

Steps of construction:

- Draw a circle of 3cm radius and O as its centre.
- From the centre, mark $OP = 7\text{cm}$.
- Draw a perpendicular bisector of the line OP such that it meets at M.
- Draw a circle with radius OM and cut the circle at Q and R.
- Joint PR and PQ.
- Hence, PR and PQ are tangents.

Question 5: $P(x) = x^3 + ax^2 - x + b$ and

[a] Find the relation between a and b for x-1 to be a factor of P(x).

[b] Find the relation between a and b for x-2 to be a factor of P(x).

[c] Find a and b so that both x-1 and x-2 are factors of P(x).

Solution:

Given $P(x) = x^3 + ax^2 - x + b$

[a] $x - 1$ is a factor of P(x)

$$P(1) = 0$$

$$P(1) = (1)^3 + a \times (1)^2 - 1 + b = 0$$

$$= 1 + a - 1 + b = 0$$

$$a + b = 0$$

[b] $x - 2$ is a factor of P(x)

$$P(2) = 0$$

$$P(2) = 2^3 + a \times 2^2 - 2 + b = 0$$

$$= 8 + 4a - 2 + b = 0$$

$$= 4a + b = -6$$

[c] We have, $a + b = 0$ and $4a + b = -6$.
Solve these two equations, $a = -2$ and $b = 2$.

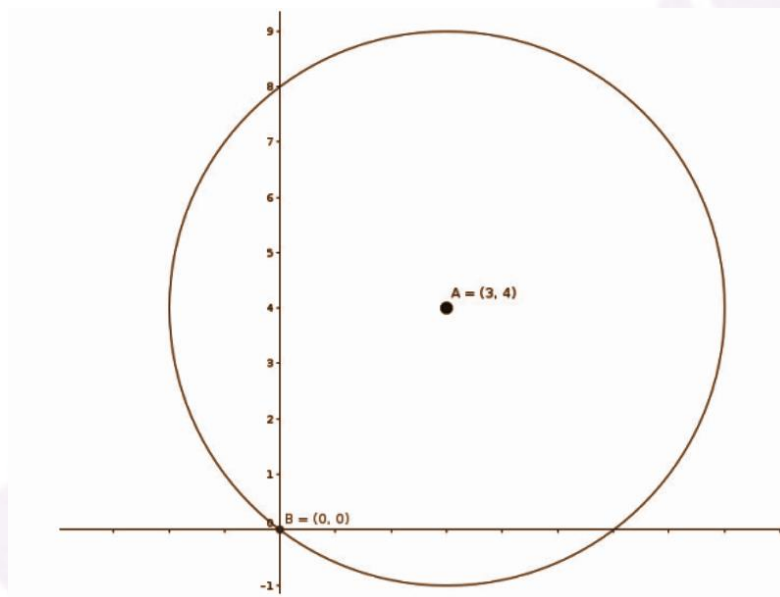
Question 6: A circle with centre (3, 4) passes through the origin.

[a] What is the radius of the circle?

[b] If a point in the circle is (x,y), write the relation between x, y?

[c] Check if the point (-2, 1) lies on this circle?

Solution:



Given , center (3, 4) ; Origin (0, 0).

[a] Radius (r) = $\sqrt{x^2 + y^2}$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5$$

[b] Equation of the circle = $(x - a)^2 + (y - b)^2 = r^2$

$$(x - 3)^2 + (y - 4)^2 = 5^2$$

$$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 = 25$$

$$\Rightarrow x^2 + y^2 - 6x - 8y + 25 - 25 = 0$$

$\Rightarrow x^2 + y^2 - 6x - 8y = 0$ is the equation of the circle.

[c] $(-2, 1)$, substitute this value in the equation of the circle, we get,

$$(-2)^2 + (1)^2 - 6 \times -2 - 8 \times 1 = 0$$

$$4 + 1 + 12 - 8 = 0$$

$$17 - 8 = 0$$

$$9 \neq 0$$

$AC >$ radius of the circle.

Hence, the point $(-2, 1)$ lies outside the circle.

Question 7: A boy saw the top of a building under construction at an elevation of 30° . The completed building was 12 meters higher and the boy saw its top at an elevation of 60° from the same spot.

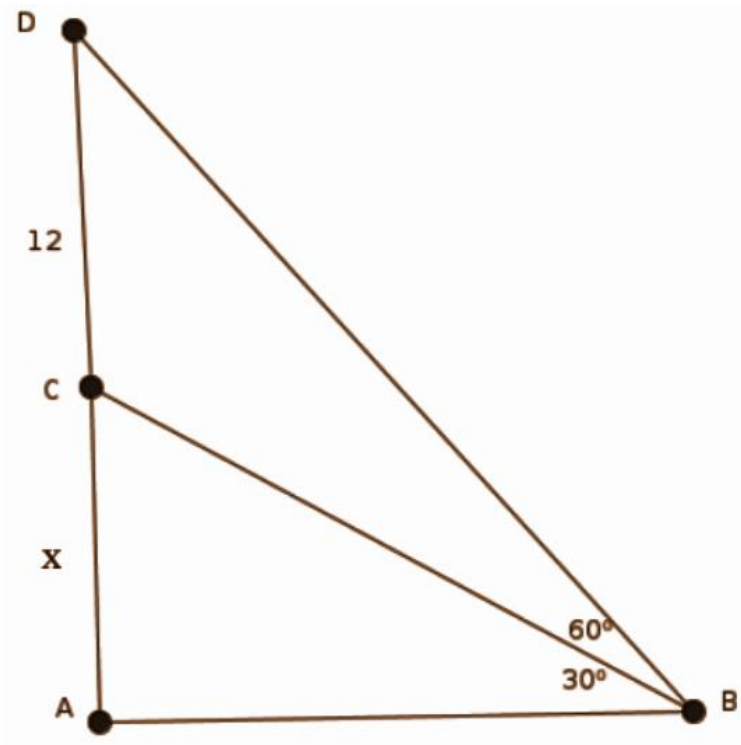
[a] Draw a rough figure based on the given details.

[b] What is the height of the building?

[c] What is the distance between the building and the boy?

Solution:

[a]



[b] Consider $AC = x$, $AD = x + 12$.

In triangle ABD, $AD / AB = \tan 60^\circ$

$$AB = AD / \tan 60^\circ$$

$$\Rightarrow AB = x + 12 / \sqrt{3} \dots\dots\dots (1)$$

In triangle ABC,

$$AC / AB = \tan 30^\circ$$

$$\Rightarrow AB = AC / \tan 30^\circ$$

$$\Rightarrow AB = x / [1 / \sqrt{3}]$$

$$= x \sqrt{3} \dots\dots\dots (2)$$

Comparing equations (1) and (2),

$$[x + 12] / [\sqrt{3}] = x \sqrt{3}$$

$$\Rightarrow 3x = x + 12$$

$$\Rightarrow 3x - x = 12$$

$$\Rightarrow 2x = 12$$

$$x = 6$$

Hence, the height of the building = $6 + 12 = 18$ m.

[c] Consider the equation (2), $AB = \sqrt{3} x$

$$= \sqrt{3} \times 6$$

$$= 1.73 \times 6$$

$$= 10.38\text{m}$$

Distance between the building and the boy = 10.38m.

Question 8: Cards marked with numbers 1, 2, 3, 4,, 20 are well shuffled and a card is drawn at random. What is the probability that the number on the card is a:

[a] prime number?

[b] divisible by 3?

[c] a perfect square?

Solution:

Total numbers = 20

[a] The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19

Favourable cases = 8

The probability of getting a prime number = $8 / 20 = 2 / 5$

[b] The numbers divisible by 3 are 3, 6, 9, 12, 15, 18

Favourable cases = 6

Probability of getting a number divisible by 3 = $6 / 20 = 3 / 10$

[c] The perfect squares are 1, 4, 9, 16

Favourable cases = 4

Probability of getting a perfect square number = $4 / 20 = 1 / 5$

Question 9: A person bought a certain number of pens for Rs. 800. If he had bought 4 pens more for the same money, he would have paid 10 less for each pen. How many pens did he buy?

Solution:

Let a person buy x number of pens.

One pen costs $800 / x$.

If he buys 4 more pens means (x + 4) pens then one pen will cost $(800 / x - 10)$.

So total money required to buy x + 4 pens is $(x + 4) * (800 / x - 10)$

$$= 800 + 3200 / x - 10x - 40$$

Multiplying above by x,

$$-10x^2 + 760x + 3200$$

$$x = -4 \text{ and } x = 80$$

80 pens are bought by the person.

Question 10: Prove that $[\cos^2 a + \tan^2 a - 1] / [\sin^2 a] = \tan^2 a$.

Solution:

$$\begin{aligned} & [\cos^2 a + \tan^2 a - 1] / [\sin^2 a] \\ &= [\cos^2 a / \sin^2 a] + [\tan^2 a / \sin^2 a] - [1 / \sin^2 a] \\ &= \cot^2 a + \sec^2 a - \operatorname{cosec}^2 a \\ &= [\cos^2 a - 1] / \sin^2 a + (1 / \cos^2 a) \\ &= 1 + \sec^2 a \\ &= \tan^2 a \end{aligned}$$

Question 11: A conical tent is to accommodate 11 persons. Each person must have 4 sq. m of the space on the ground and 20 cubic metres of air to breath. Find the height of the cone.

Solution:

Area of the base = $11 \times 4 = 44 \text{ m}^2$ and volume of the cone = $11 \times 20 = 220 \text{ m}^3$

$$[1 / 3] \times \pi R^2 h = 220 \text{ m}^3 \quad \dots(i)$$

$$\text{Area of the base} = \pi R^2$$

$$\pi R^2 = 44$$

$$R^2 = 44 / 22 \times 7$$

$$R^2 = 14$$

$$R = \sqrt{14} \quad \dots(ii)$$

By equation (i) and (ii),

$$[1 / 3] \times [22 / 7] \times \sqrt{14} \times \sqrt{14} \times h = 220$$

$$h = 220 \times [3 / 22 \times 2]$$

$$h = 30 / 2 = 15 \text{ cm}$$

Question 12: Mohan has a recurring deposit in a bank, where he deposited Rs.2500 per month for 2 years. If he gets Rs. 66,250 at the time of maturity, find:

[a] The interest paid by the bank

[b] The rate of interest

Solution:

$$I = MV - nx$$

$$I = 66250 - (2500 \times 24)$$

$$I = 66250 - 60000$$

$$I = 6,250$$

$$6250 = 2500 \times 24 \times 25 \times R / 2 \times 12 \times 100$$

$$6250 = 625R$$

$$R = 10\%$$

Question 13: The weekly wages of 40 workers in a small factory is given below. If the mean weekly wage is Rs. 145, find the value of a and b.

Daily wages	80 - 100	100 - 120	120 - 140	140 - 160	160 - 180
Number of workers	4	6	a	b	18

Solution:

Daily wages	80 - 100	100 - 120	120 - 140	140 - 160	160 - 180
Number of workers	4	6	a	b	18
Midpoint	90	110	130	150	170
$x_i f_i$	360	660	130a	150b	3060

$$\text{Mean} = 145$$

$$n = 40$$

$$4 + 6 + a + b + 18 = 40$$

$$a + b = 40 - 28$$

$$a + b = 12$$

$$a = b - 12 \text{ ---- (1)}$$

$$\text{Mean} = \frac{\sum fx}{\sum f_i}$$

$$145 = \frac{[360 + 660 + 130a + 150b + 3060]}{40}$$

$$5800 = 4080 + 130a + 150b$$

$$1720 = 130a + 150b$$

$$1720 = 130 [b - 12] + 150b$$

$$1720 = 130b - 1560 + 150b$$

$$1720 + 1560 = 280b$$

$$3280 / 280 = b$$

$$b = 12$$

$$a = b - 12$$

$$a = 12 - 12 = 0$$

Question 14: Find the value of x, given that $B^2 = A$, where

$$B = \begin{bmatrix} 2, 12 \\ 0, 1 \end{bmatrix} \text{ and } A = \begin{bmatrix} 4, x \\ 0, 1 \end{bmatrix} .$$

Solution:

$$A = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = A$$
$$\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$
$$x = 36$$

Question 15: Construct a ΔABC in which $AB = AC = 5\text{cm}$ and $BC = 3.2\text{cm}$. Using a ruler and a compass only draw the reflection $A'BC$ of ΔABC in BC . Draw lines of symmetry of the figure $ABA'C$.

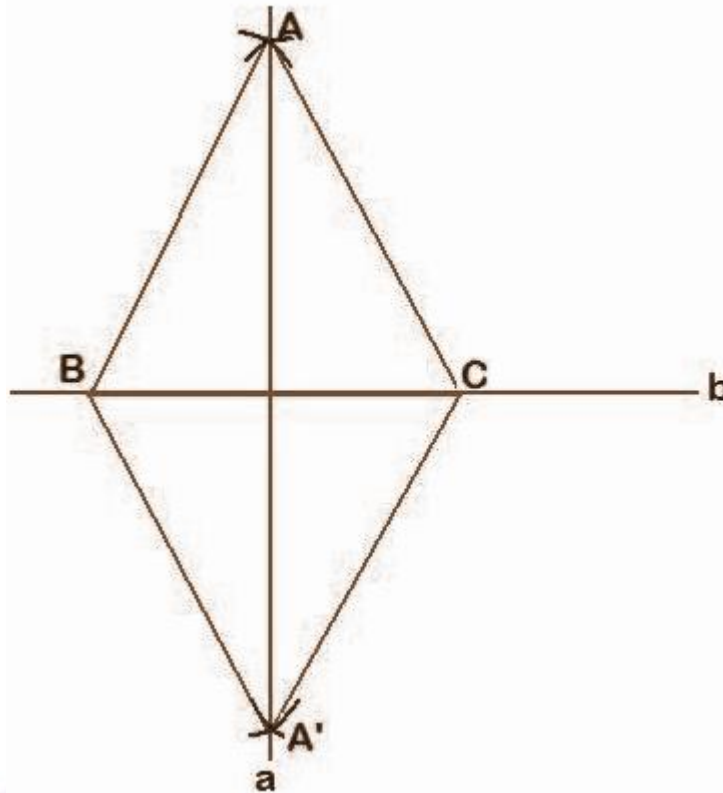
Solution:

$$AB = AC = 5 \text{ cm and } BC = 3.2 \text{ cm}$$

Step 1: Draw a line $BC = 3.2 \text{ cm}$

Step 2: Take the radius of 5 cm and centre as B and C, draw two arcs that intersect at A. The triangle ABC is obtained.

Step 3: The radius of 5 cm and centre B and C are taken, draw two arcs opposite side of point A, both the arcs intersect at A', The reflection of triangle ABC is obtained.



Question 16: [a] Write the 6th term of the arithmetic sequence 1, 25, 49, 73, 97

.....

[b] How many perfect square terms are there in the arithmetic sequence 97, 73, 49

Solution:

[a] Given an arithmetic sequence is 1, 25, 49, 73, 97 ,

First term (f) = 1

$d = 25 - 1 = 24$.

$T_6 = a + (n - 1)d$

6th term = $f + 5d$

$\Rightarrow 1 + 5 \times 24$

$$\Rightarrow 1 + 120 = 121$$

[b] Given an arithmetic sequence is 97, 73, 49,

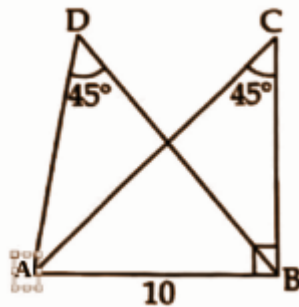
The perfect square numbers are 1, 4, 9, 16,

Hence the given sequence is 97, 73, 49, 25, 1.

From the above sequence, the perfect square numbers are 49, 25, and 1.

\therefore The number of perfect square terms are 3.

Question 17: In the figure, $\angle ABC = 90^\circ$, $\angle C = \angle D = 45^\circ$, $AB = 10$ cm.



[a] What is the length of AC?

[b] What is the radius of the circumcircle of triangle ABC?

[c] What is the radius of the circumcircle of triangle ABD?

Solution:

In right $\triangle ABC$, the angles are 45, 45, 90

$$\Rightarrow 1 : 1 : \sqrt{2}$$

$$\Rightarrow AB : BC : AC$$

$$\Rightarrow x : x : x\sqrt{2}$$

$$\Rightarrow 10 : 10 : 10\sqrt{2}$$

[a] The length of AC = $10\sqrt{2}$ cm.

[b] The radius of the circumcircle of $\triangle ABC$

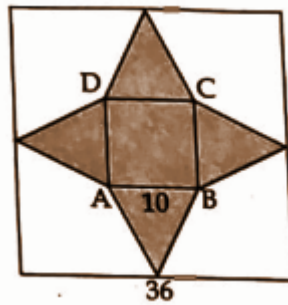
= Half of the hypotenuses AC

$$= 10\sqrt{2} / 2$$

$$= 5\sqrt{2} \text{ cm}$$

[c] The radius of the circumcircle of $\triangle ABD$
 = Half of the hypotenuses AC
 = $10\sqrt{2} / 2$
 = $5\sqrt{2}$ cm

Question 18: The figure of a square sheet paper is shown below. Length of one side of the paper sheet is 36cm and $AB = 10$ cm. The shaded portion is cut out and folded into a square pyramid.



- [a] What is the length of the base of the pyramid?
 [b] What is the slant height of the pyramid?
 [c] Find the lateral surface area of the pyramid.

Solution:

Given, Side of the paper sheet = 36cm

$AB = 10$ cm

[a] Base edge of the pyramid

$AB = 10$ cm

[b] Slant height of the pyramid

$$= [36 - 10] / 2$$

$$= 26 / 2$$

$$= 13\text{cm} [\because a + 2l = 36, \text{ side of the larger square}]$$

[c] Lateral surface area = $2al$

$$= 2 \times 10 \times 13$$

$$= 260 \text{ cm}^2$$

Question 19: In a school, the total number of students in 10 A division is equal to the number of students in 10 B. One student is to be selected from each division. The number of boys in 10 A is 20. The probability of selecting a boy from 10 A is $(2 / 5)$ and that of class B is $(3 / 5)$.

[a] How many students are there in 10 A?

[b] What is the probability of selecting a girl from 10 A?

[c] How many boys are there in 10 B?

[d] What is the probability of both the selected students being boys?

Solution:

Class	XA	XB
Boys	20	30
Girls	30	20
Total	50	50

Given the probability of boys in XA = $2 / 5$

Given the probability of boys in XB = $3 / 5$

[a] Number of boys in XA

$$= 20 \times [5 / 2]$$

$$= 50$$

[b] Probability of girl from XA

$$= 1 - [2 / 5]$$

$$= [5 - 2] / 5$$

$$= 3 / 5$$

[c] Number of boys in X B

$$= 50 \times [3 / 5]$$

$$= 10 \times 3$$

$$= 30$$

$$\begin{aligned} & \text{[d] Both being boys} \\ & = [2 / 5] \times [3 / 5] \\ & = 6 / 25 \end{aligned}$$

Question 20:

[a] If $p(x) = x^2 - 7x + 13$, what is $p(3)$?

[b] Write the polynomial $p(x) - p(3)$ as the product of two first degree polynomials.

[c] Find the solutions of the equation $p(x) - p(3) = 0$.

Solution:

[a] Given polynomial

$$p(x) = x^2 - 7x + 13$$

$$p(3) = 3^2 - 7 \times 3 + 13$$

$$= 9 - 21 + 13$$

$$= 1$$

$$\text{[b] } p(x) - p(3) = x^2 - 7x + 13 - 1$$

$$= x^2 - 7x + 12$$

$$= (x - 3)(x - 4)$$

Hence the product two first degree polynomial = $(x - 3)(x - 4)$

$$\text{[c] } p(x) - p(3) = 0$$

$$x^2 - 7x + 12 = 0$$

$$\Rightarrow (x - 3)(x - 4) = 0$$

$$\Rightarrow (x - 3) = 0 \text{ or } (x - 4) = 0$$

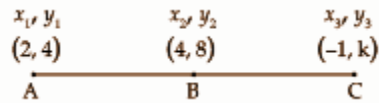
ie., $x = 3$ or $x = 4$.

Hence the solution is $x = 3$ and 4 .

Question 21: [i] If C (-1, k) is a point on the line passing through the points A (2, 4) and B (4, 8) which number is k?

[ii] What is the relation between the x coordinate and the y coordinate of any point on this line?

Solution:



[i]

Points A, B and C are collinear.

Area of triangle ABC = 0

$$(1 / 2) (x_1 [y_2 - y_3] + x_2 [y_3 - y_1] + x_3 [y_1 - y_2])$$

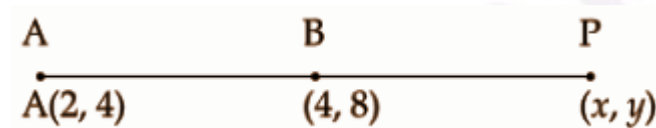
$$|2(8 - k) + 4(k - 4) + (-1)(4 - 8)| = 0$$

$$16 - 2k + 4k - 16 - 4 + 8 = 0$$

$$2k = -4$$

$$k = -2$$

[ii]



Area of triangle ABP = 0

$$(1 / 2) (x_1 [y_2 - y_3] + x_2 [y_3 - y_1] + x_3 [y_1 - y_2])$$

$$|2(8 - y) + 4(y - 4) + (x)(4 - 8)| = 0$$

$$16 - 2y + 4y - 16 - 4x = 0$$

$$2y - 4x = 0$$

$$2y = 4x$$

$$y = 2x$$

$$2x - y = 0$$

Question 22: A box contains some green and blue balls. 7 red balls are put into it. Now the probability of getting a red ball from the box is $7 / 24$ and that of the blue ball is $1 / 6$.

[i] How many balls are there in the box?

[ii] How many of them are blue?

[iii] What is the probability of getting a green ball from the box?

Solution:

Let the number of green balls be x.

The number of blue balls is y .

Number of red balls = 7

Total number of balls = $x + y + 7$

$P(\text{red ball}) = 7 / 24$

$P(\text{blue ball}) = 1 / 3$

[i] Since $P(\text{red ball}) = 7 / 24$,

$$7 / [x + y + 7] = 7 / 24$$

$$24 = x + y + 7$$

$$24 - 7 = x + y$$

$$17 = x + y \text{ ---- (1)}$$

$P(\text{blue ball}) = 1 / 3$

$$y / [x + y + 7] = 1 / 3$$

$$3y = x + y + 7$$

$$2y = x + 7$$

$$-x + 2y = 7 \text{ ---- (2)}$$

On adding equation (1) and (2),

$$17 = x + y$$

$$-x + 2y = 7$$

$$3y = 24$$

$$y = 24 / 3$$

$$y = 8$$

Put $y = 8$ in equation (1),

$$17 = x + 8$$

$$17 - 8 = x$$

$$x = 9$$

Total number of balls = $8 + 9 + 7 = 24$

[ii] Number of blue balls

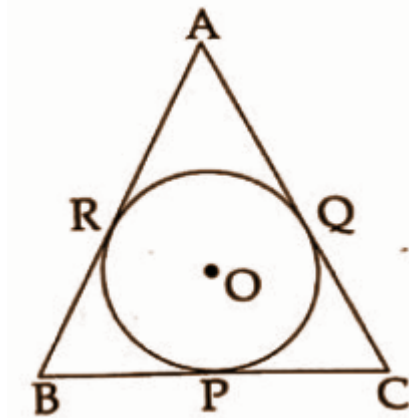
$$y / 24 = 1 / 3$$

$$3y = 24$$

$$y = 8$$

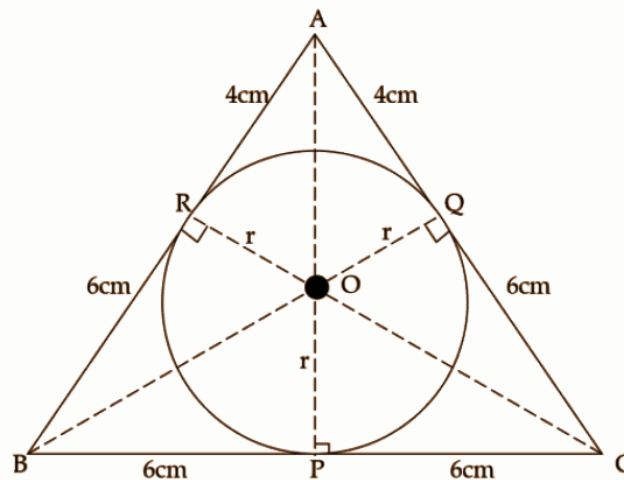
[iii] $P(\text{green ball}) = x / 24 = 9 / 24 = 3 / 8$

Question 23: Circle with centre O touches the sides of a triangle at P , Q and R , $AB = AC$, $AQ = 4\text{cm}$ and $CQ = 6\text{cm}$.



- [a] What is the length of CP ?
- [b] Find the perimeter and the area of the triangle.
- [c] What is the radius of the circle?

Solution:



[a] $CP = CQ$ [Length of external tangents are equal]

$$CP = 6\text{cm}$$

[b] Perimeter of triangle = $4 + 6 + 6 + 6 + 4 + 6 = 32\text{ cm}$

For the area of $\triangle ABC$,

$$s = [AB + BC + CA] / 2$$

$$= [10 + 12 + 10] / 2$$

$$= 16\text{cm}$$

$$\text{Area of } \triangle ABC = \sqrt{s (s - a) (s - b) (s - c)}$$

$$\begin{aligned}
 &= \sqrt{(16)(16 - 10)(16 - 12)(16 - 10)} \\
 &= \sqrt{16 * 6 * 4 * 6} \\
 &= 48\text{cm}^2
 \end{aligned}$$

[c] Area of $\triangle ABC$ = area of $\triangle AOB$ + area of $\triangle BOC$ + area of $\triangle COA$

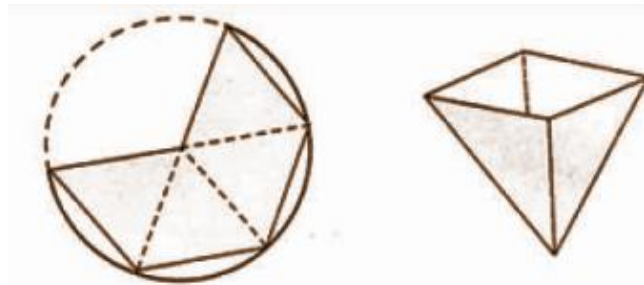
$$48 = (1/2) * 10 * r + (1/2) * 12 * r + (1/2) * 10 * r$$

$$48 * 2 = r(10 + 12 + 10)$$

$$48 * 2 = 32 * r$$

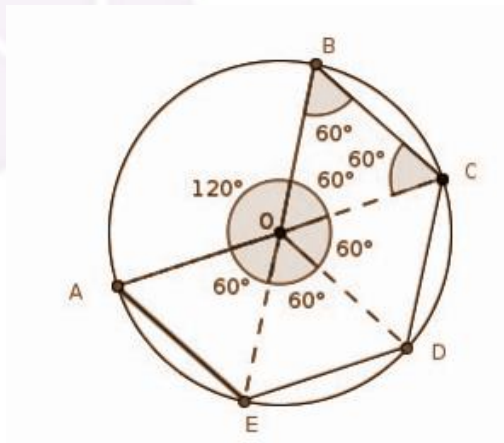
$$r = 3\text{cm}$$

Question 24:



From a tin sheet, a sector of radius 20cm and central angle 240° is divided into 4 equal parts as shown in the figure. The shaded portion is cut off. Using this, a vessel in the shape of a square pyramid is made. What is the capacity of this vessel?

Solution:



In $\triangle OBC$,

$$\angle O = \angle B = \angle C = 60^\circ$$

$$OB = BC = OC = 20 \text{ cm}$$

$$e = 20 \text{ cm}$$

$$a = 20 \text{ cm} \Rightarrow d = 20\sqrt{2}$$

$$h^2 = e^2 - (d/2)^2 = 20^2 - (10\sqrt{2})^2$$

$$= 400 - 200$$

$$= 200$$

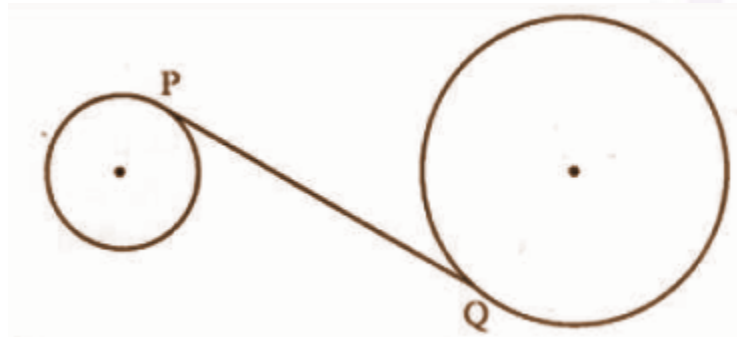
$$h = \sqrt{200} = 10\sqrt{2} \text{ cm}$$

$$\text{Volume} = [1/3] a^2 h$$

$$= [1/3] \times 20^2 \times 10\sqrt{2}$$

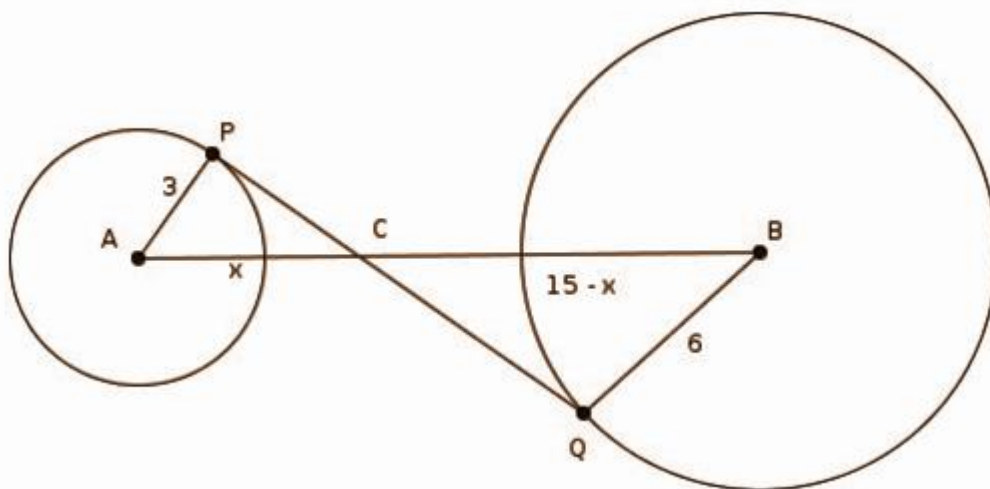
$$= 4000\sqrt{2} / 3 \text{ cm}^3$$

Question 25:



In the figure, the radius of the smaller circle is 3cm, that of the bigger circle is 6cm and the distance between the centres of the circles is 15cm. PQ is a tangent to both circles. Find its length.

Solution:



In $\triangle APC$ and $\triangle BQC$

$$\angle P = \angle Q = 90^\circ$$

$$\angle ACP = \angle BCQ$$

$$\triangle APC \sim \triangle BQC$$

$$AP / AC = BQ / BC$$

$$3 / x = 6 / 15 - x$$

$$6x = 3(15 - x)$$

$$= 45 - 3x$$

$$9x = 45$$

$$x = 5$$

$$AP = 3 \text{ cm}, AC = 5 \text{ cm}$$

$$\Rightarrow PC = 4 \text{ cm}$$

$$BQ = 6 \text{ cm}, BC = 10 \text{ cm}$$

$$\Rightarrow QC = 8 \text{ cm}$$

$$PQ = PC + QC$$

$$= 4 + 8$$

$$PQ = 12 \text{ cm}$$

