$90^o + 90^o + 70^o + \angle POR = 360^o$

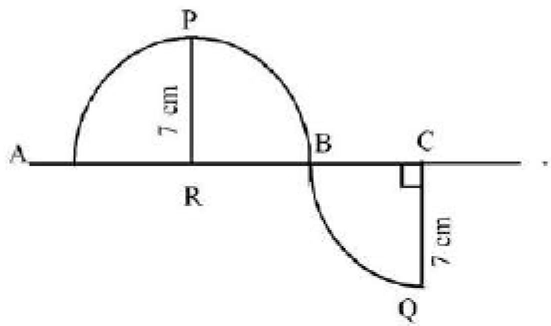
$\angle POR = 360^o - 250^o$

$\angle POR = 110^o$

Question 12: Find the circumference of a circle whose diameter is 14cm.

Solution:
The radius will be half of the diameter.
The diameter of the circle = 14 cm
Radius of the circle = 14 / 2 = 7 cm
Circumference = 2Πr
= 2 * Π * 7
= 2 * (22 / 7) * 7
= 43.98 cm

Question 13: Find the area of the shaded portion in the given figure:



Solution:
Area of the shaded portion = $(1 / 2) \pi r^2 + (1 / 4) \pi r^2$
$= (3 / 4) \pi r^2$
$= (3 / 4) * (22 / 7) * (7^2)$
$= 115.5 \text{ cm}^2$

Question 14: A CCTV camera is placed on the top of a straight 12-meter high pole in such a way that traffic can be seen beyond 13 meters of the line of sight of it. Find the distance from the foot of the pole beyond which the traffic is visible.

Solution:

Height of the pole = 12m

Length of the line of sight = 13m

$Hypotenuse^2 = Base^2 + Perpendicular^2$

$Base^2 = Hypotenuse^2 - Perpendicular^2$

$= 13^2 - 12^2$

= 169 - 144

= 25

$Base^2 = 25$

Base = 5m

Question 15: Find the square root of 6889 by using the Dwandwa Yoga Method.

Solution:

Number = 6889

83 | 6889

83 | 83

| 1

6889 = 83 * 83

$6889 = 83^2$

$\sqrt{6889} = \sqrt{83^2}$

$\sqrt{6889} = 83$

Question 16: By using the division algorithm method find the quotient and remainder when polynomial P (x) = $x^4 – 3x^2 + 4x - 3$ is divided by g (x) = $x^2 + (1 - x)$.

Solution:

$$
\begin{array}{r|rrrrr}
 & x^2 & +x & -3 & & \\
\hline
x^2 - x + 1 & x^4 & +0x^3 & -3x^2 & +4x & -3 \\
- & & & & & \\
 & x^4 & -x^3 & +x^2 & & \\
\hline
 & & x^3 & -4x^2 & +4x & -3 \\
 & - & & & & \\
 & & x^3 & -x^2 & +x & \\
\hline
 & & & -3x^2 & +3x & -3 \\
 & & - & & & \\
 & & & -3x^2 & +3x & -3 \\
\hline
 & & & & & 0
\end{array}
$$

Quotient: $x^2 + x - 3$
Remainder: 0

Question 17: Name the type of quadrilateral formed by the points (4, 5), (7, 6), (4, 3), (1, 2).

Solution:

Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D of the given quadrilateral, respectively.

AB = $\sqrt{(4-7)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

BC = $\sqrt{(7-4)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$

CD = $\sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$

AD = $\sqrt{(4-1)^2 + (5-2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$

Diagonal AC = $\sqrt{(4-4)^2 + (5-3)^2} = \sqrt{(0)^2 + (2)^2} = \sqrt{0+4} = 2$

Diagonal CD = $\sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52}$

The opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths.

Therefore, the given points are the vertices of a parallelogram.

Question 18: A vessel is in the form of a hollow hemisphere. The diameter of the hemisphere is 14 cm. Find the inner surface area of the vessel.

Solution:

D = 14cm (given)

R = 14 / 2 = 7cm

CSA of hemisphere = $2\pi r^2$

= 2 × [22 / 7] × 7 × 7

= 44 × 7

= 308 cm^2

Question 19: A car travels 260 km distance from a place A to place B, at a uniform speed, 65 km/hr passes through all thirteen green traffic signals, 4 minutes at the first signal, 7 minutes at the second signal, 10 minutes at a third signal and so on stops for 40 minutes at the thirteenth signal. How much total time does it take to reach place B? Solve by suitable Mathematical Method.

Solution:

Time sequence forms an A.P: 4, 7, 10......

a = 4

d = 7 − 4 = 3

For the 13^{th} station, time = 40

Total time is the sum of the first thirteen terms of the sequence

S = [n / 2] {2a + (n − 1) d}

S = [13 / 2] {2(4) + (13 − 1) 3}

S = [13 / 2] [8 + 36]

S = [13 / 2] × 44

S = 286

S = [286 / 60] hour

= 4 + [286 / 60]

= [240 + 286] / 60

= 526 / 60

= 8.76 hours

Question 20: Per day expenses of 25 families of the frequency distribution of a Dhani of a village is given as follows:

Per day expense (in Rs)	**25 - 35**	**35 - 45**	**45 - 55**	**55 - 65**	**65 - 75**
Number of families	**3**	**7**	**6**	**6**	**3**

Find the mean expense of families by Direct Method.

Solution:

Per day expense (in Rs)	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75
Number of families	3	7	6	6	3
Midpoint	30	40	50	60	70
$f_i x_i$	90	280	300	360	210

Mean = $\sum f_i x_i / \sum f_i$

= 1240 / 25

= 49.6

Question 21: Find the value of (tan 67°) / (cot 23°).

Solution:

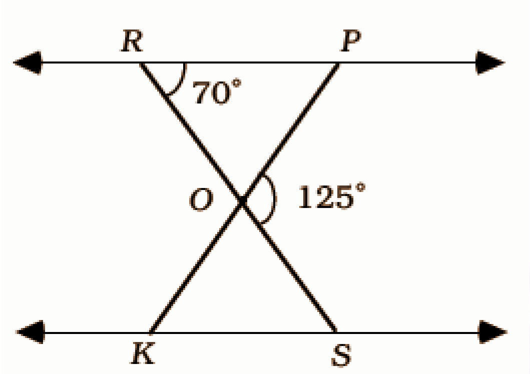
(tan 67°) / (cot 23°)

tan θ = cot (90° - θ)

tan 67^0 = cot (90° - 67°)

= cot 23°
= cot 23° / cot 23°
= 1

Question 22: In the figure, ΔOPR ~ ΔOSK , ∠POS = 125˚ and ∠PRO = 70˚. Find the values of ∠OKS and ∠ROP.



Solution:
Given that,
Δ OPR ~ Δ OSK, ∠POS = 125˚ and ∠PRO = 70˚
Sum of the supplementary angles = 180°
∠POS + ∠POR = 180°
125° + ∠ROP = 180°
∠ROP = 180° - 125°
∠ROP = 55°
∠OKS = ∠PRO [Since triangle Δ OPR ~ Δ OSK]
∠OKS = 70°

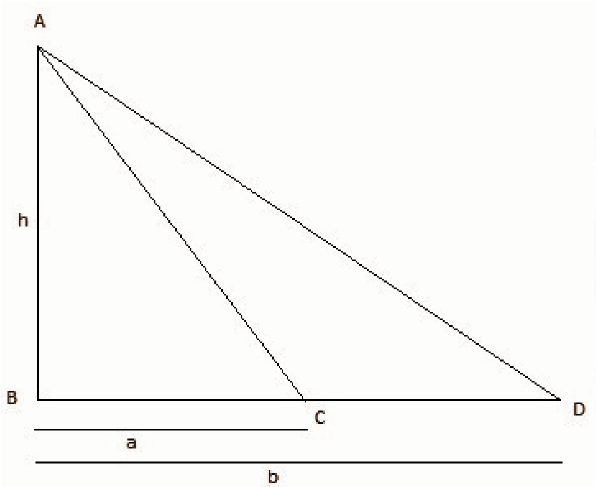
Question 23: Find quadratic polynomials whose sum and product of zeros are 8 and 12 respectively.

Solution:
A quadratic polynomial when the sum and products are given by f (x) = k, where k is constant.
Sum = 8
Product = 12

Polynomial = x^2 - (sum of zeroes) x + product of zeroes
= $x^2 - 8x + 12$

Question 24: The angle of elevation of the top of the tower from two points at a distance of a and b from the base of the tower in the same straight line with it are complementary. Prove the height of the tower is √ab.

Solution:



Height of the tower = h
Distance between the tower and point one = BC = A meter
Distance between the tower and point one = BD = B meter
The ∠ACB + ∠ADB = 90° (complementary angle) and ∠ABC = ∠ABD = 90°
∠ACB = θ
∠ADB = 90 − θ
In right-angle triangle ABC
tan θ = AB / BC --- (1)
tan θ = h / a
In right-angle triangle ABD
tan (90 - θ) = AB / BD --- (2)
tan (90 - θ) = h / b
Now multiplying equation 1 with equation 2,
tan θ * cot θ = (h / a) * (h / b)

$\tan\theta * (1 / \tan\theta) = h^2 / ab$
$1 = h^2 / ab$
$h^2 = ab$
$h = \sqrt{ab}$

Question 25: Circumference of a circle is equal to the perimeter of a square if the area of a square is 484 sq. meter, then find the area of the circle.

Solution:
If the side of square = xm, then perimeter of square $4 \times x$
Area of square = x^2
Area of square = 484 sq m^2
$x^2 = 484$
$x = \sqrt{484}$
x = 22 m
Perimeter of square = 4x
$= 4 \times 22$
= 88 m
Circumference of circle = Perimeter of square
$\Rightarrow 2\pi r = 88$
$\Rightarrow 2 \times (22 / 7) \times r = 88$
$r = [88 \times 7] / [2 \times 22]$
= 14 m
Area of the circle = πr^2
$= (22 / 7) \times 14 \times 14$
= 616 sq m.
Thus, the area of the circle = 616 sq m.

Question 26: A card is drawn from a well-shuffled pack of 52 cards. Find the probability of the following that the card is:
(a) Black
(b) Ace of Heart
(c) Spade

Solution:

(a) Out of 52 cards, 26 cards are black.

Favourable number of outcomes = 26

P (black card) = 26 / 52 = 1 / 2

(b) There are 4 aces out of which one is a heart.

Favourable number of outcomes = 1

P (ace of heart) = 1 / 52

(c) Out of 52 cards, there are 13 cards of spades

Favourable number of outcomes = 13

P (spade) = 13 / 52 = 1 / 4

Question 27: If the second and third terms of an Arithmetic Progression are 3 and 5 respectively, then find the sum of the first 20 terms of it.

Solution:

$a_2 = 3$

$a_3 = 5$

$a_3 - a_2 = 5 - 3 = 2$ [common difference]

$a_2 - d = a_1$

$3 - 2 = a_1$

$a_1 = 1$

$S_n = (n / 2) (2a + [n - 1] d)$

$S_{20} = (20 / 2) (2 * 1 + 19 * 2)$

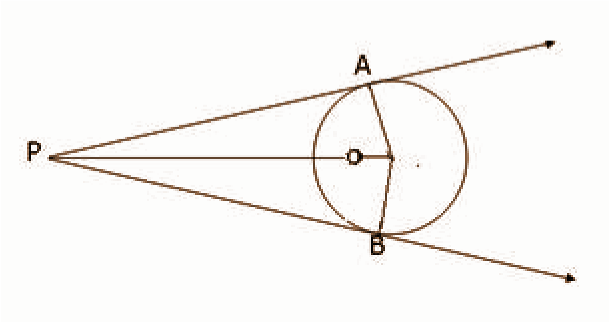
= 10 [2 + 38]

= 10 [40]

= 400

Question 28: Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Solution:



Consider a circle with centre O.

Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends $\angle AOB$ at centre O of the circle.

It can be observed that

$OA \perp PA$

$\therefore \angle OAP = 90^\circ$

Similarly, $OB \perp PB$

$\therefore \angle OBP = 90^\circ$

In quadrilateral OAPB,

Sum of all interior angles = 360º

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$

$\Rightarrow 90^\circ + \angle APB + 90^\circ + \angle BOA = 360^\circ$

$\Rightarrow \angle APB + \angle BOA = 180^\circ$

$\therefore$ The angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line−segment joining the points of contact at the centre.

Question 29: Prove that:

[i] $\sqrt{[1 + \cos\theta] / [1 - \cos\theta]} = \operatorname{cosec}\theta + \cot\theta$

[ii] $\tan\theta / [1 - \cot\theta] + \cot\theta / [1 - \tan\theta] = 1 + \tan\theta + \cot\theta$

Solution:

[i] √[1 + cos θ] / [1 − cos θ]

$$\sqrt{\frac{1+cos\theta}{1-cos\theta}} = \sqrt{\frac{1+cos\theta}{1-cos\theta} \times \frac{1+cos\theta}{1+cos\theta}}$$

$$=\sqrt{\frac{(1+cos\theta)^2}{1-cos^2\theta}}$$

$$=\sqrt{\frac{(1+cos\theta)^2}{sin^2\theta}}$$

$$=\frac{1+cos\theta}{sin\theta}$$

$$=cosec\theta + cot\theta$$

[ii] tan θ / [1 - cot θ] + cot θ / [1 - tan θ] = 1 + tan θ + cot θ

LHS = tanθ / (1 - cotθ) + cotθ / (1 - tanθ)

= tanθ / (1 - 1 / tanθ) + (1 / tanθ) / (1 - tanθ)

= $\tan^2\theta$ / (tanθ - 1) + 1 / tanθ (1 - tanθ)

= $\tan^3\theta$ / (tanθ - 1) - 1 / tanθ (tanθ - 1)

= ($\tan^3\theta$ - 1) / tanθ (tanθ - 1)

= (tanθ - 1)($\tan^2\theta$ + 1 + tanθ) / tanθ (tanθ - 1)

= ($\tan^2\theta$ + 1 + tanθ) / tanθ

= tanθ + cotθ + 1

Question 30: A well of diameter 7 m is dug and earth from digging is evenly spread out to form a platform 22 m × 14 m × 2.5 m. Find the depth of the well.

Solution:

The diameter of the well = 7 m

The radius of the well = 7 / 2 m

L * B * H = $\pi r^2 h$

22 * 14 * 2.5 = 22 / 7 × 7 / 2 × 7 / 2 × h

770 / 38.485 = h

h = 20cm

Question 31: Prove the √6 is an irrational number.

Solution:

Let √6 be a rational number, then

$\sqrt{6} = p \div q$, where p,q are integers , q not = 0 and p,q have no common factors (except 1)

=> $6 = p^2 \div q^2$

=> $p^2 = 2q^2$ ....(i)

As 2 divides $6q^2$, so 2 divides p^2 but 2 is a prime number

=> 2 divides p

Let p = 2m, where m is an integer.

Substituting this value of p in (i),

$(2m)^2 = 6q^2$

=> $2m^2 = 3q^2$

As 2 divides $2m^2$, 2 divides $3q^2$

=> 2 divides 3 or 2 divide q^2

But 2 does not divide 3, therefore, 2 divides q^2

=> 2 divides q

Thus, p and q have a common factor 2.

This contradicts that p and q have no common factors (except 1).

Hence, the supposition is wrong.

Therefore, $\sqrt{6}$ is an irrational number.

Question 32: A box contains 7 red marbles, 10 white marbles and 5 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be

(i) not red?

(ii) white?

(iii) green?

Solution:

Number of red marbles = 7

Number of white marbles = 10

Number of green marbles = 5

Total marbles = 7 + 10 + 5 = 22

Probability of getting not red marble = 15 / 22

Probability of getting white marble = 10 / 22
Probability of getting green marble = 5 / 22

Question 33: Seven spheres of equal radii are made by melting a silver-cuboid of dimensions 8cm × 9cm ×11cm. Find the radius of a silver sphere.

Solution:
The total volume before melting equals the total volume after melting.
The volume of the cuboid is equal to the volume of seven spheres.

$V_c = 7V_s$

$abc = 7 \times [4 / 3] \times \pi r^3$

$8 \times 9 \times 11 \approx 7 \times [4 / 3] \times [22 / 7] \times r^3$

$792 = 29.321r^3$

$r^3 = 792 / 29.321$

$r^3 = 27$

r = 3 cm

Question 34: (i) If the distance between the points (x, 3) and (5, 7) is 5, then find the value of x.
(ii) Find the ratio in which the line 3x + y = 9 divides the line segment joining the points (1, 3) and (2, 7).

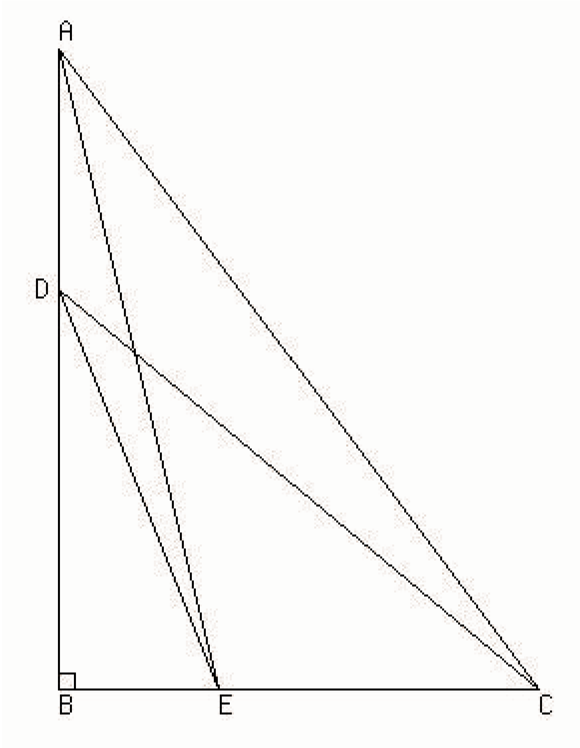
Solution:
[i] Let $(x_1, y_1) = (x, 3)$
$(x_2, y_2) = (5, 7)$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$5 = \sqrt{(5 - x)^2 + (7 - 3)^2}$
d = 5
On squaring of both sides,
$25 = (5 - x)^2 + 16$
$25 - 16 = (5 - x)^2$
$9 = (5 - x)^2$
$(5 - x) = \pm 3$

If x = 3,
5 - x = 3
5 - 3 = x
x = 2
If x = -3,
5 - (-3) = 3
x = 8
x = 2, 8

[ii] Let the line divides the points in k:1 ratio according to section formula
(2k + 1 / k + 1, 7k + 3 / k + 1) = (x, y)
It must satisfy the given equation so
3(2k + 1 / k + 1) + (7k + 3 / k + 1) = 9
6k + 3 + 7k + 3 / k + 1 = 9
13k + 6 = 9k + 9
13k - 9k = 9 - 6
4k = 3
k = 3 / 4
Hence, the ratio is 3:4.

Question 35: ABC is a right-angled triangle whose ∠B is a right angle. If points D and E are situated on the sides AB and BC respectively, then prove that $AE^2 + CD^2 = AC^2 + DE^2$.

Solution:



ΔABE is a right triangle, right-angled at B

$AB^2 + BE^2 = AE^2$……………..(1) (by the Pythagoras theorem)

ΔDBC is a right triangle, right-angled at B

$DB^2 + BC^2 = CD^2$…………….(2) (by the Pythagoras theorem)

Adding equations 1 & 2

$AE^2 + CD^2 = (AB^2 + BE^2) + (BD^2 + BC^2)$

$AE^2 + CD^2 = (AB^2 + BC^2) + (BE^2 + BD^2)$......(3) [Rearranging the terms]

ΔABC is a right triangle,

$AB^2 + BC^2 = AC^2$…………….(4) (by the Pythagoras theorem)

ΔDBE is a right triangle

$DB^2 + BE^2 = DE^2$………………(5) (by the Pythagoras theorem)

$AE^2 + CD^2 = (AB^2 + BC^2) + (BE^2 + BD^2)$

$AE^2 + CD^2 = AC^2 + DE^2$

Question 36: The diagonal of a rectangular field is 25 meters more than the shorter side. If the longer side is 23 meters more than the shorter side, find the sides of the field.

Solution:

Let the shorter side of the rectangular field be 'x' meters.
Therefore the longer side will be (x + 23) meters and the length of the diagonal will be (x + 25) meters.
The diagonal divides the rectangular into two right-angled triangles and the diagonal is the common side of the two triangles and it is also the longest side of the triangles i.e. the hypotenuse.
By Pythagoras Theorem,
$(\text{Diagonal})^2 = (\text{Smaller Side})^2 + (\text{Longer Side})^2$
$(x + 25)^2 = (x)^2 + (x + 23)^2$
$x^2 + 50x + 625 = x^2 + x^2 + 46x + 529$
$x^2 + 50x - 46x + 625 - 529 = 2x^2$
$x^2 + 4x + 96 = 2x^2$
$x^2 - 4x - 96 = 0$
$x^2 - 12x + 8x - 96 = 0$
$x(x - 12) + 8(x - 12) = 0$
x = 12, -8
x = 12m as length cannot be possible.
So the length of the shorter side is 12 meters and the length of the longer side is 12 + 23 = 35 meters.

Question 37: Prove that (tan A - sin A) / (tan A + sin A) = (sec A - 1) / (sec A + 1).

Solution:
LHS = [tan A – sin A] / [tan A + sin A]
= [sin A / cos A] - sin A / [sin A / cos A] + sin A
= sin A [(1 / cos A) - 1] / sin A [(1 / cos A) + 1]
= [sec A - 1] / [sec A + 1]

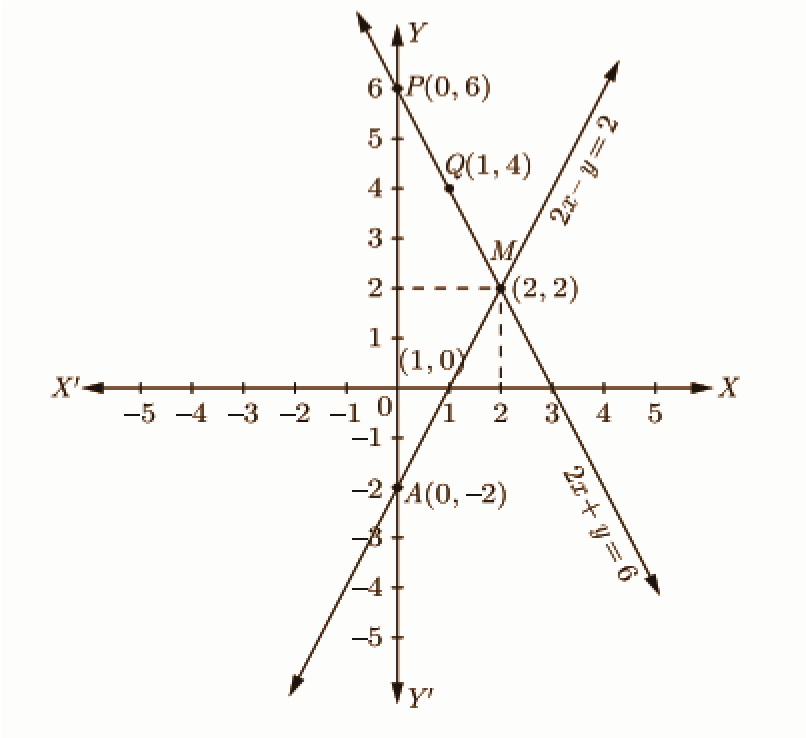
Question 38: PQRS is a trapezium in which PQ || RS and its diagonals intersect each at the point O. Prove that PO / QO = RO / SO.

Solution:
As PQ || RS and PR and QS are transversals,

$\angle$OSR = $\angle$OQP [alternate angles]
$\angle$ORS = $\angle$OPQ [alternate angles]
ΔSOR ~ ΔPOQ
SO / PO = RO / OQ = SR / PQ
SO / PO = RO / QO
QO / PO = RO / SO [by alterendo]

Question 39: Solve the following pair of linear equations by graphical method: 2x + y = 6, 2x – y = 2. Thus find the value of p in the relation 6x + 7y = p.

Solution:
2x + y = 6 ---- (1)
2x – y = 2 ---- (2)
4x = 8
x = 2
Put x = 1 in (1),
2 * 2 + y = 6
y = 6 - 4
y = 2
In 6x + 7y = p
6 * 2 + 7 * 2
= 12 + 14
= 26



Question 40: The cost of 5 apples and 3 oranges is Rs. 35 and the cost of 2 apples and 4 oranges is Rs. 28. Formulate the problem algebraically and solve it graphically.

Solution:

Let the cost of 1 apple be x and the cost of 1 orange be y.

According to question

5x + 3y = 35 - - - - - (i)

2x + 4y = 28 - - - - (ii)

Multiply by 2 in equation (i) and by 5 in equation (ii)

Now,

=> 10x + 6y = 70 - - - - (i)

=> 10x + 20y = 140 - - - (ii)

By subtraction equation (i) from (ii)

=> 14y = 70

=> y = 10 / 14

=> y = 5

Putting the value of y in equation (i)

=> 5x + (3 * 5) = 35
=> 5x = 35 - 15
=> x = 20 / 5
=> x = 4
Hence, the cost of an apple = ₹4 and the cost of an orange = ₹5.