

ICSE Class 10 Maths Mock Sample Paper 1 with Solutions

SECTION A

Attempt all questions from this section.

Question 1

(a) Find the value of a and b if $x - 1$ and $x - 2$ are factors of $x^3 - ax + b$. [3]

Solution:

Given,

$x - 1$ and $x - 2$ are factors of $x^3 - ax + b$.

Let $p(x) = x^3 - ax + b$

$$p(1) = 0$$

$$(1)^3 - a(1) + b = 0$$

$$1 - a + b = 0$$

$$a = b + 1 \dots (i)$$

$$p(2) = 0$$

$$(2)^3 - a(2) + b = 0$$

$$8 - 2a + b = 0$$

$$8 - 2(b + 1) + b = 0 \text{ [From (i)]}$$

$$8 - 2b - 2 + b = 0$$

$$6 - b = 0$$

$$b = 6$$

Substituting $b = 6$ in (i),

$$a = 6 + 1 = 7$$

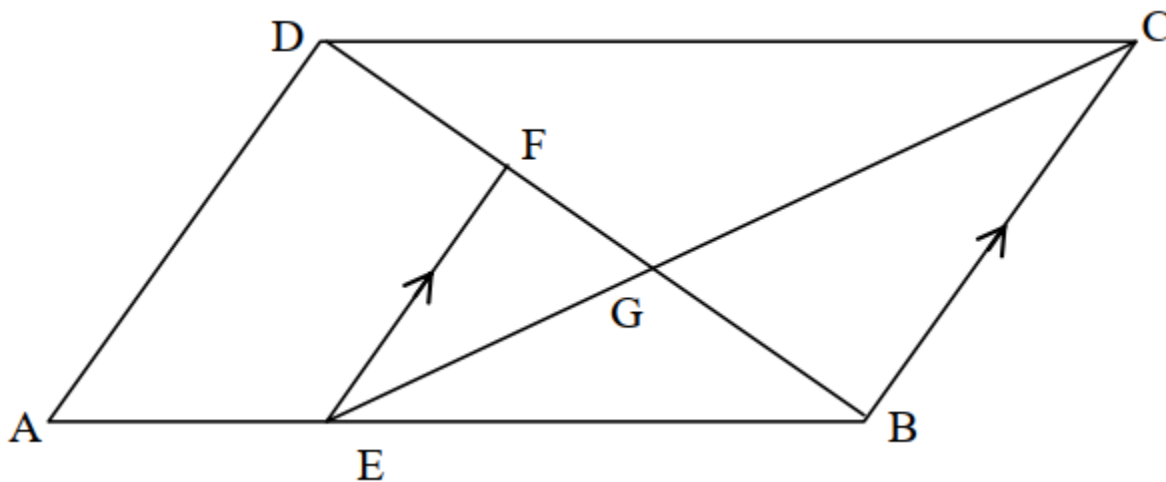
Therefore, $a = 7$ and $b = 6$.

(b) In the figure given below, ABCD is a parallelogram. E is a point on AB. CE intersects the diagonal BD at G and EF is parallel to BC.

If $AE : EB = 1 : 2$, find

(i) $EF : AD$

(ii) area of triangle BEF : area of triangle ABD



[3]

Solution:

In triangle BEF and ABD,

$$\angle EBF = \angle ABD$$

$$\angle FEB = \angle BAD$$

$$\angle ADB = \angle EFB$$

By AAA similarity criterion,

$$\triangle BEF \sim \triangle ABD$$

By BPT,

$$BE/AB = EF/AD$$

$$BE/(AE + EB) = EF/AD$$

$$2/(1 + 2) = EF/AD \quad [\text{given } AE/EB = 1/2]$$

$$EF/AD = 2/3$$

$$EF : AD = 2 : 3$$

(ii) The ratio of area of two similar triangles is equal to the ratio of squares of their corresponding sides.

$$\text{area of triangle BEF} / \text{area of triangle BAD} = EF^2/AD^2$$

$$= (2/3)^2$$

$$= 4/9$$

(c) On a certain sum of money, the difference between the compound interest for a year, payable half-yearly, and the simple interest for a year is Rs. 16. Find the sum lent out, if the rate of interest in both cases is 8%.

[4]

Solution:

Given,

Rate of interest = 8%

Let P be the sum lent out for interest.

$$SI = PTR/100$$

$$SI = (P \times 1 \times 8)/100$$

$$SI = 2P/25 \dots (i)$$

$$CI = P[1 + (R/200)]^2 - P \quad (\text{half-yearly payable})$$

$$= P[1 + (8/200)]^2 - P$$

$$= P(208/200)^2 - P$$

$$= P(43264/40000) - P$$

$$CI = 3264P/40000 \dots (ii)$$

$$CI - SI = \text{Rs. } 16 \text{ (given)}$$

$$(3264P/40000) - (2P/25) = 16$$

$$0.0816P - 0.08P = 16$$

$$0.0016P = 16$$

$$P = 16/0.0016$$

$$P = 10000$$

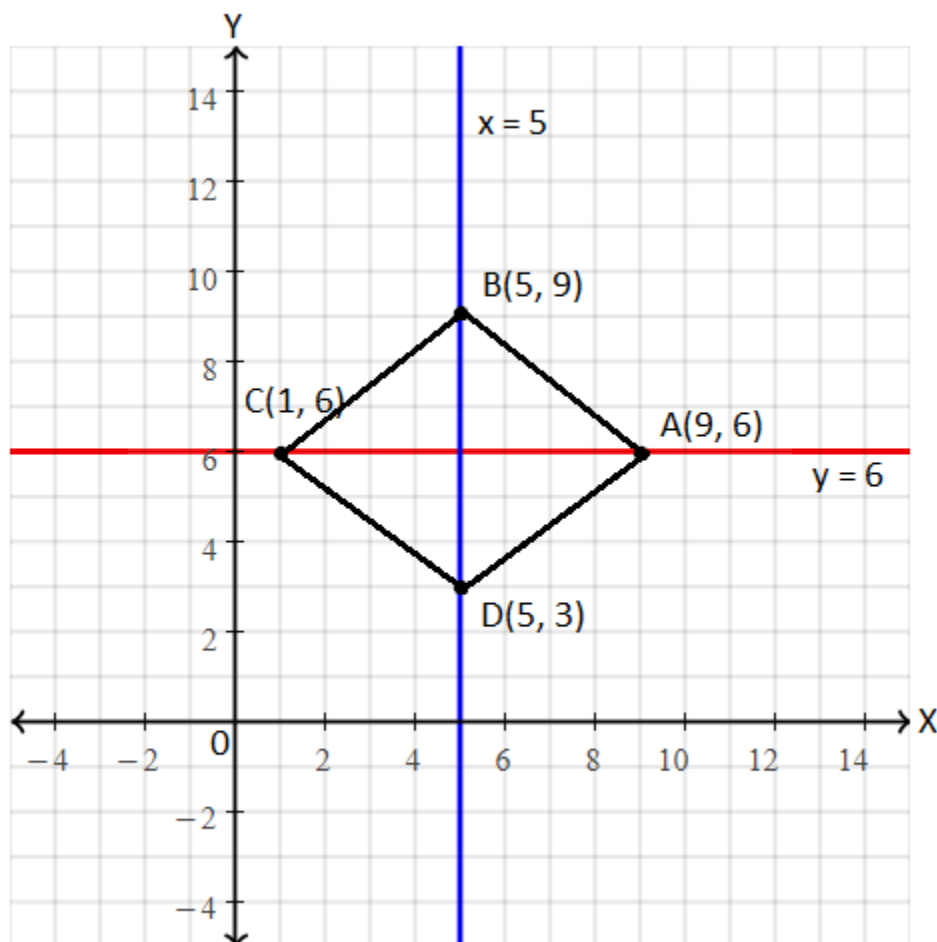
Therefore, the sum lent out is Rs. 10000.

Question 2

(a) Plot the points A(9, 6) and B(5, 9) on the graph paper. These two points are the vertices of a figure ABCD which is symmetrical about $x = 5$ and $y = 6$. Complete the figure on the graph. Write down the geometrical name of the figure.

[3]

Solution:



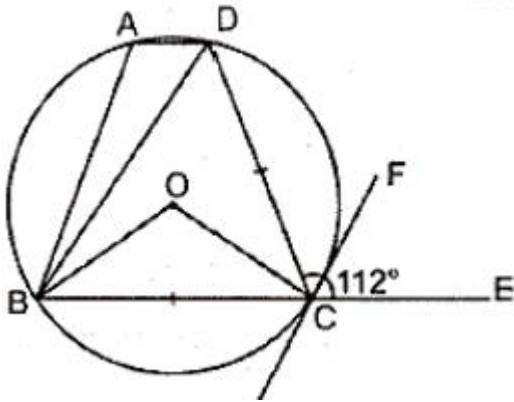
Coordinates of C(1, 6) and D(5, 3).

The obtained figure is a rhombus.

(b) In the figure, ABCD is a cyclic quadrilateral in which $BC = CD$ and CF is a tangent to the circle at C. BC is produced to E and $\angle DCE = 112^\circ$. If O is the centre of the circle, find:

(i) $\angle BOC$

(ii) $\angle DCF$ [3]



Solution:

ABCD is a cyclic quadrilateral in which $BC = CD$.

CF is a tangent to the circle at C.

BC is produced to E.

$$\angle DCE = 112^\circ$$

$$\angle ADC = \angle DCE = 112^\circ \text{ (alternate angles)}$$

AD \parallel BC and BD is the transversal.

$$\angle BDC = \frac{1}{2} (\angle ADC)$$

$$= \left(\frac{1}{2}\right) \times 112^\circ$$

$$= 56^\circ$$

$$\angle BOC = 2 \angle BDC \text{ (angles of the same segment)}$$

$$= 2 \times 56^\circ$$

$$= 112^\circ$$

(ii) CF is the transversal.

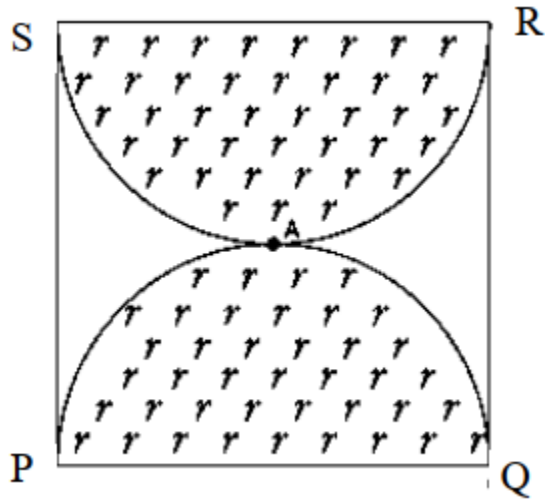
$$\angle DCF = \frac{1}{2} \angle DCE$$

$$= \left(\frac{1}{2}\right) \times 112^\circ$$

$$= 56^\circ$$

(c) PQRS is a square piece of land of side 56 m. Two semicircular grass-covered lawns are made on two of its opposite sides as shown in the figure. Calculate the area of the uncovered portion.

[4]



Solution:

Given,

Side of square PQRS = $a = 56$ m

Diameter of semicircle = 56 m

Radius of semicircle = $r = 56/2 = 28$ m

Area of the uncovered portion = Area of the square - $2 \times$ Area of the semicircle

$$= a^2 - 2 \times (\pi r^2/2)$$

$$= (56)^2 - (22/7) \times 28 \times 28$$

$$= 3136 - 2464$$

$$= 672 \text{ m}^2$$

Question 3

(a)

If $A = \begin{bmatrix} 4 & 4 \\ -2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$ find the matrix D such that

$$3A - 2B + 2D = 0$$

[3]

Solution:

Given,

$$A = \begin{bmatrix} 4 & 4 \\ -2 & 6 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$$

$$3A = 3 \begin{bmatrix} 4 & 4 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 12 & 12 \\ -6 & 18 \end{bmatrix}$$

$$2B = 2 \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 6 & -4 \end{bmatrix}$$

$$3A - 2B + 2D = 0$$

$$\begin{bmatrix} 12 & 12 \\ -6 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 6 & -4 \end{bmatrix} + 2D = 0$$

$$\begin{bmatrix} 8 & 10 \\ -12 & 22 \end{bmatrix} + 2D = 0$$

$$2D = \begin{bmatrix} -8 & -10 \\ 12 & -22 \end{bmatrix}$$

$$D = \begin{bmatrix} \frac{-8}{2} & \frac{-10}{2} \\ \frac{12}{2} & \frac{-22}{2} \end{bmatrix}$$

$$D = \begin{bmatrix} -4 & -5 \\ 6 & -11 \end{bmatrix}$$

(b) A point $P(a, b)$ is reflected in the Y-axis to $P^I(-3, 1)$

Write down the values of a and b .

P^{II} is the image of P when reflected in the X-axis.

Write down the coordinates of P^{II} .

P^{III} is the image of P when reflected in the line $X = 5$.

Write down the coordinates of P^{III} .

[3]

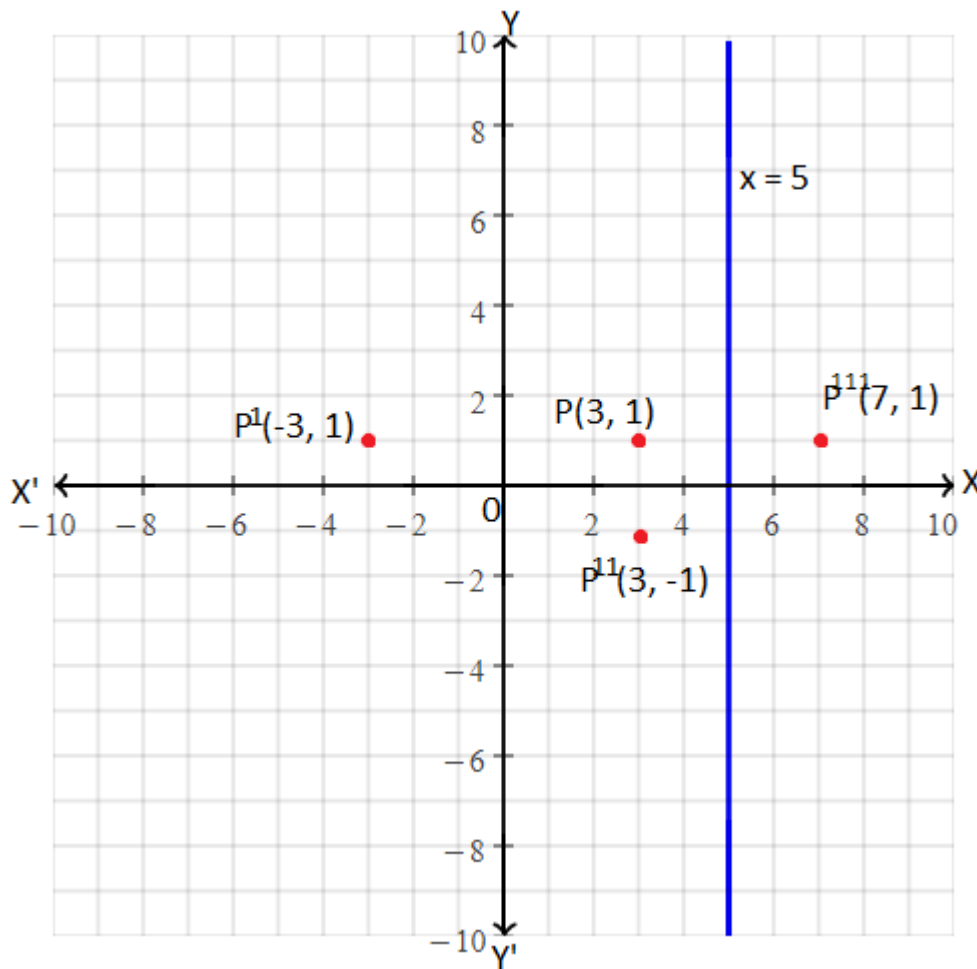
Solution:

Given,

$P(a, b)$ is reflected in the Y-axis to $P^I(-3, 1)$.

P^{II} is the image of P when reflected in the X-axis.

P^{III} is the image of P when reflected in the line $X = 5$.



Coordinates of $P(3, 1)$

Thus, $a = 3$, $b = 1$

Coordinates of $P^{11} = (3, -1)$

Coordinates of $P^{111} = (7, 1)$

(c) Given: $A = \{x : 3 < 2x - 1 < 9, x \in \mathbb{R}\}$, $B = \{x : 11 \leq 3x + 2 \leq 23, x \in \mathbb{R}\}$ where \mathbb{R} is the set of real numbers.

(i) Represent A and B on the number line

(ii) On the number line also mark $A \cap B$.

[4]

Solution:

$$A = \{x : 3 < 2x - 1 < 9, x \in \mathbb{R}\}$$

$$3 < 2x - 1$$

$$3 + 1 < 2x$$

$$4 < 2x$$

$$x > 4/2$$

$$x > 2$$

And

$$2x - 1 < 9$$

$$2x < 9 + 1$$

$$x < 10/2$$

$$x < 5$$

Therefore, $2 < x < 5$

$$A = \{3, 4\}$$

$$B = \{x : 11 \leq 3x + 2 \leq 23, x \in \mathbb{R}\}$$

$$11 \leq 3x + 2$$

$$11 - 2 \leq 3x$$

$$9 \leq 3x$$

$$3 \leq x$$

And

$$3x + 2 \leq 23$$

$$3x \leq 23 - 2$$

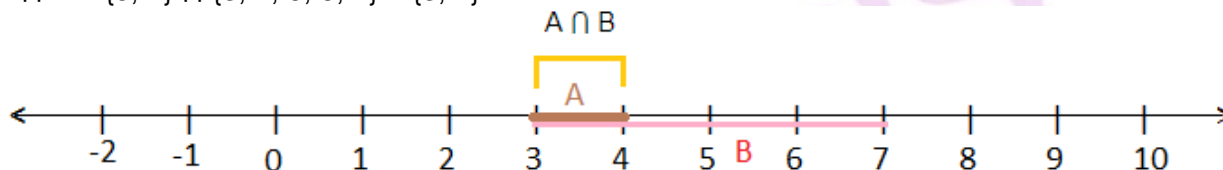
$$3x \leq 21$$

$$x \leq 7$$

Therefore, $3 \leq x \leq 7$

$$B = \{3, 4, 5, 6, 7\}$$

$$A \cap B = \{3, 4\} \cap \{3, 4, 5, 6, 7\} = \{3, 4\}$$



Question 4:

(a) Without using a trigonometric table calculate:

$$4 (\sin 32^\circ / \cos 58^\circ) + 5 (\tan 48^\circ / \cot 42^\circ) - 8 (\sec 72^\circ / \operatorname{cosec} 18^\circ)$$

[3]

Solution:

$$\begin{aligned} & 4 (\sin 32^\circ / \cos 58^\circ) + 5 (\tan 48^\circ / \cot 42^\circ) - 8 (\sec 72^\circ / \operatorname{cosec} 18^\circ) \\ &= 4 [\sin (90^\circ - 58^\circ) / \cos 58^\circ] + 5 [\tan (90^\circ - 42^\circ) / \cot 42^\circ] - 8 [\sec (90^\circ - 18^\circ) / \operatorname{cosec} 18^\circ] \\ &= 4 (\cos 58^\circ / \cos 58^\circ) + 5 (\cot 42^\circ / \cot 42^\circ) - 8 (\operatorname{cosec} 18^\circ / \operatorname{cosec} 18^\circ) \\ &= 4(1) + 5(1) - 8(1) \\ &= 4 + 5 - 8 \\ &= 1 \end{aligned}$$

(b) Mr. Jacob has a two years recurring deposit account in State Bank of India and deposits Rs. 1500 per month. If he receives Rs. 37,875 at the time of maturity, find the rate of interest. [3]

Solution:

Given,

Principal (P) = Rs. 1500

Time = 2 years

i.e. $n = 2 \times 12 = 24$ months

Let r be the rate of interest.

Amount received at the time of maturity = Total money deposited + Simple Interest

$$37875 = (P \times n) + \left[\frac{P \times n(n+1)}{(2 \times 12)} \right] \times (r/100)$$

$$37875 = (1500 \times 24) + \left[\frac{1500 \times 24 \times 25}{24} \right] \times (r/100)$$

$$37875 = 36000 + (900000 / 24) \times (r/100)$$

$$37875 - 36000 = 37500 \times (r/100)$$

$$r = (1875 \times 100) / 37500$$

$$r = 5$$

Hence, the rate of interest is 5%.

(c) Calculate the arithmetic mean, correct to one decimal place, for the following frequency distribution of marks obtained in a Geometry test.

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of students	7	13	15	12	3

[4]

Solution:

Marks	No. of students (f)	Mid-value (x)	fx
0 - 10	7	5	35
10 - 20	13	15	195
20 - 30	15	25	375
30 - 40	12	35	420
40 - 50	3	45	135
	$\Sigma f = 50$		$\Sigma fx = 1160$

$$\text{Mean} = \Sigma fx / \Sigma f$$

$$= 1160/50$$

$$= 23.2$$

Therefore, the arithmetic mean is 23.2.

SECTION B

Attempt any four questions from this section.

Question 5

(a)

$$\text{If } \begin{bmatrix} 2 & 4 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 3x \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 5 \begin{bmatrix} 4 \\ y \end{bmatrix} \text{ find the values of } x \text{ and } y.$$

[3]

Solution:

Given,

$$\begin{bmatrix} 2 & 4 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 3x \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 5 \begin{bmatrix} 4 \\ y \end{bmatrix}$$

$$\begin{bmatrix} 6x + 8 \\ 18x + 4 \end{bmatrix} + \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 20 \\ 5y \end{bmatrix}$$

$$\begin{bmatrix} 6x + 8 \\ 18x + 4 \end{bmatrix} = \begin{bmatrix} 20 \\ 5y \end{bmatrix} - \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 6x + 8 \\ 18x + 4 \end{bmatrix} = \begin{bmatrix} 20 - 6 \\ 5y - 8 \end{bmatrix}$$

$$6x + 8 = 14$$

$$6x = 14 - 8$$

$$6x = 6$$

$$x = 6/6 = 1$$

And

$$18x + 4 = 5y - 8$$

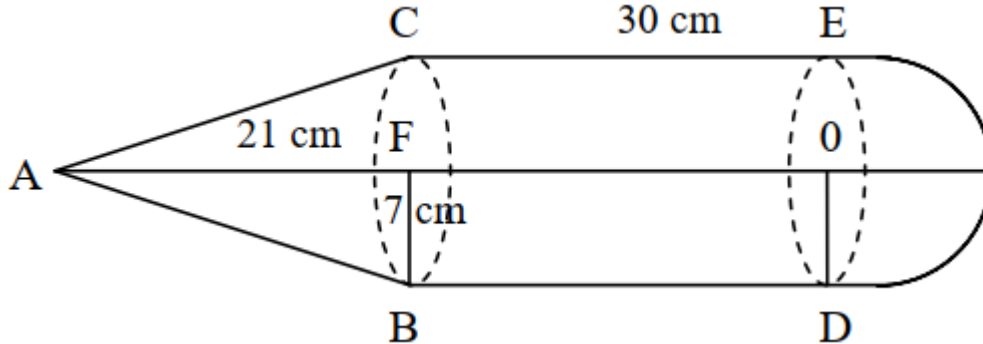
$$18(1) + 4 + 8 = 5y$$

$$5y = 30$$

$$y = 30/5 = 6$$

Therefore, $x = 1$ and $y = 6$.

(b) In the diagram given below, if $AF = 21$ cm, $CE = 30$ cm and $FB = 7$ cm. Find the volume of the figure.



[3]

Solution:

From the given figure,

Radius of cone = Radius of cylinder = Radius of hemisphere = $r = 7$ cm

Height of the cone = $h = 21$ cm

Height of the cylinder = $H = 30$ cm

Volume of the figure = Volume of cone + Volume of cylinder + Volume of hemisphere

$$= \left(\frac{1}{3}\right)\pi r^2 h + \pi r^2 H + \left(\frac{2}{3}\right)\pi r^3$$

$$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 7 \times 7 \times 21 + \left(\frac{22}{7}\right) \times 7 \times 7 \times 30 + \left(\frac{2}{3}\right) \times \left(\frac{22}{7}\right) \times 7 \times 7 \times 7$$

$$= 22 \times 7 \times 7 + 22 \times 7 \times 30 + \left(\frac{2}{3}\right) \times 22 \times 7 \times 7$$

$$= 1078 + 4620 + 718.67$$

$$= 6416.67 \text{ cm}^3$$

(c) A man bought 200 shares each of face value Rs. 10 at Rs. 12 per share. At the end of the year, the company from which he bought the shares declares a dividend of 15%. Calculate:

(i) the amount of money invested by the man

(ii) the amount of dividend he received

(iii) the percentage return on his outlay.

[4]

Solution:

Number of shares = 200

Nominal value = Rs. 10

Market value = Rs. 12

Dividend = 15%

(i) Investment = Number of shares \times Market value

$$= \text{Rs. } 200 \times 12$$

$$= \text{Rs. } 2400$$

(ii) Dividend = Number of shares \times Percentage of dividend \times Nominal value

$$= 200 \times 15\% \times 10$$

$$= 200 \times (15/100) \times 10$$

$$= \text{Rs. } 300$$

(iii) Let P be the percentage of return.

Now, Dividend percentage \times Nominal value = $p\% \times$ Market value

$$15\% \times 10 = p\% \times 12$$

$$p = (15 \times 10) / 12$$

$$p = 25/2$$

$$p = 12.5$$

Hence, the percentage of return on outlay is 12.5%.

Question 6

(a) Solve the following quadratic equation for x and give your answer correct to three significant figures.

$$2x^2 - 4x - 3 = 0$$

[3]

Solution:

$$2x^2 - 4x - 3 = 0$$

Dividing by 2,

$$x^2 - 2x - 3/2 = 0$$

$$x^2 - 2x = 3/2$$

Adding 1 on both sides,

$$x^2 - 2x + 1 = (3/2) + 1$$

$$(x - 1)^2 = (5/2)$$

$$x - 1 = \pm \sqrt{(5/2)}$$

$$x = 1 \pm \sqrt{(5/2)}$$

$$x = 1 \pm \sqrt{2.5}$$

$$x = 1 \pm 1.581$$

Now,

$$x = 1 + 1.581, x = 1 - 1.581$$

$$x = 2.581, x = -0.581$$

(b) An integer is chosen at random from 1 to 50. Find the probability that the number is:

(i) divisible by 5

(ii) a perfect cube

(iii) a prime number.

[3]

Solution:

Integers from 1 to 50: {1, 2, 3, 4, 5, ..., 49, 50}

Total number of outcomes = $n(S) = 50$

(i) Number of integers divisible by 5 = {5, 10, 15, 20, 25, 30, 35, 40, 45, 50} = 10

$P(\text{getting a number that is divisible by 5}) = 10/50 = 1/5$

(ii) Perfect cubes = {1, 8, 27} = 3

$P(\text{getting a perfect cube number}) = 3/50$

(iii) Prime numbers = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47} = 15

$P(\text{getting a prime number}) = 15/50$

$= 3/10$

(c) Find x from the following equation using properties of proportion:

$$(x^2 - x + 1)/(x^2 + x + 1) = 14(x - 1)/13(x + 1) \quad [4]$$

Solution:

Given,

$$(x^2 - x + 1)/(x^2 + x + 1) = 14(x - 1)/13(x + 1)$$

$$(x^2 - x + 1)/(x^2 + x + 1) = (14x - 14)/(13x + 13)$$

Using componendo and dividendo,

$$(x^2 - x + 1 + x^2 + x + 1)/(x^2 - x + 1 - x^2 - x - 1) = (14x - 14 + 13x + 13)/(14x - 14 - 13x - 13)$$

$$(2x^2 + 2)/(-2x) = (27x - 1)/(x - 27)$$

$$2(x^2 + 1)/(-2x) = (27x - 1)/(x - 27)$$

$$(x^2 + 1)/(-x) = (27x - 1)/(x - 27)$$

$$\Rightarrow (x^2 + 1)(x - 27) = -x(27x - 1)$$

$$\Rightarrow x^3 - 27x^2 + x - 27 = -27x^2 + x$$

$$\Rightarrow x^3 - 27 = 0$$

$$\Rightarrow x^3 = 27$$

$$\Rightarrow x^3 = (3)^3$$

$$\Rightarrow x = 3$$

Question 7

(a) Bosco wishes to start a 200 m^2 rectangular vegetable garden. Since he has only 50 m barbed wire, he fences three sides of the rectangular garden letting his house compound wall act as the fourth side of the fence. Find the dimensions of the garden. [3]

Solution:

Let x and y be the length and breadth of the garden respectively.

According to the given,

Sum of three sides = 50 m

$$x + 2y = 50 \text{ m}$$

$$x = 50 - 2y \dots (i)$$

Area of the garden = 200 m^2 (given)

$$xy = 200$$

$$(50 - 2y)y = 200 \text{ [From (i)]}$$

$$50y - 2y^2 = 200$$

$$2y^2 - 50y + 200 = 0$$

$$2(y^2 - 25y + 100) = 0$$

$$y^2 - 25y + 100 = 0$$

$$y^2 - 20y - 5y + 100 = 0$$

$$y(y - 20) - 5(y - 20) = 0$$

$$(y - 5)(y - 20) = 0$$

$$y = 5, y = 20$$

When $y = 5$ m,

$$x = 50 - 2(5)$$

$$= 50 - 10$$

$$= 40 \text{ m}$$

When $y = 20$ m,

$$x = 50 - 2(20)$$

$$= 50 - 40$$

$$= 10 \text{ m}$$

Therefore, the dimensions of the garden are 40 m and 5 m or 10 m and 20 m.

(b) Construct a triangle ABC with $AB = 6$ cm, $BC = 7$ cm and $\angle ABC = 60^\circ$. Locate by construction the point P such that

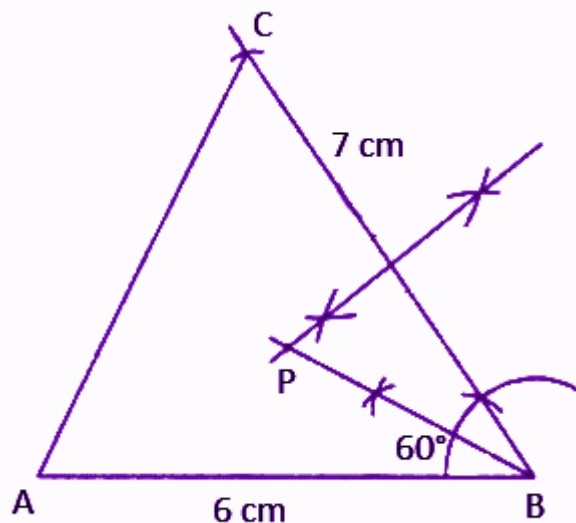
(i) P is equidistant from B and C

(ii) P is equidistant from AB and BC.

(iii) Measure and record the length of PA.

[3]

Solution:



The length of the PA is 4.5 cm. (by measure)

(c) Mr. A. Ramchander has an account with the Central Bank of India. The following entries are from his passbook.

Date	Particulars	Withdrawal	Deposits	Balance
05.01.2009	B/F			8000
20.01.2009	To self	2500		
04.02.2009	By cash		9000	

20.02.2009	By cash		3000	
04.03.2009	To self	1000		
15.04.2009	By cash		12000	

Complete the above page of his passbook and calculate the interest accumulated in four months, January to April at the rate of 3.5% per annum. If the interest is added on 30th April, find his balance on that date.

[4]

Solution:

Date	Particulars	Withdrawal	Deposits	Balance
05.01.2009	B/F			8000
20.01.2009	To self	2500		5500
04.02.2009	By cash		9000	14500
20.02.2009	By cash		3000	17500
04.03.2009	To self	1000		16500
15.04.2009	By cash		12000	28500

Rate of interest = 3.5%

Let us calculate interest on the number of days in four months from 05.01.2009 to 15.04.2009.

Interest = $[(8000 \times 15 + 5500 \times 15 + 14500 \times 16 + 17500 \times 12 + 16500 \times 42) \times 3.5] / 365 \times 100$

= $[(80 \times 15 + 55 \times 15 + 145 \times 16 + 175 \times 12 + 165 \times 42) \times 3.5] / 365$

= $(1200 + 825 + 2320 + 2100 + 6930) / 365$

= $13375 / 365$

= 36.64

Therefore, interest is Rs. 36.64.

Question 8

(a) Prove that $[1/(\sec x - \tan x)] + [1/(\sec x + \tan x)] = 2/\cos x$.

[3]

Solution:

LHS = $[1/(\sec x - \tan x)] + [1/(\sec x + \tan x)]$

= $[\sec x + \tan x + \sec x - \tan x] / [(\sec x - \tan x)(\sec x + \tan x)]$

= $2 \sec x / (\sec^2 x - \tan^2 x)$

= $2 \sec x / 1$

= $2(1/\cos x)$

= $2/\cos x$

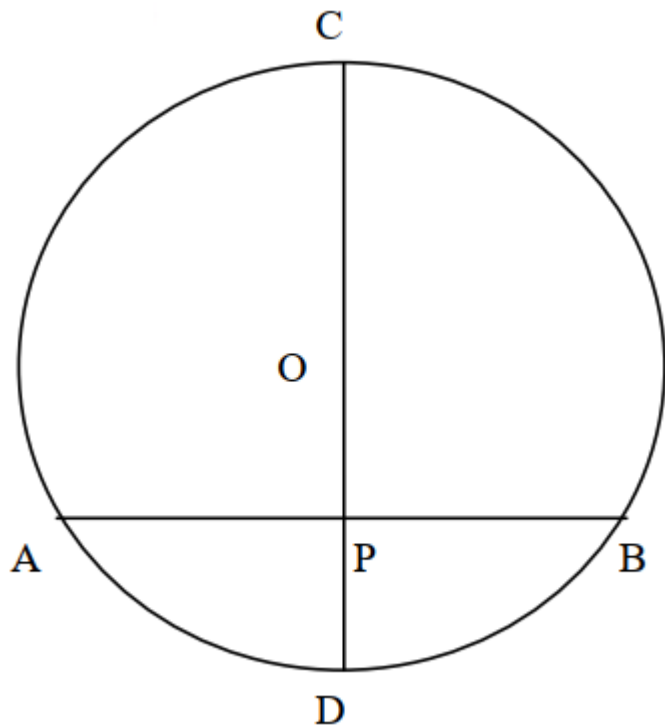
= RHS

Hence proved.

(b) In the figure given below, CD is the diameter of the circle which meets the chord AB at P such that AP = BP =

12 cm. If $DP = 8$ cm, find the radius of the circle.

[3]



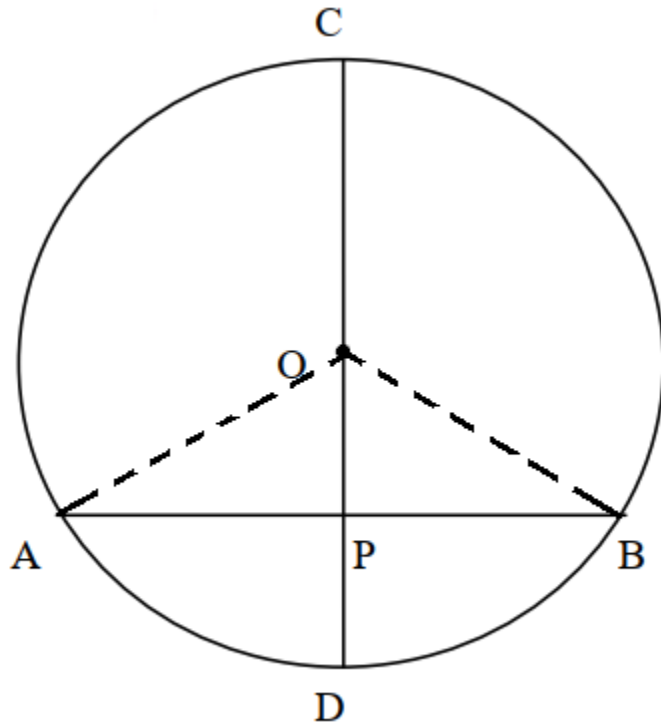
Solution:

Given,

$AP = BP = 12$ cm

$DP = 8$ cm

Join OA and OB .



Let $OP = x$

$OA = OB = OD = \text{Radius of the circle}$

$OA = OB = OD = (8 + x) \text{ cm}$

In right triangle OPA,

$$OA^2 = OP^2 + AP^2$$

$$(8 + x)^2 = x^2 + (12)^2$$

$$64 + x^2 + 16x = x^2 + 144$$

$$16x = 144 - 64$$

$$16x = 80$$

$$x = 80/16$$

$$x = 5$$

Now,

$$OD = 8 + 5 = 13 \text{ cm}$$

Therefore, the radius of the circle is 13 cm.

(c) Prove that $A(2, 1)$, $B(0, 3)$ and $C(-2, 1)$ are the three vertices of an isosceles right-angled triangle. Hence, find the coordinates of a point D, if ABCD is a square. [4]

Solution:

Given,

$A(2, 1)$, $B(0, 3)$ and $C(-2, 1)$

Using distance formula,

$$AB = \sqrt{[(0 - 2)^2 + (3 - 1)^2]}$$

$$= \sqrt{4 + 4}$$

$$= \sqrt{8}$$

$$BC = \sqrt{[(-2 - 0)^2 + (1 - 3)^2]}$$

$$= \sqrt{4 + 4}$$

$$= \sqrt{8}$$

$$AC = \sqrt{(-2 - 2)^2 + (1 - 1)^2}$$

$$= \sqrt{(16 + 0)}$$

$$= \sqrt{16}$$

$$\text{Now, } AB^2 + BC^2 = (\sqrt{8})^2 + (\sqrt{8})^2$$

$$= 8 + 8$$

$$= 16$$

$$= AC^2$$

Therefore, ABC is an isosceles right-angled triangle.

Let D(x, y) be the fourth vertex of square ABCD.

Midpoint of AC = Midpoint of BD

$$[(2 - 2)/2, (1 + 1)/2] = [(0 + x)/2, (3 + y)/2]$$

$$(0/2, 2/2) = [x/2, (3 + y)/2]$$

$$(0, 1) = [x/2, (3 + y)/2]$$

$$x/2 = 0, 3 + y = 2$$

$$x = 0, y = -1$$

Hence, the coordinates of D are (0, -1).

Question 9

(a) A fair die is rolled. Find the probability of getting

(i) 3 on the face of the dice

(ii) an odd number on the face of the dice

(iii) a number greater than 1 on the face of the dice.

[3]

Solution:

Given,

A fair die is rolled.

Sample space = $S = \{1, 2, 3, 4, 5, 6\}$

$$n(S) = 6$$

(i) Let A be the event of getting 3 on the face of the die.

$$A = \{3\}$$

$$n(A) = 1$$

$$P(A) = n(A)/n(S)$$

$$= 1/3$$

(ii) Let B be the event of getting an odd number.

$$B = \{1, 3, 5\}$$

$$n(B) = 3$$

$$P(B) = n(B)/n(S)$$

$$= 3/6$$

$$= 1/2$$

(iii) Let C be the event of getting a number greater than 1.

$$C = \{2, 3, 4, 5, 6\}$$

$$n(C) = 5$$

$$P(C) = n(C)/n(S)$$

$$= 5/6$$

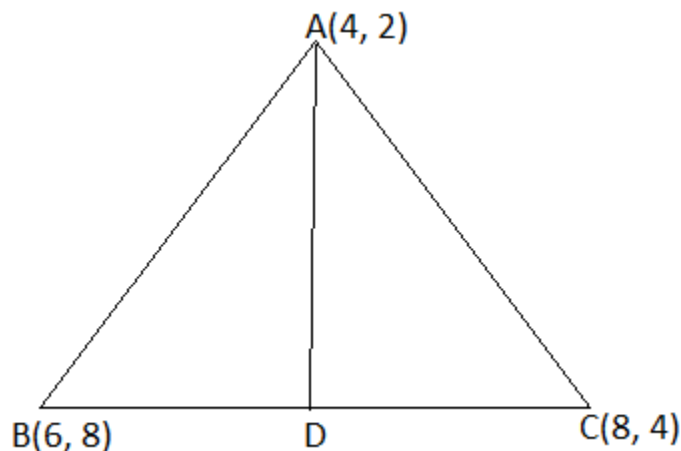
(b) A(4, 2), B(6, 8), and C(8, 4) are the vertices of a triangle ABC. Write down the equation of the median of the triangle through A. [3]

Solution:

Let AD be the median of triangle ABC.

Given,

A(4, 2), B(6, 8) and C(8, 4)



D = Midpoint of BC

$$= [(6 + 8)/2, (8 + 4)/2]$$

$$= (14/2, 12/2)$$

$$= (7, 6)$$

Let A(4, 2) = (x₁, y₁)

D(7, 6) = (x₂, y₂)

Equation of line AD is

$$(y - y_1)/(y_2 - y_1) = (x - x_1)/(x_2 - x_1)$$

$$(y - 2)/(6 - 2) = (x - 4)/(7 - 4)$$

$$(y - 2)/4 = (x - 4)/3$$

$$3(y - 2) = 4(x - 4)$$

$$3y - 6 = 4x - 16$$

$$4x - 16 - 3y + 6 = 0$$

$$4x - 3y - 10 = 0$$

Hence, the equation of the median of the triangle through A is $4x - 3y - 10 = 0$.

(c) The angle of elevation of an aeroplane from a point P on the ground is 60° . After 12 seconds from the same point P, the angle of elevation of the same plane changes to 30° . If the plane is flying horizontally at a speed of $600\sqrt{3}$ km/hr, find the height at which the plane is flying. [4]

Solution:

Given,

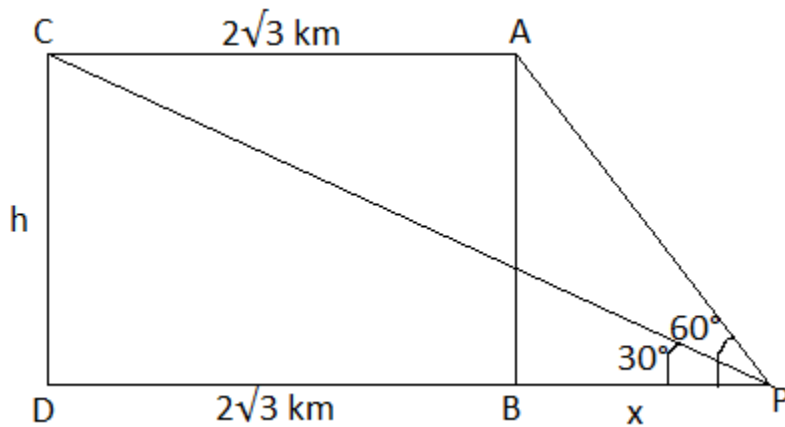
Speed = $600\sqrt{3}$ km/hr

Distance = Speed \times Time

Distance traveled by aeroplane in 12 seconds = $600\sqrt{3} \times (12/60 \times 60)$

$$= 2\sqrt{3} \text{ km}$$

Let A be the initial position of an aeroplane.



$$AC = BD = 2\sqrt{3} \text{ km}$$

$$CD = AB = h$$

In right triangle ABP,

$$\tan 60^\circ = AB/BP$$

$$\sqrt{3} = h/x$$

$$\Rightarrow h = \sqrt{3}x \dots (i)$$

In right triangle CDP,

$$\tan 30^\circ = CD/DP$$

$$1/\sqrt{3} = h/(2\sqrt{3} + x)$$

$$2\sqrt{3} + x = h\sqrt{3}$$

$$2\sqrt{3} + x = (\sqrt{3}x) \sqrt{3} \text{ [From (i)]}$$

$$2\sqrt{3} + x = 3x$$

$$\Rightarrow 3x - x = 2\sqrt{3}$$

$$\Rightarrow 2x = 2\sqrt{3}$$

$$\Rightarrow x = \sqrt{3}$$

Substituting $x = \sqrt{3}$ in (i),

$$h = (\sqrt{3})(\sqrt{3})$$

$$h = 3 \text{ km}$$

Hence, the height at which the plane is flying is 3 km.

Question 10

(a) The following table shows the distribution of the heights of a group of students:

Height (cm)	140 - 145	145 - 150	150 - 155	155 - 160	160 - 165	165 - 170	170 - 175
No. of students	8	12	18	22	26	10	4

Use a graph sheet to draw an Ogive for the distribution.

Use the Ogive to find:

(i) the interquartile range

(ii) the number of students whose height is more than 168 cm

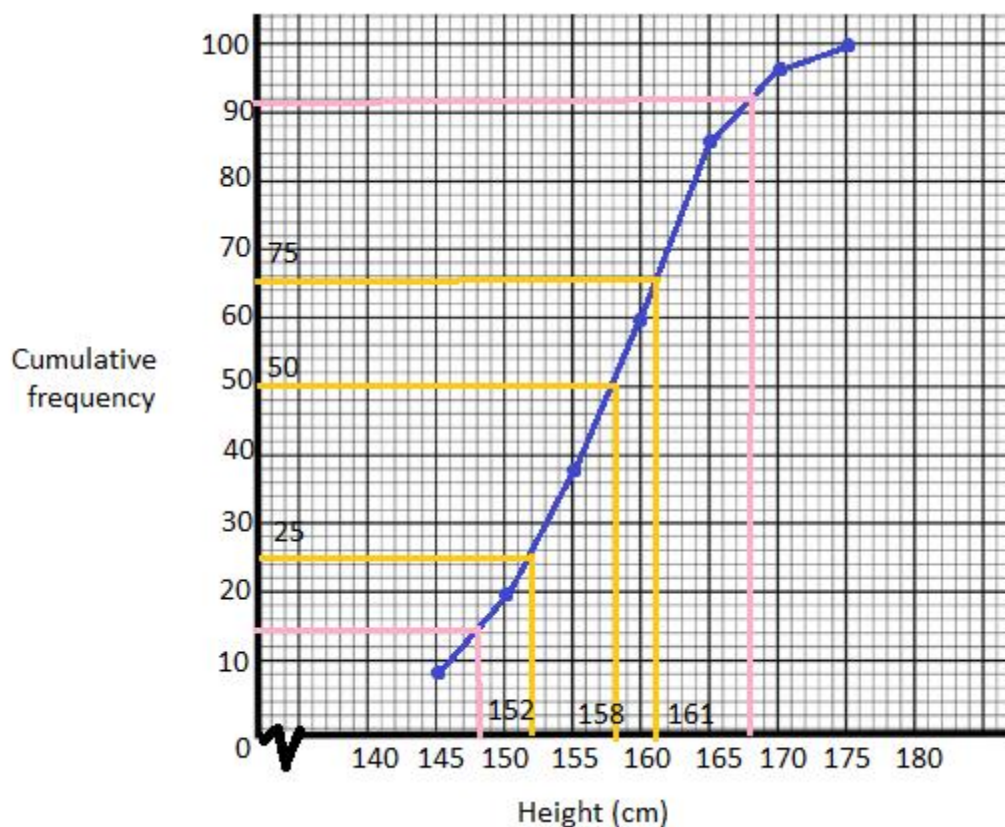
(iii) the number of students whose height is less than 148 cm.

[6]

Solution:

Height (cm)	No. of students (frequency)	Cumulative frequency
140 - 145	8	8
145 - 150	12	20
150 - 155	18	38
155 - 160	22	60
160 - 165	26	86
165 - 170	10	96
170 - 175	4	100

Ogive:



(i) Median value = $(N/2)$ th observation = $(100/2) = 50$ th observation = 158
 Lower quartile = $(N/4)$ th observation = $(100/4) = 25$ th observation = 152
 Upper quartile = $(3N/4)$ th observation = $(300/4) = 75$ th observation = 161
 Inter quartile range = Upper quartile - Lower quartile
 = 161 - 152

= 9

(ii) The number of students whose height is more than 168 cm = $100 - 92 = 8$

(iii) The number of students whose height is less than 148 cm = 14

(b) The manufacturer sold a TV to a wholesaler for Rs. 7000. The wholesaler sold it to a trader at a profit of Rs. 1000. If the trader sold it to the customer at a profit of Rs. 1500, find:

(i) the total VAT (value-added tax) collected by the state government at the rate of 5%.

(ii) the amount that the customer pays for the TV. [4]

Solution:

Cost price of TV for wholesaler = Rs. 7000

Selling price of TV to a trader from wholesaler = Rs. $(7000 + 1000) = \text{Rs. } 8000$

Selling price of TV from trader = Rs. $(8000 + 1500) = \text{Rs. } 9500$

(i) The total VAT (value added tax) collected by the state government at the rate of 5% = $(5/100) \times 9500 = \text{Rs. } 475$

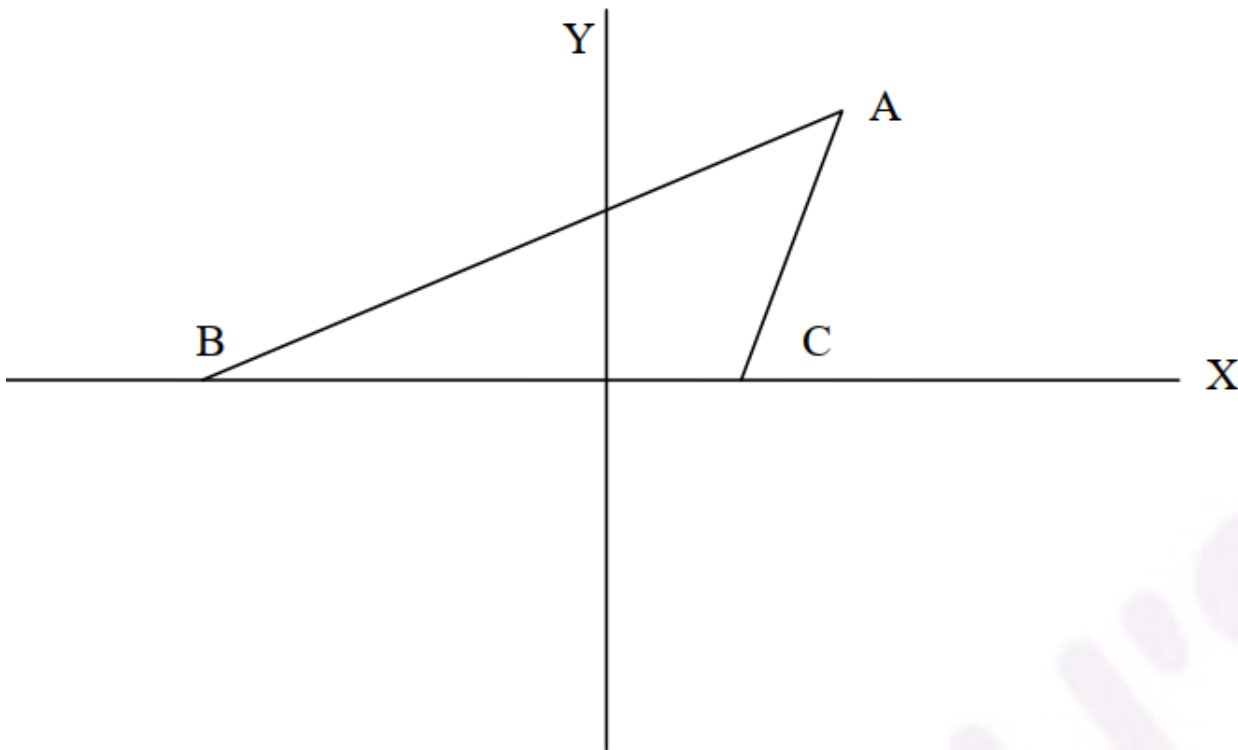
(ii) The amount the customer pays for the TV = Rs. $9500 + \text{Rs. } 475$

<https://www.timesnownews.com/education/article/cbse-syllabus-reduced-by-30-for-9th-to-12th-core-concepts-retained-hrd-minister/617823>

= Rs. 9975

Question 11:

(a) In the diagram given below, the equation of AB is $x - \sqrt{3}y + 1 = 0$ and the equation of AC is $x - y - 2 = 0$.



(i) Write down the angles that the lines AC and AB make with the positive direction of X-axis.

(ii) Find $\angle BAC$. [3]

Solution:

(i) Given,

The equation of AB is $x - \sqrt{3}y + 1 = 0$.

$$\sqrt{3}y = x + 1$$

$$\text{Slope} = 1/\sqrt{3}$$

$$\tan \theta = 1/\sqrt{3}$$

$$\tan \theta = \tan 30^\circ$$

$$\theta = 30^\circ$$

$$\text{i.e. } \angle ABC = 30^\circ$$

The equation of AC is $x - y - 2 = 0$.

$$y = x - 2$$

$$\text{Slope} = 1$$

$$\tan \theta = 1$$

$$\tan \theta = \tan 45^\circ$$

$$\theta = 45^\circ$$

$$\text{i.e. } \angle ACX = 45^\circ$$

$$(ii) \angle ACB = 180^\circ - \angle ACX = 180^\circ - 45^\circ = 135^\circ$$

In triangle ABC,

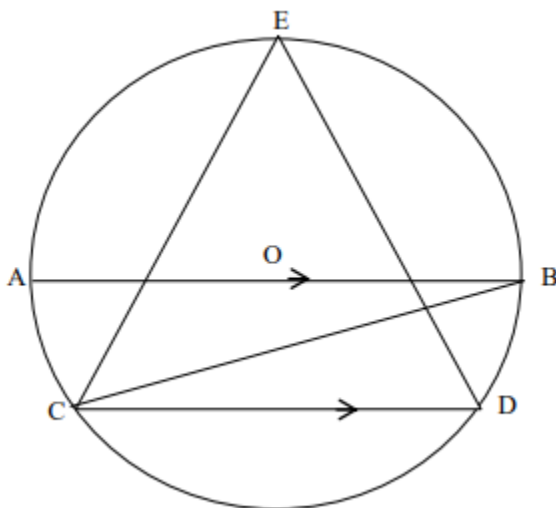
$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

$$30^\circ + 135^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 30^\circ - 135^\circ$$

$$\angle BAC = 15^\circ$$

(b) In the figure given below, O is the centre of the circle. Chord CD is parallel to the diameter AB. If $\angle ABC = 35^\circ$, calculate $\angle CED$. [3]



Solution:

Given,

Chord CD is parallel to the diameter AB.

$$\angle ABC = 35^\circ$$

The angle subtended by the chord at the centre is twice the angle subtended by it on the corresponding segment of the circle.

$$\angle AOC = 2\angle ABC$$

$$= 2 \times 35^\circ$$

$$= 70^\circ$$

$$\angle AOC = \angle BOD = 70^\circ$$

By linear pair axiom,

$$\angle AOC + \angle COD + \angle BOD = 180^\circ$$

$$70^\circ + \angle COD + 70^\circ = 180^\circ$$

$$\angle COD = 180^\circ - 70^\circ - 70^\circ$$

$$\angle COD = 140^\circ$$

Now,

$$\angle COD = 2\angle CED$$

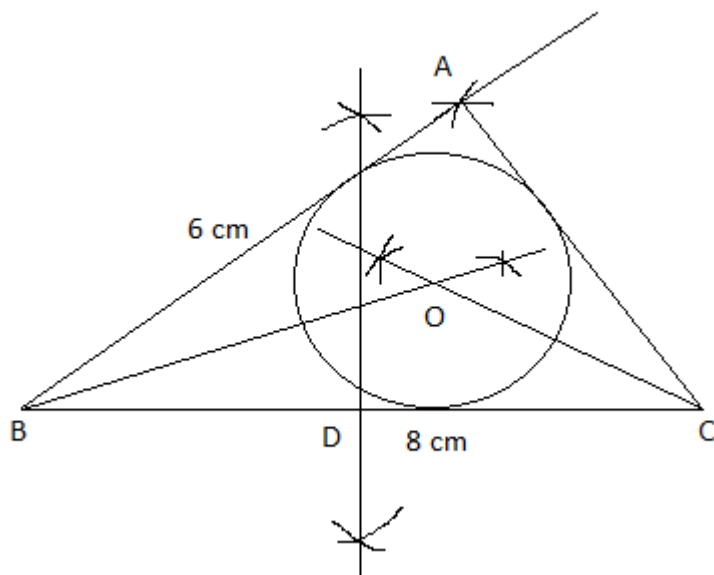
$$140^\circ = 2\angle CED$$

$$\angle CED = 140^\circ / 2$$

$$\angle CED = 70^\circ$$

(c) Construct a triangle ABC, given that AB = 6 cm, BC = 8 cm and median AD = 5 cm. Construct an incircle to triangle ABC and measure its radius. [4]

Solution:



Measure of radius of incircle = 2 cm (approx)