

Rajasthan Board Class 12 Maths Important Questions

Question 1: If the points $(x, -2)$, $(5, 2)$, $(8, 8)$ are collinear, then find the value of x .

Solution:

If $(x, -2)$, $(5, 2)$, $(8, 8)$ are collinear, then

$$\begin{vmatrix} x & -2 & 1 \\ 5 & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} x & -2 & 1 \\ 5 & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} x & -2 & 1 \\ 5-x & 4 & 0 \\ 8 & 8 & 1 \end{vmatrix} = \begin{vmatrix} x & -2 & 1 \\ 5-x & 4 & 0 \\ 8-x & 10 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 5-x & 4 \\ 8-x & 4 \end{vmatrix}$$

$$= 50 - 10x - 32 + 4x$$

$$= 18 - 6x$$

$$x = 3$$

Question 2: Find the unit vector along with the sum of vectors $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

Solution:

The sum of the given vectors is vector $(\mathbf{a} + \mathbf{b})$ (= vector \mathbf{c} , say) $= 4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $|\text{vector } \mathbf{c}| = \sqrt{(4^2 + 3^2 + (-2)^2)} = \sqrt{29}$

Thus, the required unit vector is

$$\mathbf{c} = [1 / |\mathbf{c}|] [\mathbf{c}] = (1 / \sqrt{29}) [4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}]$$

$$= [4 / \sqrt{29}] \mathbf{i} + [3 / \sqrt{29}] \mathbf{j} - [2 / \sqrt{29}] \mathbf{k}$$

Question 3: If $2 P(A) = P(B) = 5 / 13$ and $P(A / B) = \frac{2}{5}$ then find $P(A \cup B)$.

Solution:

$$2 P(A) = P(B) = 5 / 13$$

$$P(A) = (1 / 2) * (5 / 13) = 5 / 26$$

$$P(B) = 5 / 13$$

$$P(A / B) = \frac{2}{5}$$

$$P(A / B) = P(A \cap B) / P(B)$$

$$(2 / 5) = P(A \cap B) / (5 / 13)$$

$$P(A \cap B) = (2 / 5) * (5 / 13)$$

$$P(A \cap B) = 2 / 13$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= (5 / 26) + (5 / 13) - (2 / 13)$$

$$= (5 / 26) + (6 / 26)$$

$$= 11 / 26$$

Question 4: Find A, if $2A - \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ 0 & 1 \end{bmatrix}$.

Solution:

$$2A - \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} 1 & 5 \\ 3 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 1 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 4 \\ 0 & 6 \end{bmatrix}$$

$$A = \frac{1}{2} \begin{bmatrix} 4 & 4 \\ 4 & 4 \\ 0 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 0 & 3 \end{bmatrix}$$

Question 5: Find the direction cosines of the line $(x - 2) / 2 = (y + 1) / -2 = (z - 1) / 1$.

Solution:

The line is $(x - 2) / 2 = (y + 1) / -2 = (z - 1) / 1$ --- (i)

$(a, b, c) = (2, -2, 1)$

The direction ratios are $\pm a / \sqrt{a^2 + b^2 + c^2}, \pm b / \sqrt{a^2 + b^2 + c^2}, \pm c / \sqrt{a^2 + b^2 + c^2}$

$= \pm 2 / \sqrt{4 + 4 + 1}, \pm (-2) / \sqrt{4 + 4 + 1}, \pm 1 / \sqrt{4 + 4 + 1}$

$= \pm 2 / 3, \pm 2 / 3, \pm 1 / 3$

Question 6: Find $\int [\tan x / \cot x] dx$.

Solution:

$\int [\tan x / \cot x] dx$

$\tan x = 1 / \cot x$

$= \int \tan^2 x dx$

$= \int \sec^2 x dx - \int 1 dx$

$= \tan x - x + c$

Question 7: Find the angle between planes $r \cdot (i - j + k) = 5$ and $r \cdot (2i + j - k) = 7$.

Solution:

Let the normal of the planes be $a = i - j + k$ and $b = 2i + j - k$.

$a \cdot b = |a| \cdot |b| \cos \theta$

$\cos \theta = (a \cdot b) / |a| \cdot |b|$

$\cos \theta = [i - j + k] [2i + j - k] / |i - j + k| |2i + j - k|$

$= [2 - 1 - 1] / \sqrt{1 + 1 + 1} \sqrt{4 + 1 + 1}$

$\cos \theta = 0$

$\theta = \cos^{-1} 0$

$\theta = \pi / 2$

Question 8: From a pack of 52 cards, two cards are drawn randomly one by one without replacement. Find the probability that both of them are red.

Solution:

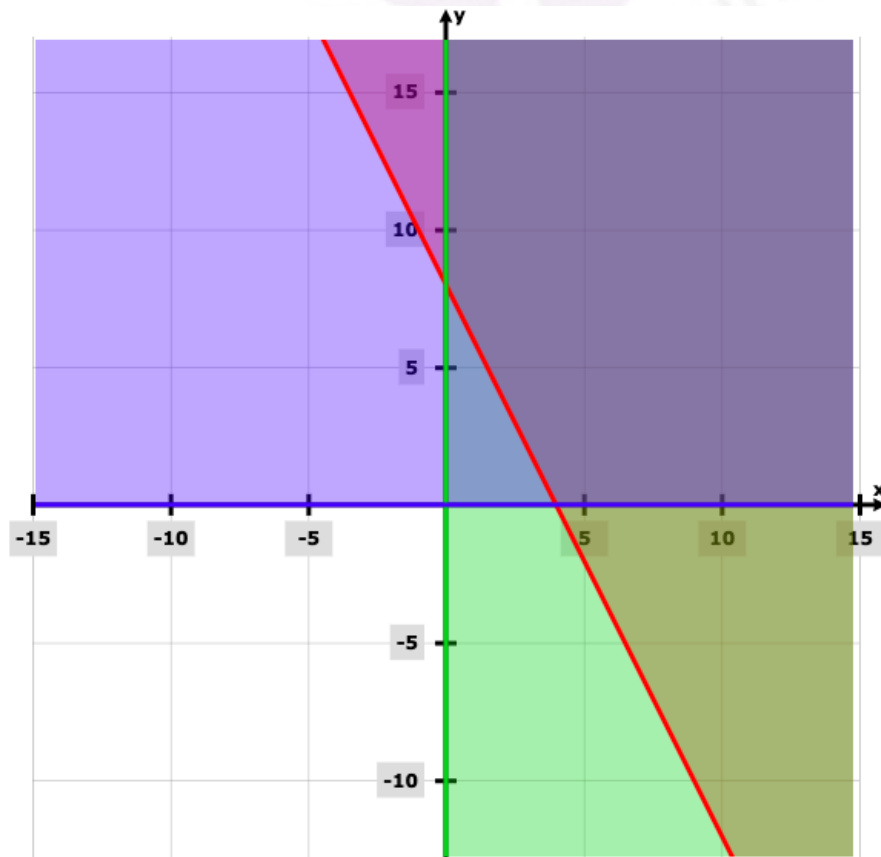
The probability of getting the first card as red is $26 / 52$ as there are 26 red cards in a deck.

Now the probability of getting the second card as red is $25 / 51$, as 25 red cards are left and the total number of cards is 51.

$$\begin{aligned} P(\text{both the cards are red}) &= [26 \times 25] / [52 \times 51] \\ &= 650 / 2652 \\ &= 25 / 102 \end{aligned}$$

Question 9: Show the region of a feasible solution under the following constraints $2x + y \geq 8$, $x \geq 0$, $y \geq 0$ in the answer book.

Solution:



Question 10: Examine the continuity of function $f(x) = \begin{cases} x+5 & x \leq 1 \\ x-5 & x > 1 \end{cases}$ at point $x = 1$.

Solution:

$$f(x) = \begin{cases} x+5 & x \leq 1 \\ x-5 & x > 1 \end{cases}$$

$$\text{Left hand limit} = \lim_{x \rightarrow 1^-} (x + 5) = 6$$

$$\text{Right hand limit} = \lim_{x \rightarrow 1^+} (x - 5) = -4$$

Since $\text{LHL} \neq \text{RHL}$, $f(x)$ is discontinuous at $x = 1$.

Question 11: Find $\int (1 / 1 + \sin x) dx$.

Solution:

$$\begin{aligned} & \int (1 / 1 + \sin x) dx \\ &= \int (1 - \sin x) / (1 + \sin x) (1 - \sin x) dx \\ &= \int (1 - \sin x) / (1^2 - \sin^2 x) dx \\ &= \int (1 - \sin x) / \cos^2 x dx \\ &= \int (1 / \cos^2 x) dx - (\sin x / \cos^2 x) dx \\ &= \int [\sec^2 x - \sec x \tan x] dx \\ &= \tan x - \sec x + c \end{aligned}$$

Question 12: If $A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & -2 \\ 1 & 2 \end{bmatrix}$, then find $2A^2 - 3B$.

Solution:

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -5 & -2 \\ 1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1-4 & -2-6 \\ 2+6 & -4+9 \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 5 \end{bmatrix}$$

$$3B = 3 \begin{bmatrix} -5 & -2 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} -15 & -6 \\ 3 & 6 \end{bmatrix}$$

$$2A^2 - 3B = 2 \begin{bmatrix} -3 & -8 \\ 8 & 5 \end{bmatrix} = \begin{bmatrix} -6 & -16 \\ 16 & 10 \end{bmatrix} - \begin{bmatrix} -15 & -6 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} 9 & -10 \\ 13 & 4 \end{bmatrix}$$

Question 13: Examine the continuity of function f defined by

$$f(x) = \begin{cases} \frac{e^{1/x}}{1 + e^{1/x}}; & x \neq 0 \\ 0 & ; \quad x = 0 \end{cases} \quad \text{at } x = 0.$$

Solution:

$$f(0) = 0$$

$$f(0 - 0) = \lim_{h \rightarrow 0} f(0 - h)$$

$$= \lim_{h \rightarrow 0} \frac{e^{(1/0) - h}}{[1 + e^{(1/0) - h}]}$$

$$= \lim_{h \rightarrow 0} \frac{e^{-1/h}}{[1 + e^{-1/h}]}$$

$$= \frac{e^{-\infty}}{[1 + e^{-\infty}]}$$

$$e^{-\infty} = 1 / [e^{\infty}] = 1 / \infty = 1 / (1 / 0) = 0 / 1 = 0$$

$$f(0 - 0) = 0 / [1 + 0] = 0 / 1 = 0$$

$$f(0 + 0) = \lim_{h \rightarrow 0} f(0 + h)$$

$$= \lim_{h \rightarrow 0} \frac{e^{1/h}}{[1 + e^{1/h}]}$$

$$= \lim_{h \rightarrow 0} \frac{e^{1/h}}{e^{1/h} [e^{-1/h} + 1]}$$

$$= \lim_{h \rightarrow 0} \frac{1}{[e^{-1/h} + 1]}$$

$$= \frac{1}{[1 + e^{-\infty}]}$$

$$= \frac{1}{0 + 1}$$

$$= 1$$

$$f(0) = f(0 - 0) \neq f(0 + 0)$$

So, $f(x)$ is not continuous at $x = 0$.

Question 14: Prove that the relation R in a set of real numbers R defined as $R = \{(a, b) : a \geq b\}$ is reflexive and transitive but not symmetric.

Solution:

$$R = \{(a, b) : a \geq b\}$$

Reflexive

$$R = \{(a, a) : a > a\}$$

Hence, R is reflexive.

Symmetric

$$\text{Let } (a, b) \in R$$

$$aRb \Leftrightarrow a \geq b$$

$$bRa \Leftrightarrow b \geq a \text{ not true}$$

$$(b, a) \notin R$$

Hence, R is not symmetric.

Transitive

$$\text{Let } (a, b) \in R \text{ and } (b, c) \in R$$

$$aRb \Leftrightarrow a \geq b$$

$$bRc \Leftrightarrow b \geq c$$

$$a \geq c$$

$$(a, c) \in R$$

Hence, R is transitive.

So, R is reflexive and transitive but not symmetric.

Question 15: Solve $2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x)$, $0 < x < \pi/2$.

Solution:

$$2 \tan^{-1}(\sin x) = \tan^{-1}(2 \sec x)$$

$$2 \tan^{-1} x = \tan^{-1} (2x / (1 - x))$$

$$\tan^{-1} [(2 \sin x) / (1 - \sin^2 x)] = \tan^{-1} [2 \sec x]$$

$$(2 \sin x) / (1 - \sin^2 x) = 2 \sec x$$

$$\sin x / \cos^2 x = \sec x$$

$$\sin x \sec x = 1$$

$$\sin x / \cos x = 1$$

$$\tan x = 1$$

$$x = \pi / 4$$

Question 16: Find the intervals in which the function f given by $f(x) = x^2 - 6x + 5$ is

- a) Strictly increasing
- b) Strictly decreasing

Solution:

$$f(x) = x^2 - 6x + 5$$

$$f'(x) = 2x - 6$$

$$\text{Put } f'(x) = 0$$

$$2x - 6 = 0$$

$$x = 3$$

Then divide the given interval into two parts as $(-\infty, 3)$ and $(3, \infty)$.

[i] Take the interval $(3, \infty)$

$$f'(x) = 2x - 6 > 0$$

The given function is strictly increasing in the interval $(3, \infty)$.

[ii] Take the interval $(-\infty, 3)$

$$f'(x) = 2x - 6 < 0$$

The given function is strictly decreasing in the interval $(-\infty, 3)$.

Question 17: Prove that the relation R defined on set Z as $a R b \Leftrightarrow a - b$ is divisible by 3, is an equivalence relation.

Solution:

$aRb \Leftrightarrow a - b$ is divisible by 3

(i) Reflexive : $aRa \Rightarrow a - a = 0$ is divisible by 3

$\therefore R$ is reflexive.

(ii) Symmetric: Let $(a, b) \in R$

$\Rightarrow aRb \Leftrightarrow a - b$ is divisible by 3

$\Rightarrow bRa \Leftrightarrow b - a$ is divisible by 3

$\Rightarrow (b, a) \in R$

$\therefore R$ is symmetric

(iii) Transitive: Again $(a, b) \in R$ and $(b, c) \in R$

$aRb \Leftrightarrow a - b$ is divisible by 3 and $bRc \Leftrightarrow b - c$ is divisible by 3

$\Rightarrow a - b$ is divisible by 3 and $b - c$ is divisible by 3

$\Rightarrow (a - b) + (b - c)$ is divisible by 3

$\Rightarrow a - c$ is divisible by 3

$\Rightarrow (a, c) \in R$

$\therefore R$ is transitive, hence R is an equivalence relation.

Question 18: Prove that $\cos^{-1} (63 / 65) + 2 \tan^{-1} (1 / 5) = \sin^{-1} (3 / 5)$.

Solution:

LHS = $\cos^{-1} (63 / 65) + 2 \tan^{-1} (1 / 5)$

Take $\theta = \cos^{-1} (63 / 65)$

$\cos \theta = 63 / 65$

$\sin \theta = \sqrt{1 - \cos^2 \theta}$

$= \sqrt{1 - (63 / 65)^2}$

$\sin \theta = 16 / 65$

$\theta = \sin^{-1} (16 / 65)$

$$\begin{aligned}
&= \sin^{-1} (16 / 65) + \tan^{-1} (2 * (1 / 5)) / (1 - (1 / 25)) \\
&= \sin^{-1} (16 / 65) + \tan^{-1} (10 / (25 - 1)) \\
&= \sin^{-1} (16 / 65) + \tan^{-1} (5 / 12) \\
\theta &= \tan^{-1} (5 / 12) \\
\tan \theta &= 5 / 12 \\
\sec^2 \theta &= 1 + \tan^2 \theta \\
&= 1 + (25 / 144) \\
&= 169 / 144 \\
\sec \theta &= 13 / 12 \\
\cos \theta &= 12 / 13 \\
\sin \theta &= \sqrt{1 - (12 / 13)^2} \\
&= \sqrt{1 - [144 / 169]} \\
&= \sqrt{25 / 169} \\
&= 5 / 13 \\
&= \sin^{-1} (16 / 65) + \sin^{-1} (5 / 13) \\
\sin^{-1} x + \sin^{-1} y &= \sin^{-1} [x + \sqrt{1 - y^2} + y \sqrt{1 - x^2}] \\
&= \sin^{-1} [(16 / 65) (\sqrt{1 - (5 / 13)^2}) + (5 / 13) (\sqrt{1 - (16 / 65)^2})] \\
&= \sin^{-1} ((16 / 65) * (12 / 13) + (5 / 13) * (63 / 65)) \\
&= \sin^{-1} ([192 + 315] / (63 / 65)) \\
&= \sin^{-1} [507 / (65 * 13)] \\
&= \sin^{-1} (3 / 5)
\end{aligned}$$

Question 19: If the radius of a sphere is measured as 7 cm with an error of 0.01 cm, then find the approximate error in calculating its volume.

Solution:

$$r = 7\text{cm}$$

$$\Delta r = 0.01 \text{ cm}$$

$$V = (4 / 3) \pi r^3$$

$$dv / dr = (4 / 3) \pi 3r^2$$

$$= 4\pi r^2$$

$$= 4\pi * 49$$

$$= 196 \pi \text{ cm}^3 / \text{cm}$$

$$\Delta v = [dv / dr] * (\Delta r)$$

$$\Delta v = [196 \pi] * (0.01) \\ = 1.96 \pi$$

Question 20: Find two positive numbers x and y, the sum of them is 60 and xy^3 is maximum.

Solution:

$$\text{Let } P = xy^3$$

$$\text{It is given that } x + y = 60$$

$$\Rightarrow x = 60 - y$$

$$P = (60 - y) y^3 \text{ [Putting value of } x]$$

$$= 60y^3 - y^4$$

$$\Rightarrow dP / dy = 180y^2 - 4y^3$$

$$d^2P / dy^2 = 360y - 12y^2$$

For maximum or minimum values of y, P we have

$$dy / dP = 0$$

$$\Rightarrow 180y^2 - 4y^3 = 0$$

$$\Rightarrow 4y^2(45 - y) = 0$$

$$\Rightarrow y = 0$$

$$45 - y = 0$$

$$y = 45$$

$$(d^2P / dy^2)_{y=45} = 360 \times 45 - 12(45)^2$$

$$= 12 \times 45 - (30 - 45)$$

$$= -8100 < 0$$

P is maximum when $y = 45$

When $y = 45$,

$$x + y = 60$$

$$\Rightarrow x = 60 - 45$$

$$x = 15$$

The numbers are 15 and 45.

Question 21: Solve the equation $\cos^{-1} x + \cos^{-1} 2x = (2\pi) / 3$.

Solution:

$$\cos^{-1} x + \cos^{-1} 2x = 2\pi / 3$$

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1} [xy - \sqrt{1-x^2} \sqrt{1-y^2}]$$

$$\cos^{-1} (x * 2x - \sqrt{1-x^2} \sqrt{1-(2x)^2}) = 2\pi / 3$$

$$\cos^{-1} (2x^2 - \sqrt{1-x^2} \sqrt{1-(2x)^2}) = 2\pi / 3$$

$$2x^2 - \sqrt{1-x^2} \sqrt{1-(2x)^2} = 2\pi / 3$$

$$2x^2 - \sqrt{1-x^2} \sqrt{1-4x^2} = 2\pi / 3$$

$$2x^2 + (1/2) = \sqrt{1-x^2} \sqrt{1-4x^2}$$

On squaring both sides,

$$(2x^2 + (1/2))^2 = (1-x^2)(1-4x^2)$$

$$4x^4 + 2x^2 + (1/4) = 1 - 4x^2 - x^2 + 4x^4$$

$$7x^2 = 1 - (1/4)$$

$$7x^2 = 3/4$$

$$x^2 = 3/28$$

$$x = \pm \sqrt{3/28}$$

The solution of $\cos^{-1} x + \cos^{-1} 2x = 2\pi / 3$ is $\pm \sqrt{3/28}$.

Question 22: Solve the following system of equations by using Cramer's rule.

$$5x - 4y = 7$$

$$x + 3y = 9$$

Solution:

$$5x - 4y = 7 \text{ ---- (1)}$$

$$x + 3y = 9 \text{ ---- (2)}$$

$$\Delta = \begin{vmatrix} 5 & -4 \\ 1 & 3 \end{vmatrix} = (5 * 3 - (1)(-4)) = 15 + 4 = 19$$

$$\Delta_1 = \begin{vmatrix} 7 & -4 \\ 9 & 3 \end{vmatrix} = (7 * 3 - (9)(-4)) = 21 + 36 = 57$$

$$\Delta_2 = \begin{vmatrix} 5 & 7 \\ 1 & 9 \end{vmatrix} = (5 * 9 - (7)(1)) = 45 - 7 = 38$$

$$x = \Delta_1 / \Delta = 57 / 19 = 3$$

$$y = \Delta_2 / \Delta = 38 / 19 = 2$$

Question 23: Using integration, find the area of a triangular region whose sides have the equations $y = x + 1$, $y = 2x + 1$ and $x = 2$. (Draw the figure in answer book)

Solution:

Consider $y = x + 1$ and $y = 2x + 1$

Equate both the equations and solve for x

$$x + 1 = 2x + 1$$

$$x = 0$$

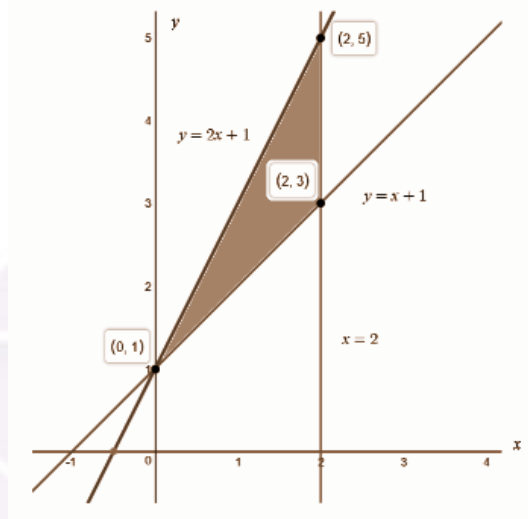
Substitute $x = 0$ in $y = x + 1$,

$$y = 1$$

Substitute $x = 2$ in $y = 2x + 1$

$$y = 5$$

Graph of $y = x + 1$, $y = 2x + 1$ and $x = 2$ is given by



$$\begin{aligned} \text{Area} &= \int_0^2 [2x + 1 - x - 1] dx \\ &= \int_0^2 x dx \\ &= [x^2 / 2]_0^2 \\ &= 2 \text{ square units} \end{aligned}$$

Question 24: Find the area of the triangle whose vertices are A (1, 1, 1), B (1, 2, 3) and C (2, 3, 1).

Solution:

A (1, 1, 1), B (1, 2, 3) and C (2, 3, 1)

A (1, 1, 1), B (1, 2, 3)

$$\begin{aligned} \vec{AB} &= (1 - 1)\mathbf{i} + (2 - 1)\mathbf{j} + (3 - 1)\mathbf{k} \\ &= 0\mathbf{i} + 1\mathbf{j} + 2\mathbf{k} \end{aligned}$$

A (1, 1, 1), C (2, 3, 1)

$$\begin{aligned} \vec{AC} &= (2 - 1)\mathbf{i} + (3 - 1)\mathbf{j} + (1 - 1)\mathbf{k} \\ &= 1\mathbf{i} + 2\mathbf{j} + 0\mathbf{k} \end{aligned}$$

$$\begin{aligned} \vec{AB} \times \vec{AC} &= \mathbf{i} [(1 \times 0) - (2 \times 2)] - \mathbf{j} [(0 \times 0) - (1 \times 2)] + \mathbf{k} [(0 \times 2) - (1 \times 1)] \\ &= -4\mathbf{i} + 2\mathbf{j} - 1\mathbf{k} \end{aligned}$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{16 + 4 + 1} = \sqrt{21}$$

The area of the triangle ABC = $(1/2) * (\vec{AB} \times \vec{AC})$

$$= (1/2) * \sqrt{21}$$

$$= \sqrt{21} / 2$$

Question 25: If $x^2 + y^2 = t - (1/t)$ and $x^4 + y^4 = t^2 + (1/t^2)$, then prove that $x \left(\frac{d^2y}{dx^2}\right) + 2 \left(\frac{dy}{dx}\right) = 0$.

Solution:

$$x^2 + y^2 = t - (1/t)$$

$$x^4 + y^4 + 2x^2y^2 = t^2 + (1/t^2) - 2t * (1/t)$$

$$t^2 + (1/t^2) + 2x^2y^2 = t^2 + (1/t^2) - 2$$

$$x^2y^2 = -1$$

$$2xy^2 + 2x^2y \left(\frac{dy}{dx}\right) = 0$$

$$xy \left(y + x \left(\frac{dy}{dx}\right)\right) = 0$$

$$xy \neq 0$$

$$y + x \left(\frac{dy}{dx}\right) = 0$$

$$\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right) + x \left(\frac{d^2y}{dx^2}\right) = 0$$

$$x \left(\frac{d^2y}{dx^2}\right) + 2 \left(\frac{dy}{dx}\right) = 0$$

Question 26: A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is 6. Find the probability that it is actually 6.

Solution:

Let E_1 be an event of a man telling the truth and E_2 be an event of a man not telling the truth.

A = probability on 6 in the coin

$$P(E_1) = 2/3$$

$$P(E_2) = 1/3$$

$$P(A/E_1) = 1/6$$

$$P(A/E_2) = 5/6$$

$$P(E_1/A) = [(2/3) * (1/6)] / [(2/3) * (1/6) + (1/3) * (5/6)] \\ = 2/7$$

Question 27: Bag A contains 2 red and 3 black balls while another bag B contains 3 red and 4 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag B.

Solution:

Consider the events E and F as follows:

E : Bag A is selected

F : Bag B is selected

A : Ball drawn is red

Since there are only two bags.

Bag A contains 2 red and 3 black balls

$$P(A/E) = 2/5$$

Bag B contains 3 red and 4 black balls

$$P(A/F) = 3/7$$

The required probability is

$$P(F/A) = [P(F) P(A/F)] / [P(E) P(A/E) + P(F) P(A/F)] \\ = (1/2) (3/7) / [(1/2) * (2/5) + (1/2) * (3/7)] \\ = (8/35) / [(3/14) + (8/35)] \\ = (3/14) / (29/70) \\ = 15/29$$

Question 28: Prove that if a plane has the intercepts a , b , c and is at distance p units from the origin, then prove that $(1/a^2) + (1/b^2) + (1/c^2) = (1/p^2)$.

Solution:

The plane is $(x/a) + (y/b) + (z/c) = 1$

Perpendicular distance from origin $(0, 0, 0)$

$$P = |0 + 0 + 0 - 1| / \sqrt{(1/a^2 + 1/b^2 + 1/c^2)}$$

$$= 1 / \sqrt{(1/a^2 + 1/b^2 + 1/c^2)}$$

$$(1/a^2 + 1/b^2 + 1/c^2) = 1/p^2$$

Question 29: A dice is thrown twice and the sum of the numbers appearing is observed to be 7. Find the conditional probability that the number 2 has appeared at least once.

Solution:

A die is thrown twice and the sum of numbers appearing is observed to be 7.

When a dice is rolled twice, the outcome is as follows:

$(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)$

$(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)$

$(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)$

$(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)$

$(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)$

$(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)$

Probability = Favourable outcome / Total number of outcomes

Favorable outcome that number 2 has appeared at least once :

$(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) (1, 2) (3, 2) (4, 2) (5, 2) (6, 2)$

Favorable outcomes = 11

Total number of outcome = 36

$P(\text{number 2 has appeared at least once}) = 11 / 36$

Question 30: Prove that the lines $r = i + j - k + \lambda (3i - j)$ and $r = 4i - k + \mu (2i + 3k)$ are intersecting, also find the point of intersection.

Solution:

The position vectors of arbitrary points on the given lines are

$$(i + j - k) + \lambda (3i - j) = (3\lambda + 1)i + (1 - \lambda)j - k$$

If the lines intersect, then they have a common point. So, for some values of λ and μ ,

$$(3\lambda + 1)i + (1 - \lambda)j - k = (2\mu + 4)i + 0j + (3\mu - 1)k$$

$$(3\lambda + 1) = (2\mu + 4); 1 - \lambda = 0; -1 = (3\mu - 1)$$

Solving the last two of these three equations, $\lambda = 1$ and $\mu = 0$.

These values of λ and m satisfy the first equation.

So, the given lines intersect.

Putting $\lambda = 1$ in first line,

$r = (i + j - k) + (3i - j) = 4i + 0j - k$ which is the position vector of the point of intersection.

Thus, the coordinates of the point of intersection are $(4, 0, -1)$.

