Second PUC Mathematics March, 2015 Question Paper

(English Version)

Instructions : *i) The question paper has five Parts –* ***A, B, C, D*** *and* ***E****.*
Answer ***all*** *the Parts.*
ii) ***Use*** *the* ***Graph Sheet*** *for the question on linear programming in Part –* ***E****.*

PART – A

Answer **all** the **ten** questions : **(10×1=10)**

1. Let $*$ be a operation defined on the set of rational numbers by $a * b = \frac{ab}{4}$, find the identity element.

2. Write the values of x for which $2\tan^{-1} x = \cos^{-1}\frac{1-x^2}{1+x^2}$, holds.

3. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $\frac{1}{2}|-3i+j|$.

4. Find the values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.

5. Find $\frac{dy}{dx}$, if $y = \sin(x^2 + 5)$.

6. Evaluate : $\int e^x \left(\frac{x-1}{x^2}\right) dx$.

7. Define negative of a vector.

8. Write the direction cosines of x-axis.

9. Define feasible region in LPP.

10. If $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

PART – B

Answer **any ten** questions : **(10×2=20)**

11. Show that if $f : A \rightarrow B$ and $g : B \rightarrow C$ are one-one, then $gof : A \rightarrow C$ is also one-one.

12. Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1}x$ for $\frac{-1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$.

13. Show that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.

14. If the area of the triangle with vertices (– 2, 0), (0, 4) and (0, k) is 4 square units, find the values of k using determinants.

15. Differentiate $\left(x + \frac{1}{x}\right)^x$ with respect to x.

16. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$, $x \neq 2$ at $x = 10$.

17. Find $\frac{dy}{dx}$, if $x^2 + xy + y^2 = 100$.

18. Evaluate : $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$.

19. Evaluate : $\int \frac{dx}{x - \sqrt{x}}$.

20. Find the order and degree, if defined of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\frac{dy}{dx} + 1 = 0.$$

21. Find $|\vec{b}|$, if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$.

22. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

23. Find the angle between the pair of lines given by
$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$.

24. Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values of x, has the following form, where K is some constant

$$P(X = x) = \begin{cases} 0,1 & , \text{ if } x = 0 \\ Kx & , \text{ if } x = 1 \text{ or } 2 \\ K(5 - x), & \text{ if } x = 3 \text{ or } 4 \\ 0 & , \text{ otherwise} \end{cases}$$

find the value of K.

PART – C

Answer **any ten** questions :

(10×3=30)

25. Determine whether the relation R in the set A = {1, 2, 3, ... 13, 14} defined as R = {(x, y) : 3x – y = 0}, is reflexive, symmetric and transitive.

26. If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then find the values of x.

27. If A and B are invertible matrices of the same order, then prove that $(AB)^{-1}=B^{-1}A^{-1}$.

28. Verify Rolles theorem for the function $f(x)=x^2+2x-8,\ x\in[-4, 2]$.

29. If $x=\sqrt{a^{\sin^{-1}t}}$ and $y=\sqrt{a^{\cos^{-1}t}}$ then prove that $\frac{dy}{dx}=\frac{-y}{x}$.

30. Find the two positive numbers whose sum is 15 and sum of whose squares is minimum.

31. Evaluate : $\int x\tan^{-1}x\,dx$.

32. Evaluate $\int_0^2 e^x\,dx$ as a limit of sum.

33. Find the area of the region bounded by the curve $y^2=4x$ and the line $x=3$.

34. Show that the position vector of the point P, which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio m : n is $\frac{m\vec{b}+n\vec{a}}{m+n}$.

35. Show that the four points with position vectors $4\hat{i}+8\hat{j}+12\hat{k}$, $2\hat{i}+4\hat{j}+6\hat{k}$, $3\hat{i}+5\hat{j}+4\hat{k}$ and $5\hat{i}+8\hat{j}+5\hat{k}$ are coplanar.

36. Find the equation of the plane passing through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z + 2 = 0$ and the point (2, 2, 1).

37. Form the differential equation of the circles touching the x-axis at origin.

38. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured person meets with an accident. What is the probability that he is a scooter driver ?

PART – D

Answer **any six** questions :

(6×5=30)

39. Let R_+ be the set of all non-negative real numbers. Show that the function $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$ is invertible and write the inverse of f.

40. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = 0$.

41. Solve by matrix method :

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3.$

42. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$.

43. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y – coordinate is changing 8 times as fast as the x – coordinate.

44. Find the integral of $\frac{1}{\sqrt{x^2 - a^2}}$ with respect to x and hence evaluate

$\int \frac{dx}{\sqrt{x^2 + 6x - 7}}$.

45. Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

46. Solve the differential equation $\frac{dy}{dx} + y \sec x = \tan x, 0 \le x < \frac{\pi}{2}$.

47. Derive the equation of the line in space passing through a point and parallel to a vector both in vector and Cartesian form.

48. A die is thrown 6 times. If 'getting an odd number' is a success, what is probability of :

a) 5 successes ?

b) at least 5 successes ?

c) at most successes ?

PART – E

Answer **any one** question : **(1×10=10)**

49. a) Prove that $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_{0}^{a} f(x)\,dx, & \text{if } f(x) \text{ is an even function} \\ 0, & \text{if } f(x) \text{ is an odd function} \end{cases}$

and hence evaluate $\int_{-1}^{1} \sin^5 x \cos^4 x\,dx$.

b) Prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2.$$

50. a) A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for finishing and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 8,000 on each piece of model A and Rs. 12,000 on each piece of model B. How many pieces of model A and model B should be manufactured per week to realize a maximum profit ? What is the maximum profit per week ?

b) Find the value of K so that the function $f(x) = \begin{cases} Kx + 1, & \text{if } x \le 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$

at x = 5 is a continuous function.