# Karnataka Class 10 Maths Important Questions

**Question 1: Let U = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}.**
**Find A′, B′, A′ ∩ B′, A ∪ B and hence show that ( A ∪ B )′ = A′∩ B′.**

**Solution:**
U = {1, 2, 3, 4, 5, 6} , A = {2, 3} and B = {3, 4, 5}
A' = {1, 4, 5, 6}
B' = {1, 2, 6}
A′ ∩ B′ = {1, 6}
(A U B) = {2, 3, 4, 5}
(A ∪ B )′ = {1, 6}
A′ ∩ B′ = {1, 6}
(A ∪ B)′ = A′ ∩ B′

**Question 2: Prove that 2 + √3 is an irrational number.**

**Solution:**
If (2 + √3) is rational, then,
(2 + √3) = a / b (say) where a and b are integers and b ≠ 0
⇒ √3 = a / b – (2)
⇒ √3 = [a – 2b] / b ...(1)
∵ a and b are integers
∴ a – 2b is also an integer.
⇒ (a – 2b) / b is rational.
The L.H.S. of equation (1) is the square root of a prime number.
So, it is irrational and R.H.S. is rational.
It is a contradiction because a rational number and an irrational number can never be equal.
So, our assumption of 2 + √3 being rational is wrong.

Hence, $2 + \sqrt{3}$ is an irrational number.

**Question 3: Find the sum of all 2 digit natural numbers that are divisible by 5.**

**Solution:**

The two-digit natural numbers divisible by 5 are 10, 20 ...... 90, 95

The series forms an arithmetic progression.

The last term is $a_n = a + (n - 1) d$

$95 = 10 + (n - 1) 5$

$95 - 10 = (n - 1) 5$

$85 / 5 = n - 1$

$17 = n - 1$

$17 + 1 = 18 = n$

$n = 18$

Sum of n numbers in a series is given by,

$S_n = (n / 2) (a + a_n)$

$= (18 / 2) (10 + 95)$

$= 9 * 105$

$= 945$

**Question 4: If $2 ({}^nP_2) + 50 = {}^{2n}P_2$, then find the value of n.**

**Solution:**

${}^nP_r = n! / (n - r)!$

$2 ({}^nP_2) + 50 = {}^{2n}P_2$

$2 * (n! / (n - 2)!) + 50 = 2n! / (2n - 2)!$

$2 * [n (n - 1) (n - 2)! / (n - 2)!] + 50 = 2n (2n - 1) (2n - 2)! / (2n - 2)!$

$2n (n - 1) + 50 = 2n (2n - 1)$

$2n^2 - 2n + 50 = 4n^2 - 2n$

$2n^2 = 50$

$n^2 = 50 / 2$

$= 25$

$n = \sqrt{25}$

$n = 5$

**Question 5: Rationalise the denominator and simplify [3√2] / [√5 − √2].**

**Solution:**

[3√2] / [√5 − √2]

= {[3√2] / [√5 − √2]} * {[√5 + √2] / [√5 + √2]}

= [3√2 √5 + 3√2 √2] / [$(\sqrt{5})^2 - (\sqrt{2})^2$]

= [3√10 + 6] / 3

= 3 (√10 + 2) / 3

= √10 + 2

**Question 6: A box has 4 red and 3 black marbles. Four marbles are picked up randomly. Find the probability that two marbles are red.**

**Solution:**

There are 7 marbles, out of these 4 marbles can be drawn in $^7C_4$ = 35 ways.

∴ n (S) = 35

Two marbles out of 4 red marbles can be drawn in $^4C_2$ = 6 ways.

The remaining 2 marbles must be black and they can be drawn in $^3C_2$ = 3 ways.

∴ n (A) = $^4C_2$ $^3C_2$ = 6 * 3 = 18

P (A) = n (A) / n (S)

= 18 / 35

**Question 7: Calculate the standard deviation for the following scores: 5, 6, 7, 8, 9.**

**Solution:**

| **x** | **$x^2$** |
|---|---|
| 5 | 25 |
| 6 | 36 |

| | |
|---|---|
| 7 | 49 |
| 8 | 64 |
| 9 | 81 |
| $\sum x = 35$ | $\sum x^2 = 255$ |

Standard deviation = $\sqrt{(\sum x^2 / N) - (\sum x / N)^2}$

= $\sqrt{(255 / 5) - (35 / 5)^2}$

= $\sqrt{51 - 49}$

= $\sqrt{2}$

= 1.414

**Question 8: Find the radius of a circle whose centre is (– 5, 4) and which passes through the point (– 7, 1).**

**Solution:**

$(x_1, y_1) = (-5, 4)$

$(x_2, y_2) = (-7, 1)$

$\therefore\ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Radius of the circle = $\sqrt{[-7 - (-5)]^2 + (1 - 4)^2}$

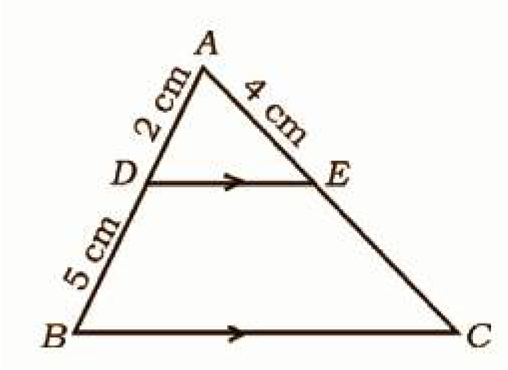
= $\sqrt{(-7 + 5)^2 + (-3)^2}$

= $\sqrt{(-2)^2 + (-3)^2}$

= $\sqrt{4 + 9}$

$r = \sqrt{13}$

**Question 9: In Δ ABC, DE | |BC, if AD = 2 cm, DB = 5 cm and AE = 4 cm, find AC.**




**Solution:**

In Δ ABC, DE || BC

∴ AD / DB = AE / EC [BPT]

2 / 5 = 4 / EC

EC = (4 * 5) / 2

= 10 cm

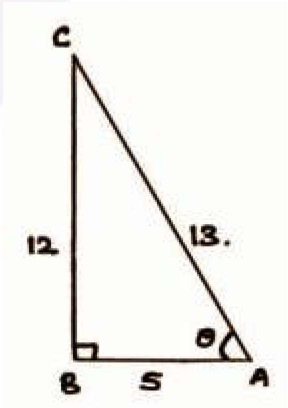
∴ AC = AE + EC

= 4 + 10

= 14 cm

**Question 10: If cos θ = 5 / 13, then find the value of (sin θ + cos θ) / (sin θ - cos θ).**

**Solution:**




cos θ = 5 / 13 = AB / AC

In triangle ABC, B = $90^o$.

$BC^2 = AC^2 - AB^2$

$BC^2 = 13^2 - 5^2$

$BC = \sqrt{169 - 25}$

$= \sqrt{144}$

= 12

$\sin \theta = 12 / 13$

$(\sin \theta + \cos \theta) / (\sin \theta - \cos \theta)$

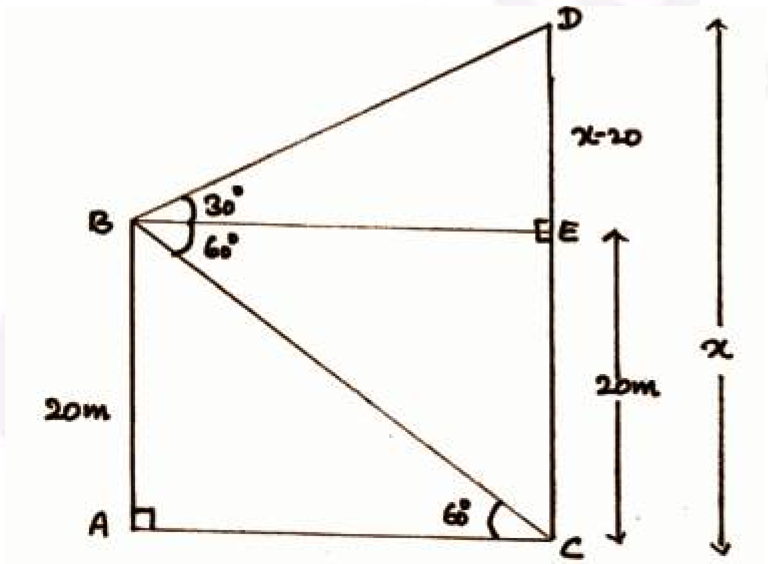
$= [(12 / 13) + (5 / 13)] / [(12 / 13) - (5 / 13)]$

$= (17 / 13) * (13 / 7)$

$= (17 / 7)$

**Question 11: From the top of a building 20 m high, the angle of elevation of the top of a vertical pole is 30° and the angle of depression of the foot of the same pole is 60°. Find the height of the pole.**

**Solution:**




In $\triangle BED$, $DBE = 30^o$

$\tan 30^o = DE / BE$

$1 / \sqrt{3} = (x - 20) / BE$

$BE = \sqrt{3} (x - 20)$

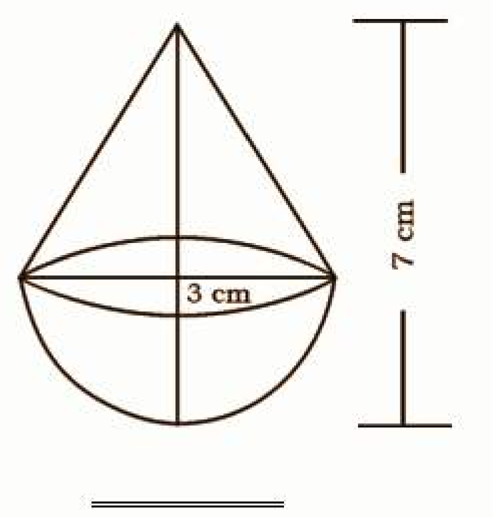
In $\triangle ABC$, $ACB = 60^o$

$\tan 60^o = AB / AC$

$\sqrt{3} = 20 / [\sqrt{3} (x - 20)]$

3 (x - 20) = 20
3x - 60 = 20
3x = 80
x = 80 / 3 = 26.6 cm
Height of the pole = 26.6 cm

**Question 12: A solid metallic cylinder of diameter 12 cm and height 15 cm is melted and recast into toys in the shape of a right circular cone mounted on a hemisphere as shown in the figure. If radii of the cone and hemisphere are each equal to 3 cm and the height of the toy is 7 cm, calculate the number of such toys that can be formed.**




**Solution:**
Height of the cone = h = 24 cm
Slant height of the cone = l = 25 cm.
∴ Lateral Surface Area of the cone = $\pi rl$
= $\pi \times 7 \times 25$ sq.cm
= 175 $\pi$ sq.cm.
Lateral Surface Area of the cylinder = $2\pi rh$
= $2\pi \times 7 \times 30$ sq.cm
= 420 $\pi$ sq.cm.
Lateral Surface Area of the hemisphere = $2\pi r^2$
= $2\pi \times 7^2$
= $98\pi$ sq.cm.

Total Surface Area of the solid = Lateral Surface Area of the cone + Lateral Surface Area of the cylinder + Lateral Surface Area of the hemisphere

$= (175 \pi + 420 \pi + 98 \pi)$ sq.cm.

$= (22 / 7) * 693$

= 2178 sq.cm.

Cost of painting = (2178 * 10) / 100

= Rs. 217·8

**Question 13: Find three consecutive positive integers such that the sum of the square of the first integer and the product of the other two is 92.**

**Solution:**

Let x, (x + 1) and (x + 2) be the three consecutive positive integers.

According to the given,

$x^2 + (x + 1)(x + 2) = 92$

$x^2 + x^2 + 2x + x + 2 = 92$

$2x^2 + 3x + 2 – 92 = 0$

$2x^2 + 3x – 90 = 0$

$2x^2 – 12x + 15x – 90 = 0$

$2x (x – 6) + 15 (x – 6) = 0$

$(x – 6)(2x + 15) = 0$

$x – 6 = 0, 2x + 15 = 0$

$x = 6, x = - 15 / 2$

The value of x cannot be negative.

Therefore, x = 6

Hence, the required three consecutive positive integers are 6, 7 and 8.

**Question 14: At constant pressure, a certain quantity of water at 24°C is heated. It was observed that the rise in temperature was found to be 4°C per minute. Calculate the time required to raise the temperature of water to 100°C at sea level by using formula.**

**Solution:**

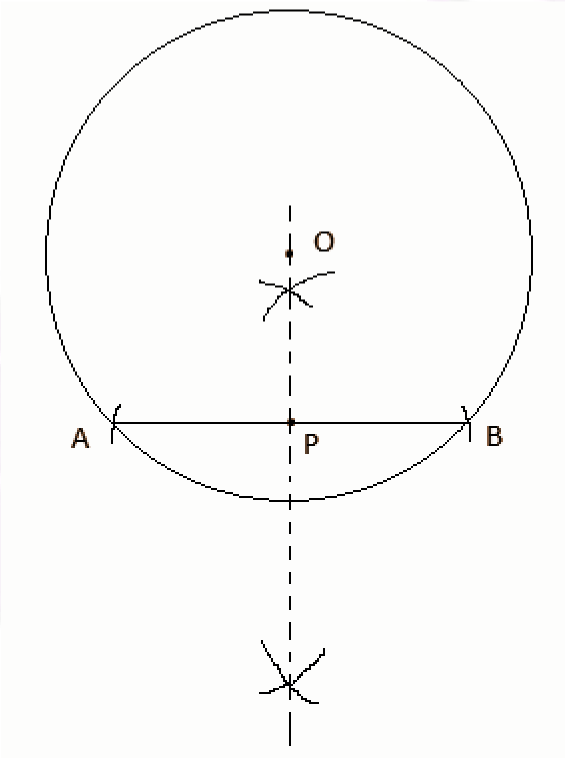
The temperature of the water is at 24 degrees and raised by 4 degrees per minute.

This forms an arithmetic progression 24, 28 , 32, 36, ....100

a = 24
d = 4
$T_n = a + (n - 1)d$
100 = 24 + (n - 1) 4
76 = (n - 1) 4
76 / 4 = (n - 1)
19 = (n - 1)
19 + 1 = n
20 = n

**Question 15: Draw a chord of length 6 cm in a circle of radius 5 cm. Measure and write the distance of the chord from the centre of the circle.**

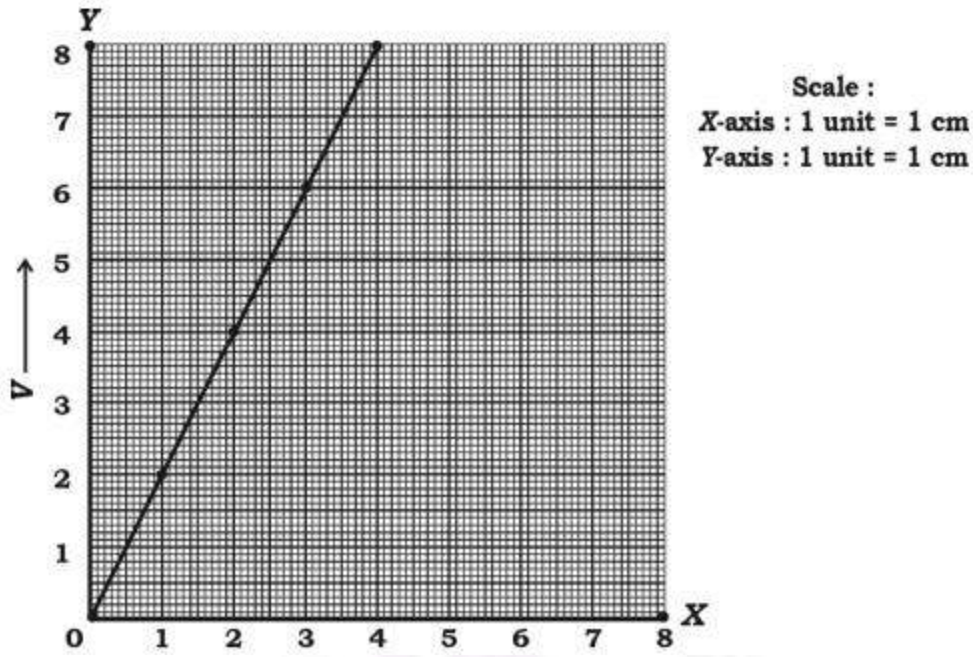
**Solution:**




Radius = 5 cm
Length of chord AB = 6 cm
The distance of the chord from the centre O is OC = 4 cm (by measure)

**Question 16: A student while conducting an experiment on Ohm's law, plotted the graph according to the given data. Find the slope of the line obtained.**

| X-axis I | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Y-axis V | 2 | 4 | 6 | 8 |




**Solution:**

From the given graph:

$(x_1, y_1) = (1, 2)$

$(x_2, y_2) = (2, 4)$

$(x_3, y_3) = (3, 6)$

$(x_4, y_4) = (4, 8)$

Slope = $(y_4 - y_1) / (x_4 - x_1)$

$= (8 - 2) / (4 - 1)$

$= 6 / 3$

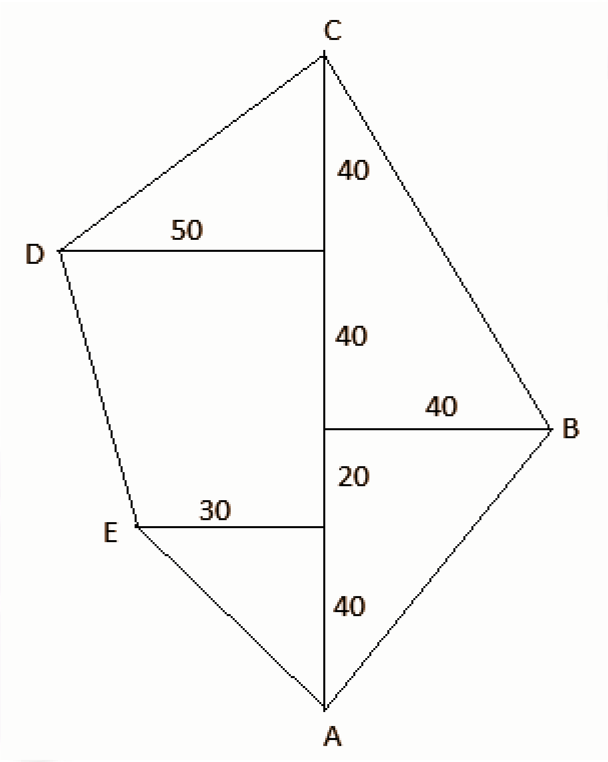
$= 2$

Hence, the slope of the line obtained is 2.

**Question 17: Draw the plan for the information given below: (Scale 20 m = 1 cm).**

| | Metre To C | |
|---|---|---|
| To D 50 | 140<br>100<br>60 | 40 to B |
| To E 30 | 40 | |
| | From A | |

**Solution:**




**Question 18: A dealer sells an article for Rs. 16 and loses as much per cent as the cost price of the article. Find the cost price of the article.**

**Solution:**

Let x be the cost price of the article.

Loss percentage = x%

The selling price of the article = Rs. 16

Loss percentage = (Loss/CP) × 100

x = (loss/x) × 100

$x^2$ = 100 × loss

⇒ Loss = $x^2$ / 100

⇒ x – 16 = $x^2$ / 100

⇒ $x^2$ = 100x – 1600

⇒ $x^2$ – 100x + 1600 = 0

⇒ $x^2$ – 80x – 20x + 1600 = 0

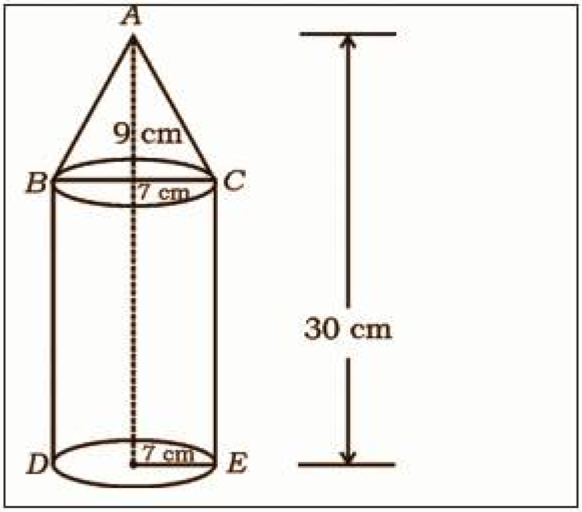
⇒ x (x – 80) – 20 (x – 80) = 0

⇒ (x – 80) (x – 20) = 0

⇒ x = 80, x = 20

Therefore, the cost price of the article is Rs. 20 or Rs. 80.

**Question 19: A solid is in the form of a cone mounted on a right circular cylinder, both having the same radii as shown in the figure. The radius of the base and height of the cone is 7 cm and 9 cm respectively. If the total height of the solid is 30 cm, find the volume of the solid.**




**Solution:**

The radius of the circular base of the cylinder and cone = r = 7 cm

Height of the cone = h = 9 cm

The total height of the solid = 30 cm

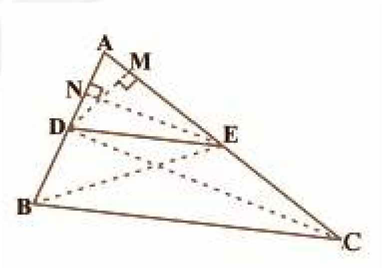
Thus, height of the cylinder = H = 30 – 9 = 21 cm
Volume of cone = (1 / 3) $\pi r^2 h$
= (1 / 3) × (22 / 7) × 7 × 7 × 9
= 22 × 7 × 3
= 462 $cm^3$
Volume of cylinder = $\pi r^2 H$
= (22 / 7) × 7 × 7 × 21
= 22 × 7 × 21
= 3234 $cm^3$
Therefore, the total volume of the solid = 462 + 3234 = 3696 $cm^3$.

**Question 20: State and prove Basic Proportionality (Thale's) theorem.**

**Solution:**

**Statement:** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

**Proof:**
In triangle ABC, a line parallel to side BC intersects other two sides namely AB and AC at D and E respectively.
Join BE and CD.
Also, draw DM ⊥ AC and EN ⊥ AB.




Area of △ ADE = 1 / 2 (AD × EN)

Area (△ ADE) = 1 / 2 (AD × EN)
Similarly,
Area (△ BDE) = 1 / 2 (DB × EN)

Area (Δ ADE) = 1 / 2 (AE × DM)
Area (Δ DEC) = 1 / 2 (EC × DM)
Area (Δ ADE) / Area (Δ BDE) = [1 / 2 AD × EN)] / [1 / 2 (DB × EN)]
= AD / DB ….(i)
Area (Δ ADE) / Area (Δ DEC) = [1 / 2 (AE × DM)] / [1 / 2 (EC × DM)]
= AE / EC ….(i)
Triangle BDE and DEC are on the same base DE and between the same parallels.
Therefore, area (Δ BDE) = area (Δ DEC) ….(iii)
From (i), (ii) and (iii),
AD / DB = AE / EC
Hence proved.