Q1: A compressive force, F is applied at the two ends of a long thin steel rod. It is heated, simultaneously, such that its temperature increases by ΔT. The net change in its length is zero. Let L be the length of the rod, A is its area of cross-section. Y is Young's modulus, and α is its coefficient of linear expansion. Then, F is equal to

(a) $L^2 Y\alpha\Delta T$

(b) $AY/\alpha\Delta T$

(c) $AY\alpha\Delta T$

(d) $LAY\alpha\Delta T$

Solution:

Thermal expansion, $\Delta L = L\alpha\Delta T$ -------(1)

Compression ΔL' produced by applied force is given by,

$Y = FL/A\Delta L' \Rightarrow F = YA\Delta L'/L$-------------(2)

Net change in length = 0 $\Rightarrow \Delta L' = \Delta L$ ------(3)

From (1),(2) and (3)

$F = YA \times (L\alpha\Delta T)L = YA\alpha\Delta T$

Answer: (c) AYαΔT

Q2: A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1mm. Then the elastic energy stored in the wire is

(a) 0.2 J

(b) 10 J

(c) 20 J

(d) 0.1 J

Solution

Elastic energy per unit volume = ½ x stress x strain

Elastic Energy = ½ x stress x strain x volume

$= \frac{1}{2} \times F/A \times (\Delta L/L) \times (AL)$

$= \frac{1}{2} \times F\Delta L$

$= \frac{1}{2} \times 200 \times 10^{-3}$

Elastic Energy = 0.1 J

Answer: (d) 0.1 J

Q3: A rod of length L at room temperature and uniform area of cross-section A, Is made of a metal having a coefficient of linear expansion α. It is observed that an external compressive force F is applied to each of its ends, prevents any change in the length of the rod when its temperature rises by ΔT K. Young's modulus, Y for this metal is

(a) $F/A\alpha\Delta T$

(b) $F/A\alpha(\Delta T - 273)$

(c) $F/2A\alpha\Delta T$
(d) $2F/A\alpha\Delta T$

Solution:
Young's Modulus Y = stress/strain = $F/A(\Delta l/l)$
Substituting the coefficient of linear expansion
$\alpha = \Delta l/(l\Delta T)$
$\Delta l/l = \alpha\Delta T$
$Y = F/A(\alpha\Delta T)$
Answer: (a) $F/A\alpha\Delta T$

Q4: Young's moduli of two wires A and B are in the ratio 7:4. Wire A is 2m long and has radius R. Wire B is 1.5 m long and has a radius 2mm. If the two wires stretch by the same length for a given load, then the value of R is close to
(a) 1.5 mm
(b) 1.9 mm
(c) 1.7 mm
(d) 1.3 mm

Solution:
$\Delta_1 = \Delta_2$
$(Fl_1/\pi r_1^2 y_1) = (Fl_2/\pi r_2^2 y_2)$
$2/(R^2 \times 7) = 1.5/(2^2 \times 4)$
R= 1.75 mm
Answer: (c) 1.7 mm

Q5: The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400 N load without exceeding its elastic limit?
(a) 1 mm
(b) 1.15 mm
(c) 0.90 mm
(d) 1.36 mm

Solution
Stress = F/A
Stress = $400 \times 4/\pi d^2$
$= 379 \times 10^6$ N/m^2
$d^2 = (400 \times 4)/(379 \times 10^6\pi)$
d = 1.15 mm

Answer: (b) 1.15 mm

Q6: A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of Elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion, of the material of the rod is nearly equal to

(a) $9F/(\pi r^2 YT)$
(b) $6F/(\pi r^2 YT)$
(c) $3F/(\pi r^2 YT)$
(d) $F/(3\pi r^2 YT)$

Solution

$Y = (F/\pi r^2) \times L/\Delta L$
$\Delta L = Fl/\pi r^2 Y$--------(1)
Change in length due to temperature change
$\Delta L = L\propto\Delta T$------------(2)
From equa (1) and (2)
$L \propto\Delta T = FL/AY$
$\propto = F/AY\Delta T$
$\propto = F/\pi r^2 YT$
Coefficient of volume expansion
$3\propto = 3F/\pi r^2 YT$

Answer: (c) $3F/\pi r^2 YT$

Q7: The following four wires are made of the same material. Which of these will have the largest extension when the same tension is applied?

(a) length = 200 cm, diameter = 2 mm
(b) length = 300 cm, diameter = 3 mm
(c) length = 50 cm, diameter = 0.5 mm
(b) length = 100 cm, diameter = 1 mm

Solution:

Since all four wires are made from the same material Young's modulus will be the same.
$\Delta L \propto L/D^2$
In (a) $L/D^2 = 200/(0.2)^2 = 5 \times 10^3$ cm^{-1}
In (b) $L/D^2 = 300/(0.3)^2 = 3.3 \times 10^3$ cm^{-1}
In (c) $L/D^2 = 50/(0.05)^2 = 20 \times 10^3$ cm^{-1}
In (d) $L/D^2 = 100/(0.1)^2 = 10 \times 10^3$ cm^{-1}

Answer: (c) length = 50 cm, diameter = 0.5 mm

Q8: A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains the same, the stress in the leg will change by a factor of

(a) 1/9
(b) 81
(c) 1/81
(d) 9

Solution

Stress = Force/Area
Stress = Force/L^2
Now, dimensions increases by a factor of 9
Now, S = (volume x density) x g /L^2
$S = L^3 \times \rho g / L^2 = L \rho g$
Stress $S \propto L$
$S_2/S_1 = L_2/L_1 = 9L_1/L_1 = 9$

Answer: (d) 9

Q9. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of the area a floats on the surface of the liquid, covering an entire cross-section of the cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere is Mg/αAB. Find the value of α.

(a) 4
(b) 5
(c) 3
(d) 2

Solution:

Increase in pressure is $\Delta p = Mg/A$
Bulk modulus is $B = \Delta p / (\Delta V/V)$
$\Delta V/V = \Delta p / B = Mg/AB$------(1)
The volume of the sphere is $V = (4/3)\pi R3$
$\Delta V/V = 3(\Delta R/R)$
From equation (1) we get
$Mg/AB = 3(\Delta R/R)$
$\Delta R/R = Mg/3AB$
Therefore $\alpha = 3$

Answer: (c) 3

Q10: A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that g = 3.1πm/s2, what will be the tensile stress that would be developed in the wire?

(a) 4.8×10^6 N/m^2
(b) 3.1×10^6 N/m^2
(c) 6.2×10^6 N/m^2
(d) 5.2×10^6 N/m^2

Solution:
Tensile stress = Force/Area
Tensile stress = $(4)(3.1\pi)/\pi(2 \times 10^{-3})^2$
Tensile stress = 3.1×10^6 Nm^{-2}
Answer: (b) 3.1×10^6 N/m^2

Q11: A steel rail of length 5m and area of cross-section 40cm^2 is prevented from expanding along its length while the temperature rises by 10^{0}C. If the coefficient of linear expansion and Young's modulus of steel is 1.2×10^{-5} K^{-1} and 2×10^{11} Nm^{-2} respectively, the force developed in the rail is approximately

(a) 2×10^9 N
(b) 3×10^{-5} N
(c) 2×10^7 N
(d) 1×10^5 N

Solution:
A = 40 cm^2 = 4×10^{-3} m^2
$\Delta T = 10^0$C
Y = 2×10^{11} Nm^{-2}
$\alpha = 1.2 \times 10^{-5}$ K^{-1}
Force = $YA\alpha\Delta T$
Force = $(2 \times 10^{11})(4 \times 10^{-3})(1.2 \times 10^{-5})(10) = 9.6 \times 10^4$ N
Force ≈ 1×10^5 N
Answer: (d) 1×10^5 N

Q12: If S is the stress and Y is Young's Modulus of the material of the wire, the energy stored in the wire per unit volume is

(a) 2Y/S
(b) S/2Y
(c) $2S^2Y$
(d) $S^2/2Y$

Solution:
Energy stored per unit volume = ½ x stress x strain
= Stress x Stress/2Y = $S^2/2Y$

Answer: (d) $S^2/2Y$

Q13: A wire fixed at the upper end stretches by length *l* by applying a force F. The work done in stretching is

(a) F/2*l*
(b) F*l*
(c) 2F*l*
(d) F*l*/2

Solution
Young's Modulus Y = FL/A*l*
Therefore, F = YA*l*/*L*
dW = Fd*l* = YA*l*(d*l*)/L
[latex]\int dW = \frac{YA}{L}\int_{0}^{1}ldl = YAl^{2}/2L[/latex]

Work done = $YAl^2/2L$
Work done = F*l*/2
Answer: (d) F*l*/2

Q14: Two wires are made of the same material and have the same volume. However, wire 1 has a cross-sectional area A and wire 2 has cross-sectional area 3A. If the length of the wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount?

(a) F
(b) 4F
(c) 6F
(d) 9F

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Solution
For the same material, Young's modulus is the same and it is given that the volume is the same and the area of the cross-section for the wire L1 is and that of L2 is 3A
$V = V_1 = V_2$
$V = A \times L_1 = 3A \times L_2 \Rightarrow L_2 = L_1/3$
$Y = (F/A)/(\Delta L/L)$
$F_1 = YA(\Delta L_1/L_1)$
$F_2 = Y3A(\Delta L_2/L_2)$

Given $\Delta L_1 = \Delta L_2 = \Delta x$ (for the same extension)
$F_2 = Y3A(\Delta x/(L_1/3)) = 9.\ (YA\Delta x/L_1) = 9F_1 = 9F$

Answer: (d) 9F

Q15: A wire elongates by *l* mm when a load W is hanged from it. If the wire goes over a pulley and two weighs W each is hung at the two ends, the elongation of the wire will be (in mm)

(a) $l/2$

(b) l

(c) $2l$

(d) Zero

Solution

Y = (Force x L)/(A x l) = WL/A l

l = WL/AY

Due to the arrangement of the pulley, the length of wire is L/2 on each side and so the elongation will be $l/2$. For both sides elongation = l

Answer: (b)