z = 0

has a non zero solution (x, y, z) then $xz/y^2$ is equal to

(a) -10 (b) 10 (c) -30 (d) 30

**Answer: (b)**

**Solution:**

Given system of linear equations

x + ky + 3z = 0

3x + ky - 2z = 0 and

2x + 4y - 3z = 0

System has non zero solution, so

D = 0

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

1(-3k + 8) - k(-9 + 4) + 3(12 - 2k) = 0

Solving above equation, we have -4k = -44

or k = 11

x + 11y + 3z = 0 ...(i)

3x + 11y - 2z = 0 .....(ii)

2x + 4y - 3z = 0 ....(iii)

Solving (i) and (iii)

x = -5y

Using x = -5y in (iii), -10y + 4y - 3z = 0

-6y - 3z = 0

or z = -2y

Now, $xz/y^2 = (-5y)(-2y)/y^2 = 10$

**Question 8:** The number of real values of λ for which the system of linear equations

2x + 4y − λz = 0

4x + λy + 2z = 0

λx + 2y + 2z = 0

has infinitely many solutions, is :

(a) 0 (b) 1 (c) 2 (d) 3

**Answer: (b)**

**Solution:**
For infinitely many solutions, D = 0, $D_x$ = 0, $D_y$ = 0 and $D_z$ = 0

$$D = \begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{vmatrix}$$

$$= 2(2\lambda - 4) - 4(8 - 2\lambda) - \lambda(8 - \lambda^2)$$

$$= \lambda^3 + 4\lambda - 40$$

If D = 0, then

$$\lambda^3 + 4\lambda - 40 = 0$$

Again, $D_x$ = 0

$$D_x = \begin{vmatrix} 0 & 4 & -\lambda \\ 0 & \lambda & 2 \\ 0 & 2 & 2 \end{vmatrix} = 0$$

Similarly, $D_y$ = 0 and $D_z$ = 0
The equation, $\lambda^3 + 4\lambda - 40$ has only one solution.
Since $\lambda(-\infty) = -\infty$ and $\lambda(\infty) = \infty$
Here α is only one solution, so λ(α) = 0
[Using intermediate value property]
Now, differentiating $\lambda^3 + 4\lambda - 40$ w.r.t. λ we get
$3\lambda^2 + 4 > 0$
The equation can not have y, m, so
λ(m) = 0 and λ(y) = 0
Thus, the number of real values of λ is 1.

**Question 9:** If x = a, y = b, z = c is a solution of the system of linear equations
x + 8y + 7z = 0
9x + 2y + 3z = 0
x + y + z = 0
such that the point (a, b, c) lies on the plane x + 2y + z = 6, then 2a + b + c equals :
(a) −1 (b) 0 (c) 1 (d) 2

**Answer: (c)**

**Solution:**

Given system of linear equations

$x + 8y + 7z = 0$ ...(i)

$9x + 2y + 3z = 0$ ....(ii)

$x + y + z = 0$ ....(iii)

Operate: (ii) - 3 x (iii)

$6x - y = 0$ or $y = 6x$ .....(iv)

Using (iv) in (i)

$x + 8(6x) + 7z = 0$

$z = -7x$ ......(v)

Since $x = a, y = b, z = c$ (Given)

$b = 6a$ and $c = -7a$

Also, (a, b, c) lies on the plane $x + 2y + z = 6$.

Therefore, $a + 2b + c = 6$ .....(vi)

Putting the values of b and c in (vi),

$a + 2(6a) – 7a = 6$

=> $a = 1$

Also, we get $b = 6$ and $c = -7$

Now, $2a + b + c = 2(1) + 6 - 7 = 1$

**Question 10:** It S is the set of distinct values of 'b' for which the following system of linear equations $x + y + z = 1$ $x + ay + z = 1$ $ax + by + z = 0$ has no solution, then S is :

(a) an empty set
(b) an infinite set
(c) a finite set containing two or more elements
(d) a singleton

**Answer: (d)**

**Solution:**

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix}$$

$= -(a - 1)^2 = 0$

$\Rightarrow a = 1$

We get first two planes co-incident for a = 1.

$x + y + z = 1$

$x + y + z = 1$

$x + by + z = 0$

If b = 1, the system will be inconsistent and hence no solution.

If $b \neq 1$, the system will produce infinite solutions.

Hence, for no solution, S has to be a singleton set {1}.