KBPE Class 11th Maths Important Questions

Question 1: Find sum to "n" terms of the sequence 4 + 44 + 444 + _________.

Solution:

4 + 44 + 444 + ... to n terms

= 4 [1 + 11 + 111 + ... to n terms]

= (4 / 9) [9 + 99 + 999 + ... to n terms]

= (4 / 9) [$(10 - 1) + (10^2 - 1) + (10^3 - 1) + ... + (10^n - 1)$]

= (4 / 9) [$(10 + 10^2 + 10^3 + ... + 10^n) - (1 + 1 + 1 + ...$ n times)]

= (4 / 9) {10 [$(10^n - 1) / (10 - 1)$] - n(1)}

= (4 / 9) [(10 / 9) ($10^n - 1$) - n]

Question 2: Solve sin 2x + sin 4x + sin 6x = 0.

Solution:

(sin 2x + sin 6x) + sin 4x = 0

2 sin 4x . cos2x + sin 4x = 0

sin 4x (2 cos 2x + 1) = 0

sin 4x = 0 or 2 cos 2x + 1= 0

sin 4x = 0 or cos 2x = - 1 / 2 = - cos **π** / 3 = cos (**π** - **π** / 3)

Using sinx = 0 ⇒ x = n**π**

sin 4x = 0

4x = n**π**

The general solution is x = (n**π**) / 4 using cos x = cos α

⇒ x = 2mx ± α

cos 2x = cos ((2**π**) / 3)

2x = 2m**π** ± (2**π**) / 3

The general solution is x = m**π** ± **π** / 3 where m,n in z.

Question 3: One card is drawn at random from a pack of 52 playing cards. Find the probability that:

[a] The card drawn is black
[b] The card drawn is a face card
[c] The card drawn is a blackface card

Solution:
[a] There are 26 black cards in a deck.
Let E_1 = event of getting a black card
Number of favourable outcomes = 26
P (E_1) = Number of favourable outcomes / total number of outcomes
P (E_1) = 26 / 52 = 1 / 2

[b] There are 12 face cards.
Let E_2 = Event of getting a face card
Number of favourable outcomes to E_2 = 12
P(E_2) = Number of favourable outcomes / total number of outcomes
P(E_2) = 12 / 52 = 3 / 13

[c] The probability of choosing a black face card is 6 / 52.

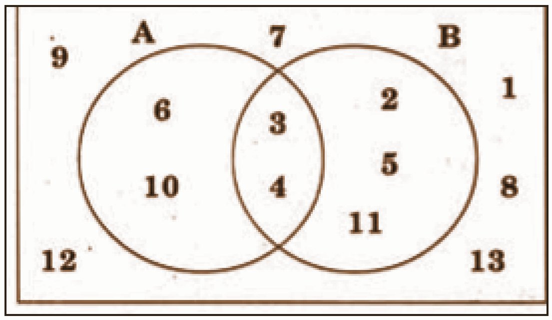
Question 4: (a) If A= {a, b, c}, then write the power set P (A).
(b) If the number of subsets with 2 elements of a set P is 10, then find the total number of elements in set P.
(c) Find the number of elements in the power set of P.

Solution:
[a] P(A) = {○ , {a} , {b} , {c} , {a , b} , {b , c} , {c , a} , {a , b , c} }

[b] Number of subsets with 2 elements = $^nC_2$
$^nC_2 = 10$
n (n - 1) ÷ 2 = 10
n (n - 1) = 20
n (n - 1) = 5 (5 - 1)
n = 5

[c] 2^n

Question 5: Consider the Venn diagram of the Universal Set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}



[a] Write sets A and B in Roster form

[b] Verify (A ∪ B)′ = A′ ∩ B′

Solution:

[a] A = {6, 10, 3, 4}

B = {2, 3, 4, 5, 11}

[b] A ∪ B = {2, 3, 4, 5, 6, 10, 11}

(A ∪ B)' = {1, 7, 8, 9, 12, 13}

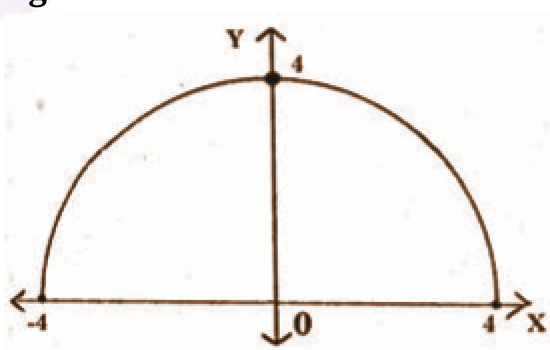
A' = {1, 2, 5, 7, 8, 9, 12, 13}

B' = {1, 6, 7, 8, 9, 10, 12, 13}

A′ ∩ B′ = {1, 7, 8, 9, 12, 13}

(A ∪ B)′ = A′ ∩ B′

Question 6: The figure shows the graph of a function f(x), which is a semi-circle centred at the origin:



[a] Write the domain and range of the function.

[b] Define the function f (x).

Solution:

[a] Domain = [- 4, 4]

Range = [0, 4]

[b] $x^2 + y^2 = 16$

$y^2 = 16 - x^2$

$y = \sqrt{16 - x^2}$

$f(x) = \sqrt{16 - x^2}$

Question 7: Consider a point A (4, 8, 10) in space.

[a] Find the distance of the point A from XY – plane.

[b] Find the distance of point A from the x-axis.

[c] Find the ratio in which the line segment joining the point A and B (6, 10, -8) is divided by YZ- plane.

Solution:

[a] 10

[b] Let P (4, 0, 0) be a line on the x-axis.

Distance = $\sqrt{(4 - 4)^2 + (8 - 0)^2 + (10 - 0)^2} = 2\sqrt{41}$

[c] $[mx_2 + nx_1] / [m + n]$

$[mx_2 + nx_1] = 0$

$m / n = - x_1 / x_2 = - 2 / 3$

m:n = - 2:3

Question 8: Consider the quadratic equation, $x^2 + x + 1 = 0$

[a] Solve the quadratic equation.

[b] Write the polar form of one of the roots

[c] If the two roots of the given quadratic are α and β, show that $\alpha^2 = \beta$.

Solution:

[a] $x^2 + x + 1 = 0$

$x = -b \pm \sqrt{b^2 - 4ac} / 2a$

$x = -1 \pm \sqrt{3}i / 2$

[b] Let $Z = (-1/2) + i\sqrt{3}/2$

$x = -1/2$ and $y = \sqrt{3}/2$

$r = \sqrt{x^2 + y^2} = \sqrt{(1/4) + (3/4)} = 1$

$\theta = \pi - (\pi/3) = 2\pi/3$

$Z = r(\cos\theta + i\sin\theta)$

$= 1(\cos(2\pi/3) + i\sin(2\pi/3))$ is the polar form

Question 9: Consider the following data:

Class	10-20	20-30	30-40	40-50	50-60
Frequency	6	15	13	7	9

[a] Calculate the mean of the distribution.
[b] Find the standard deviation of the distribution.
[c] Find the coefficient of variation of the distribution.

Solution:

[a]

Class	f_i	x_i	f_ix_i	x_i^2	$f_ix_i^2$
10 - 20	6	15	90	225	1350
20 - 30	15	25	375	625	9375
30 - 40	13	35	455	1225	15925
40 - 50	7	45	315	2025	14175
50 - 60	9	55	495	3025	27225
	50		**1730**		**68050**

[a] Mean $= \sum f_ix_i / \sum f_i$

$= 1730 / 50$

$= 34.6$

[b] Variance = $\sum f_i x_i^2 / \sum f_i$ - (mean)^2
= [68050 / 50] - [34.6]^2
= 163.84
Standard deviation = 12.8

[c] Coefficient of variation = [SD / (mean)] * 100
= (12.8 / 34.6) * 100
= 36.99

Question 10: Consider the statement "10^{2n-1} + 1 is divisible by 11". Verify that P(1) is true and then prove the statement by using mathematical induction.

Solution:
P (1) = $10^{2n-1} + 1 = 10^{2-1} + 1 = 10 + 1 = 11$ is divisible by 11.
Hence P (1) is true.
P (k) = 10^{2k-1} + 1 is divisible by 11
$10^{2k-1} + 1 = 11d$
$10^{2k-1} = 11d - 1$ ---- (1)
To prove that P (k + 1) is true
P (k + 1) = $10^{2(k+1)-1} + 1$
$10^{2k+2-1} + 1 = 10^{2k-1+2} + 1$
= $10^{2k-1} . 10^2 + 1$
= $100 . 10^{2k-1} + 1$
= 100 [11d - 1] + 1
= 100 [11d] - 100 + 1
= 100 [11d] - 99
= 11 [100d - 9]
= 11d
P (k + 1) is true.
Hence P(n) is true for all n $\in$ N.

Question 11[a]: Solve the inequality x / 3 > x / 2 + 1

(b) Solve the system of inequalities graphically:

2x + y > 6

3x + 4y ≤ 12

Solution:

[a] x / 3 > x / 2 + 1

On multiplying by 6,

6 (x / 3) > 6 (x / 2) + 6 (1)

2x > 3x + 6

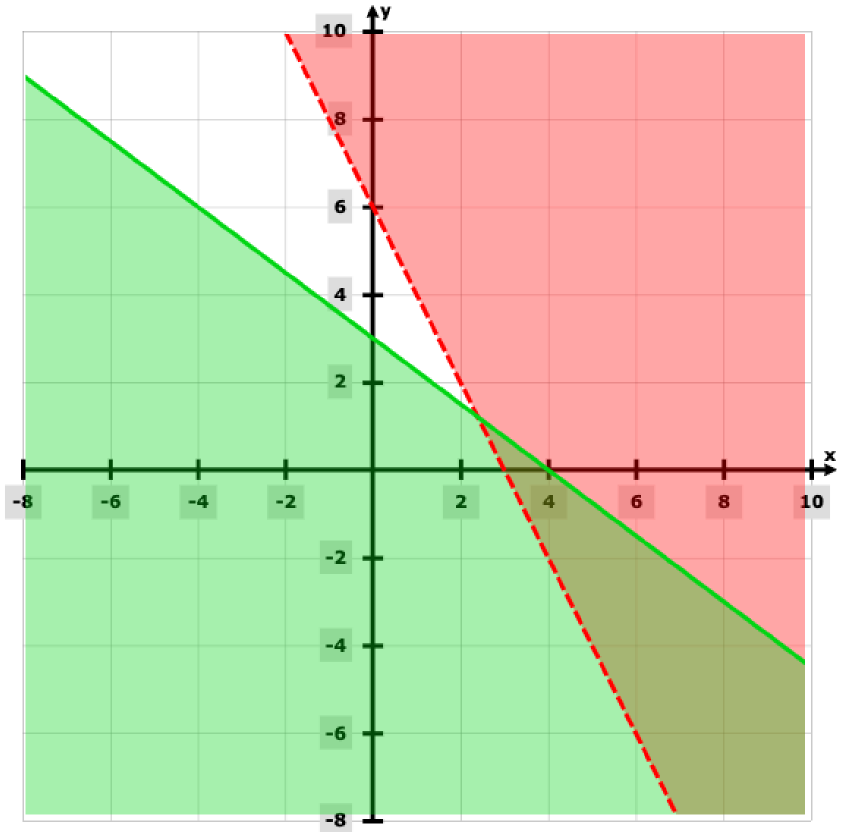
2x - 3x > 6

- x > 6

x < -6

$x = (-\infty, -6)$

[b]



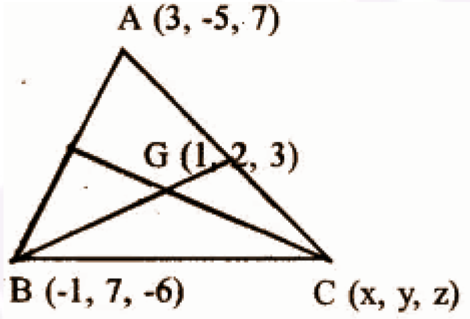
Question 12[a]: The distance between the points (1, -2, 3) and (4, 1, 2) is ______

[i] √12
[ii] √19
[iii] √11
[iv] √15

[b] The centroid of a triangle ABC is at the point (1, 2, 3). If the coordinates of A and B are (3, -5, 7) and (-1, 7, -6) respectively. Find the coordinate points of C.

Solution:
[a] Answer: [b]

[b]



[3 + (-1) + x] / 3
⇒ 2 + x = 3
⇒ x = 3 - 2 = 1
[-5 + 7 + y] / 3
⇒ 2 + y = 6
⇒ y = 6 - 2 = 4
[7 + (-6) + z] / 3
⇒ 1 + z = 9
⇒ z = 9 - 1 = 8
The coordinates of C are (1, 4, 8).

Question 13: [a] Find the variance for the observations 2,4,6,8 and 10.

(b) Consider the frequency distribution

x	5	10	15	20	25
f	7	4	6	3	5

[i] Find the mean.
[ii] Find the mean deviation about the mean.

Solution:
[a]

x	5	10	15	20	25
f	7	4	6	3	5
fx	35	40	90	60	125
x - (mean)	9	4	1	6	11
f [x - (mean)]	63	16	6	18	55

[i] Mean = $\sum f_i x_i / \sum f_i$
= 350 / 25
= 14

[ii] Mean deviation about mean = $\sum f_i |x - (\text{mean})| / N$
= 158 / 25
= 6.32

Question 14[i]: sin 225°__________
(a) $1 / \sqrt{2}$
(b) $\sqrt{3} / 2$
(c) $-1 / \sqrt{2}$
(d) 1 / 2

(ii) Find the principal and general solutions of sin x = − √3 / 2

(iii) Prove that tan [A − B / 2] = [a − b] / [a + b] cot c / 2.

Solution:

[i] Answer: (c)

[ii] $\sin x = -\sqrt{3} / 2$

$= -\sin(\pi / 3)$

$= \sin(\pi + \pi / 3)$

$= \sin(4\pi / 3)$

$\sin x = \sin(4\pi / 3)$

$x = n\pi + (-1)^n (4\pi / 3)$

[c] From the law of cosines a = R sin A, b = R sin B

(a - b) / (a + b) = [2R sin A - 2R sin B] / [2R sin A + 2R sin B]

= [sin A - sin B] / [sin A + sin B]

= [2 cos (A + B) / 2 * sin (A - B) / 2] / [2 sin (A + B) / 2 * cos (A - B) / 2]

= cot (A + B) / 2 * tan (A - B) / 2

= cot (90 - C / 2) * tan (A - B) / 2

= tan C / 2 * tan (A - B) / 2

= 1 / cot (C / 2) * tan (A - B) / 2

tan (A - B) / 2 = [(a - b) / (a + b)] cot C / 2

Question 15[a]: Find the equation of the line passing through the points (3, -2) and (-1, 4).

(b) Reduce the equation √3x + y − 8 = 0 into normal form.

(c) If the angle between two lines is π / 4 and the slope of one of the lines is 1 / 2, find the slope of the other line.

Solution:

[a] $y - y_1 = [y_2 - y_1 / x_2 - x_1] (x - x_1)$

y + 2 = [4 + 2] / [- 1 - 3] [x - 3]

y + 2 = (- 3 / 2) (x - 3)

2y + 4 = - 3x + 9

3x + 2y - 5 = 0

[b] $\sqrt{3}x + y - 8 = 0$

$\sqrt{3}x + y = 8$

Dividing by $(\sqrt{3})^2 + 1^2 = 2$

$(\sqrt{3} / 2)\ x + (1 / 2)\ y = 4$

[c] $\cos 30^o\ x + \sin 30^o\ y = 4$

$m_1 = (1 / 2)$

$m_2 = m_1$

$\tan \theta = |(m_2 - m_1) / (1 + m_1 m_2)|$

$\tan \pi / 4 = |(m_2 - (1 / 2)) / 1 + (1 / 2)\ m_2|$

m = 3 or (- 1 / 3)

Question 16: [a] Which one among the following is the interval corresponding to the inequality: $-2 < x \leq 3$?

(a) [-2, 3]

(b) [-2, 3)

(c) (-2, 3]

(d) (-2, 3)

(b) Solve the following inequalities graphically:

$2x + y \geq 4$

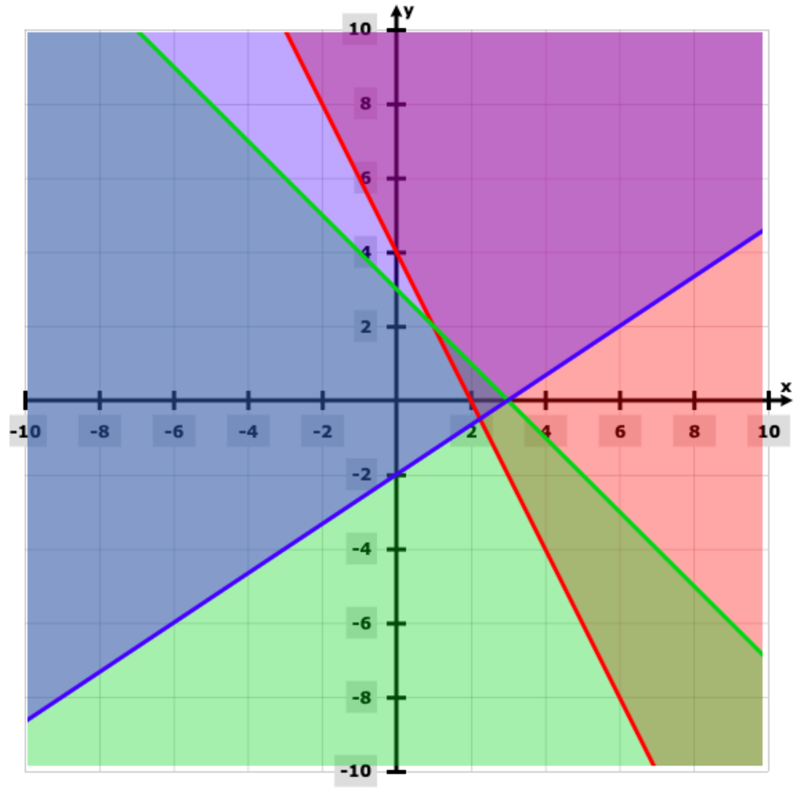
$x + y \leq 3$

$2x - 3y \leq 6$

Solution:

[a] (-2, 3]

[b]



Question 17: (a) Write the negation of the statement :
"Every natural number is greater than zero"
(b) Verify by the method of contradiction :
"P: √13 is irrational".

Solution:
[a] Negation: It is false that every natural number is greater than 0. There exists a natural number which is not greater than 0.

[b] Assume that √13 is rational, √13 = (a / b), where a and b are coprime.
$13b^2 = a^2$
13 divides a
There exists an integer k such that a = 13k.
$a^2 = 169k^2$
$13b^2 = 169k^2$
$13k^2$ = 13 divides b

13 divides both a and b which is a contradiction to our assumption that $\sqrt{13}$ is rational. So, $\sqrt{13}$ is irrational.

Question 18: (a) A= { X / X is a prime number, X ≤ 6
(i) Represent A in the Roster form
(ii) Write the Powerset of A
(b) Out of the 25 members in an office, 17 like to take tea, 16 like to take coffee. Assume that each takes at least one of the two drinks.
How many like:
(i) Both Coffee and Tea?
(ii) Only Tea and not Coffee?

Solution:
[a] [i] Roster form = {2,3,5}
[ii] Power set = {( ), (2), (3), (5), (2, 3), (2, 5) (5, 3), (2, 3, 5)}

[b] [i] Let -
A = {people drinking tea}
B = {people drinking coffee}
$A \cap B$ = {people drinking tea / coffee}
$A \cup B$ = {people drinking both tea / coffee}
n (A) = 17
n (B) = 16
$n(A \cup B) = 25$
Number of people drinking both tea & coffee,
$n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$n(A \cap B) = 17 + 16 - 25$
$n(A \cap B) = 8$

[ii] Number of people drinking only tea,
$n(A / B) = n(A) - n(A \cap B)$
n (A / B) = 17 - 8
n (A / B) = 9

Therefore, the number of people drinking both tea & coffee is 8 and those drinking only tea are 9.

Question 19: (a) Number of terms in the expansion of $[x + 1 / x]^{10}$ is ______
(i) 10
(ii) 9
(iii) 11
(iv) 12
(b) Find the term independent of x in the above expansion.

Solution:
[a] Answer: 11

[b] a = x, b = (1 / x), n = 10
$t_{r+1} = {}^{n}C_r\, a^{n+r}\, b^r = {}^{10}C_r\, x^{10+r}\, (1 / x)^r$
$= {}^{10}C_r\, x^{10-r}\, (x)^{-r}$
$= {}^{10}C_r\, x^{10-r-r}$
$= {}^{10}C_r\, x^{10-2r}$
Put 10 - 2r = 0
10 = 2r
r = 5
$t_{5+1} = {}^{10}C_5\, x^{10-2*5}$
$t_6 = 252$

Question 20: (i) If the n^{th} term of the sequence is $n (n^2 + 5) / 4$, then find its first two terms.
[ii] How many terms of an AP -6, (11 / 2), -5, …… are needed to give the sum of -25.
[iii] Find the 10^{th} term of a GP whose 3^{rd} term is 24 and 6^{th} term is 192.

Solution:
[i] $a_1 = 3 / 2$; $a_2 = 9 / 2$

[ii] $S_n = (n / 2) (2a + [n - 1] d)$
-25 = (n / 2) [2 * (-6) + [n - 1] (1 / 2)]

n = 20, 5

[iii] $ar^2 = 24$, $ar^5 = 192$
a = 6, r = 2
$a_{10} = ar^9$
$6\,(2)^9 = 3072$

Question 21: Find the coordinates of the foci vertices, eccentricity and the length of the latus rectum of the ellipse $100x^2 + 25y^2 = 2500$.

Solution:
$x^2 / 25 + y^2 / 100 = 1$
The major axis is vertical.
a = 10; b = 5
$c^2 = a^2 - b^2 = 75$
The foci are at $(0, \pm \sqrt{75})$.