

ICSE Class 10 Maths Mock Sample Paper 2 with Solutions

SECTION A

Attempt all questions from this section.

Question 1

(a) Find the remainder when $2x^3 - 3x^2 + 7x - 8$ is divided by $x - 1$.

[3]

Solution:

$$\text{Let } p(x) = 2x^3 - 3x^2 + 7x - 8$$

$$g(x) = x - 1$$

$$\begin{array}{r}
 \quad \quad \quad 2x^2 \quad -x \quad +6 \\
 x-1 \overline{) 2x^3 \quad -3x^2 \quad +7x \quad -8} \\
 \underline{2x^3 \quad -2x^2} \\
 \quad -x^2 \quad +7x \quad -8 \\
 \quad \underline{-x^2 \quad +x} \\
 \quad \quad 6x \quad -8 \\
 \quad \quad \underline{6x \quad -6} \\
 \quad \quad -2
 \end{array}$$

Remainder = -2

(b) Find the value of $\operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta)$.

[3]

Solution:

$$\operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta)$$

$$= \operatorname{cosec}[90^\circ - (25^\circ - \theta)] - \sec(25^\circ - \theta)$$

$$= \sec(25^\circ - \theta) - \sec(25^\circ - \theta)$$

$$= 0$$

(c) A shopkeeper buys an article at a discount of 25% from the wholesaler. The printed price of the article is Rs. 5000 and the rate of sales tax is 10%. The shopkeeper sells it to the customer at a discount of 5% of the printed price and charges the sales tax at the same rate. Find:

(i) the amount paid by the customer

(ii) the VAT (Value Added Tax) paid by the shopkeeper.

[4]

Solution:

Given,

The printed price of the article = Rs. 5000

Discount = 25%

The purchase price for shopkeeper = Rs. 5000 - (25% of Rs. 5000)

= Rs. 5000 - $(\frac{25}{100}) \times \text{Rs. } 5000$

= Rs. 5000 - 1250

= Rs. 3750

Tax paid by the shopkeeper = 10% of Rs. 3750 = Rs. 375

Price paid by the shopkeeper for article = Rs. 3750 + Rs. 375 = Rs. 4125

(i) Discount given to the customer = 5% of Rs. 5000 = Rs. 250

Price of article after discount = Rs. 5000 - Rs. 250 = Rs. 4750

Tax paid by the customer = 10% of Rs. 4750 = Rs. 475

Price paid by the customer = Rs. 4750 + Rs. 475 = Rs. 5225

(ii) VAT paid by the shopkeeper = Tax charged when selling - Tax paid when buying

= Rs. 475 - Rs. 375

= Rs. 100

Question 2

(a) Manoj opened a recurring deposit account with Punjab National Bank and deposited Rs. 500 per month for 3 years. The bank paid him Rs. 20220 on maturity. Find the rate of interest paid by the bank. [3]

Solution:

Given,

Monthly installment = P = Rs. 500

n = 3 years = 36 months

Let r be the rate of interest.

Maturity value = $(P \times n) + P \times \frac{[n(n+1)]}{2 \times 12} \times \frac{r}{100}$

$20220 = (500 \times 36) + 500 \times \frac{[(36 \times 37)]}{(2 \times 12)} \times \frac{r}{100}$

$20220 - 18000 = 500 \times \frac{(111/2)}{1} \times \frac{r}{100}$

$2220 = 555r/2$

$r = \frac{(2220 \times 2)}{555}$

r = 8%

Hence, the rate of interest paid by the bank is 8%.

(b) The volume and curved surface of a cylinder are equal numerically. If the height is $3\frac{1}{2}$ times the radius of the base, find the radius. [3]

Solution:

Let r be the radius and h be the height of a cylinder.

Given,

$h = 3\frac{1}{2} \times 2 = 7r/2$

The volume of cylinder = Curved surface area of the cylinder (given)

$\pi r^2 h = 2\pi r h$

$r^2 = 2r$

r = 2 units

Hence, the radius of the base of the cylinder is 2 units.

(c) A solid consisting of a right circular cone, standing on a hemisphere, is placed upright in a right circular cylinder full of water and touches the bottom. Find the volume of water left in the cylinder having given that the radius of the cylinder is 3 cm and its height is 6 cm, the radius of the hemisphere is 2 cm and the height of the cone is 4 cm. Give your answer to the nearest cubic centimeters. [4]

Solution:

Given,

Height of the cylinder = $H = 6$ cm

Radius of the cylinder = $R = 3$ cm

Height of the cone = $h = 4$ cm

Radius of the hemisphere = Radius of cone = $r = 2$ cm

Volume of water in the cylinder when it is full = $\pi r^2 h$

$$= \pi \times 3 \times 3 \times 6$$

$$= 54\pi \text{ cm}^3$$

Volume of water displaced = volume of cone + volume of hemisphere

$$= (1/3) \pi r^2 h + (2/3) \pi r^3$$

$$= (1/3) \pi r^2 (h + 2r)$$

$$= (1/3) \pi \times 2 \times 2 (4 + 2 \times 2)$$

$$= (1/3) \pi \times 4 \times 8$$

$$= 32\pi/3 \text{ cm}^3$$

Volume of water left in the cylinder = $54\pi - (32\pi/3)$

$$= (162\pi - 32\pi)/3$$

$$= 130\pi/3$$

$$= (130/3) \times (22/7)$$

$$= 136.19$$

Hence, the volume of water left in the cylinder is 136 cm^3 . (approx.)

Question 3

(a) Using factor theorem, show that $(x - 3)$ is a factor of $x^3 - 7x^2 + 15x - 9$. Hence, factorize the given expression completely. [3]

Solution:

$$\text{Let } p(x) = x^3 - 7x^2 - 15x - 9$$

For checking $(x - 3)$ is a factor of $p(x)$ or not, substitute $x = 3$ in $p(x)$,

$$p(3) = (3)^3 - 7(3)^2 + 15(3) - 9$$

$$= 27 - 63 + 45 - 9$$

$$= 72 - 72$$

$$= 0$$

Therefore, $(x - 3)$ is a factor of the given polynomial.

By dividing $p(x)$ by $(x - 3)$, the quotient = $x^2 - 4x + 3$

$$x^3 - 7x^2 + 15x - 9 = (x - 3)(x^2 - 4x + 3)$$

$$= (x - 3)(x^2 - 3x - x + 3)$$

$$= (x - 3) [x(x - 3) - 1(x - 3)]$$

$$= (x - 3)(x - 3)(x - 1)$$

$$= (x - 3)^2(x - 1)$$

(b) Find the 2×2 matrix X which satisfies the equation:

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

[3]

Solution:

Given,

$$\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 + 35 & 6 + 21 \\ 0 + 20 & 4 + 12 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} + 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} - \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -34 & -32 \\ -24 & -10 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} \frac{-34}{2} & \frac{-32}{2} \\ \frac{-24}{2} & \frac{-10}{2} \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -17 & -16 \\ -12 & -5 \end{bmatrix}$$

(c) Solve the inequation:

$$12 + 1 \frac{5}{6} x \leq 5 + 3x, x \in \mathbb{R}$$

Represent the solution on a number line.

[4]

Solution:

Given,

$$12 + 1 \frac{5}{6} x \leq 5 + 3x$$

$$12 + (11/6)x \leq 5 + 3x$$

$$12 - 5 \leq 3x - (11x/6)$$

$$7 \leq (18x - 11x)/6$$

$$7 \leq 7x/6$$

$$x \geq (7 \times 6)/7$$

$$x \geq 6$$

Therefore, $\{x : x \in \mathbb{R} \text{ and } \geq 6\}$

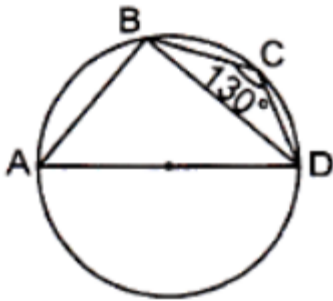


Question 4

(a) In the figure, AD is the diameter of the circle. If $\angle BCD = 130^\circ$, calculate:

(i) $\angle DAB$

(ii) $\angle ADB$



[3]

Solution:

Given,

AD is the diameter of the circle.

$$\angle BCD = 130^\circ$$

(i) ABCD is a cyclic quadrilateral. (from the given figure)

We know that the sum of the opposite angles of a cyclic quadrilateral is supplementary.

$$\angle DAB + \angle BCD = 180^\circ$$

$$\angle DAB + 130^\circ = 180^\circ$$

$$\angle DAB = 180^\circ - 130^\circ$$

$$\angle DAB = 50^\circ$$

(ii) The angle in a semicircle is a right angle.

Thus, in triangle ABD, $\angle ABD = 90^\circ$

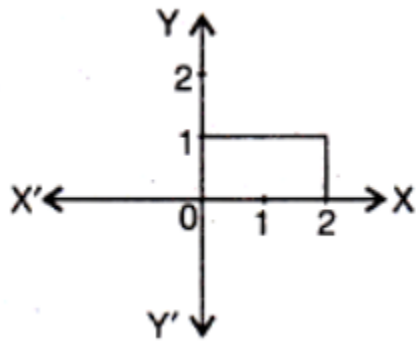
$$\angle ABD + \angle ADB + \angle DAB = 180^\circ \text{ (by the angle sum property of a triangle)}$$

$$90^\circ + \angle ADB + 50^\circ = 180^\circ$$

$$\angle ADB = 180^\circ - 90^\circ - 50^\circ$$

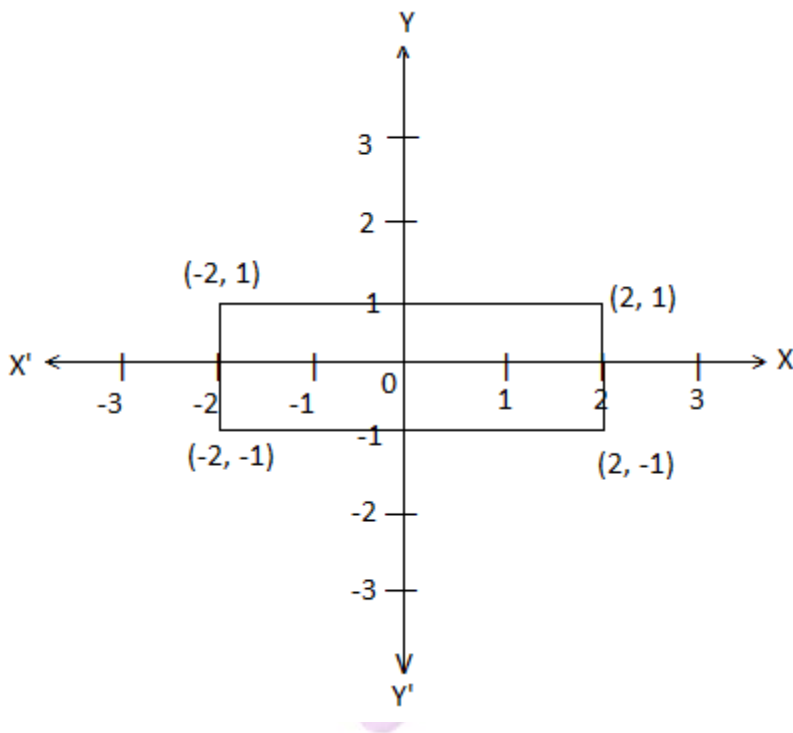
$$\angle ADB = 40^\circ$$

(b) Part of a geometrical figure is given in the diagram alongside. Complete the figure so that both the x-axis and y-axis are lines of symmetry of the completed figure. [3]



Solution:

The completed figure that shows both the x-axis and y-axis are lines of symmetry is:



(c) Calculate the mean wage correct to the nearest rupee for the following data:

Category	A	B	C	D	E	F	G
Wages in Rs per day	50	60	70	80	90	100	110
No. of workers	2	4	8	12	10	6	8

[4]

Solution:

Category	Wages in Rs per day (x)	No. of workers (f)	fx
A	50	2	100
B	60	4	240
C	70	8	560
D	80	12	960
E	90	10	900
F	100	6	600
G	110	8	880
		$\Sigma f = 50$	$\Sigma fx = 4240$

$$\begin{aligned} \text{Mean} &= \frac{\Sigma fx}{\Sigma f} \\ &= \frac{4240}{50} \\ &= 84.5 \end{aligned}$$

Hence, the approximate mean wage is Rs. 85.

SECTION - B

Attempt any four questions from this section.

Question 5

(a) Solve using the quadratic formula: $6x^2 + (12 - 8a)x - 16a = 0$

[3]

Solution:

Given,

$$6x^2 + (12 - 8a)x - 16a = 0$$

Comparing with the standard form $ax^2 + bx + c = 0$

$$a = 6, b = 12 - 8a, c = -16a$$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(12 - 8a) \pm \sqrt{(12 - 8a)^2 - 4(6)(-16a)}}{2(6)}$$

$$= \frac{[(8a - 12) \pm \sqrt{(144 + 64a^2 - 192a + 384a)}]}{12}$$

$$= \frac{[(8a - 12) \pm \sqrt{(144 + 64a^2 + 192a)}]}{12}$$

$$= \frac{[(8a - 12) \pm \sqrt{(12 + 8a)^2}]}{12}$$

$$= \frac{[(8a - 12) \pm (12 + 8a)]}{12}$$

$$x = \frac{(8a - 12 + 12 + 8a)}{12}, x = \frac{(8a - 12 - 12 - 8a)}{12}$$

$$x = \frac{16a}{12}, x = \frac{-24}{12}$$

$$x = \frac{4a}{3}, x = -2$$

(b) Deepika opened a savings bank account in a bank. Her passbook entries are shown below:

Date	Particulars	Withdrawals	Deposits Rs	Balance Rs

Jan 1	B/F	–	–	6360
Jan 12	By cash	–	750	7110
Feb 15	To self	5000	–	2110
June 6	To cheque	354	–	1756
July 18	By cheque	–	543	2299

She closed the account on 29 July and received Rs. 2354.20 as balance. Calculate the rate of interest. [3]

Solution:

Month	Qualifying amount for interest
Jan	Rs. 7110
Feb	Rs. 2110
Mar	Rs. 2110
Apr	Rs. 2110
May	Rs. 2110
June	Rs. 1756
Total	Rs. 17306

Let R be the rate of interest.

$$\begin{aligned} \text{Interest} &= \text{PTR}/100 \\ &= (17306 \times 1 \times R)/100 \times 12 \\ &= 17306 R/1200 \end{aligned}$$

Given that, Deepika received Rs. 2354.20 as balance.

$$2354.20 = 2299 + (17306R/1200)$$

$$2354.20 - 2299 = 17306 R/1200$$

$$55.2 = 17306R/1200$$

$$R = (55.2 \times 1200)/17306$$

$$R = 3.8\%$$

(c) Use graph paper for this question.

(i) Plot the points A(3, 5) and B(-2, -4). Use 1 cm = 1 unit on both the axes.

(ii) A' is the image of A when reflected in the x-axis. Write down the coordinates of A' and plot it on the graph paper.

(iii) B' is the image of B when reflected in the y-axis followed by a reflection in the origin. Write down the coordinates of B' and plot it on the graph paper.

(iv) Write down the geometrical name of the figure AA'BB'. [4]

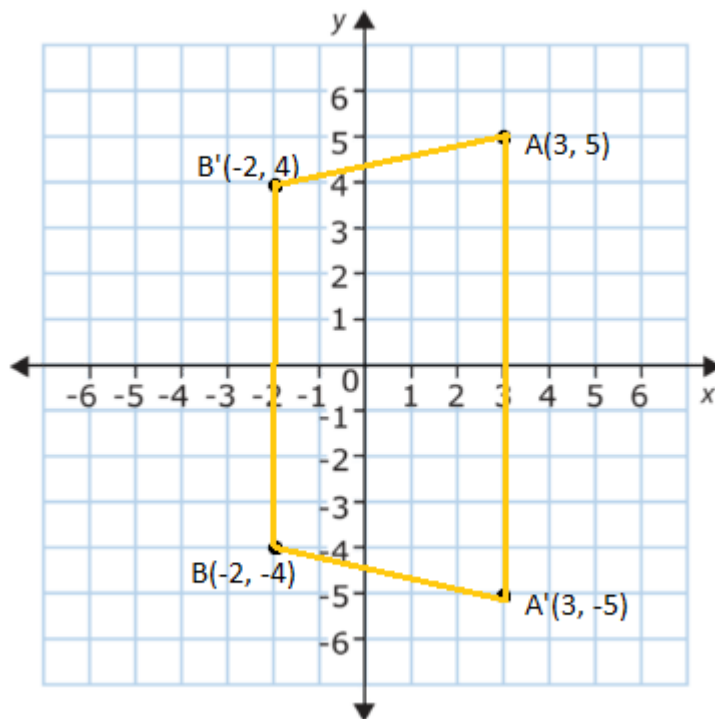
Solution:

Given,

A(3, 5) and B(-2, -4)

A' is the image of A when reflected in the x-axis.

B' is the image of B when reflected in the y-axis followed by a reflection in the origin.

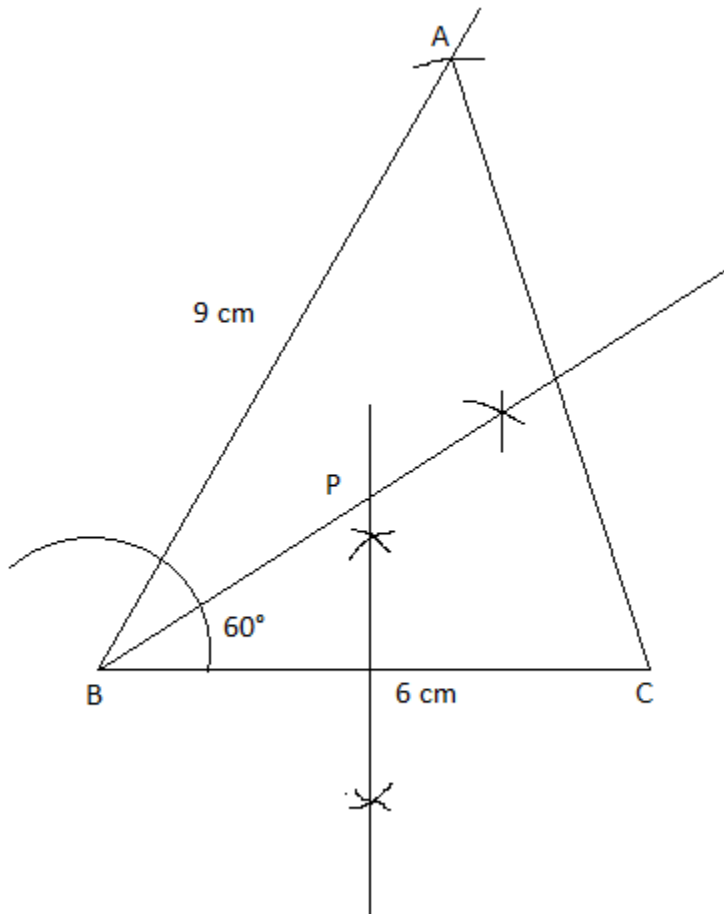


- (ii) Coordinates of A'(3, -5)
- (iii) Coordinates of B'(-2, 4)
- (iv) The geometrical name of the figure AA'BB' is an isosceles trapezium since $AA' \parallel BB'$ and $AB' = A'B$.

Question 6

(a) Construct a triangle ABC in which $BC = 6$ cm, $AB = 9$ cm, and $\angle ABC = 60^\circ$. Construct the locus of all points inside triangle ABC, which are equidistant from B and C. [3]

Solution:



P is the locus of all points inside triangle ABC which is equidistant from B and C.

(b) The compound interest, calculated yearly on a certain sum of money for the second year is Rs. 880 and for the third year, it is Rs. 968. Calculate the rate of interest and the sum of money. [3]

Solution:

Given,

CI for the second year = Rs. 880

CI for the third year = Rs. 968

Now,

Simple interest (SI) on Rs. 880 for one year = Rs. 968 - Rs. 880 = Rs. 88

$SI = \frac{PTR}{100}$

$R = \frac{(88 \times 100)}{(880 \times 1)}$

$R = 10\%$

Let p be the actual sum.

Amount after 2 years - Amount after 1 year = CI for the second year

$$p(1 + 10/100)^2 - p(1 + 10/100) = 880$$

$$p(110/100)^2 - p(110/100) = 880$$

$$p[(11/10)^2 - (11/10)] = 880$$

$$p[(121/100) - (11/10)] = 880$$

$$p[(121 - 110)/ 100] = 800$$

$$p(11/100) = 880$$

$$p = 8000$$

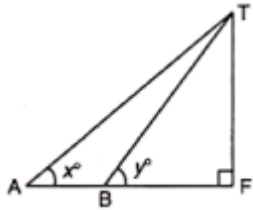
Hence, the rate of interest is 10% and the actual sum of money is Rs. 8000.

(c) In the figure, not drawn to scale, TF is a tower. The elevation of T from A is x° , where $\tan x = \frac{2}{5}$ and $AF = 200$ m. The elevation of T from B, where $AB = 80$ m, is y° . Calculate:

(i) the height of the tower TF.

(ii) the angle y , correct to the nearest degree.

[4]



Solution:

Given,

TF is a tower.

The elevation of T from A is x° , where $\tan x = \frac{2}{5}$ and $AF = 200$ m.

The elevation of T from B, where $AB = 80$ m, is y° .

(i) In right triangle AFT,

$$\tan x = \frac{TF}{AF}$$

$$\frac{2}{5} = \frac{TF}{200}$$

$$TF = \frac{(200 \times 2)}{5}$$

$$TF = 80 \text{ m}$$

Therefore, the height of the tower TF is 80 m.

(ii) $BF = AF - AB = 200 - 80 = 120$ m

$$TF = 80 \text{ m}$$

In right triangle BFT,

$$\tan y = \frac{TF}{BF}$$

$$\tan y = \frac{80}{120}$$

$$\tan y = \frac{2}{3}$$

$$y = \tan^{-1}\left(\frac{2}{3}\right)$$

$$y = 33.7$$

Therefore, the angle $y = 34^\circ$ (approx.)

Question 7

(a) A bag contains 8 red, 6 white, and 4 black balls. A ball is drawn at random from the bag. Find the probability that the drawn ball is:

(i) red or white

(ii) not black

(iii) neither white nor black.

[3]

Solution:

Given,

A bag contains 8 red, 6 white, and 4 black balls.

$$\text{Total number of outcomes} = n(S) = 18$$

$$\text{i.e. } 8 + 6 + 4 = 18$$

(i) Let A be the event of getting a red or white ball.

$$\text{Number of outcomes favourable to A} = n(A) = 14$$

i.e. $8 + 6 = 14$

$P(A) = n(A)/n(S) = 14/18 = 7/9$

(ii) Let B be the event of getting a ball that is not black.

Number of outcomes favourable to B = 14

i.e. Total number of balls - Black balls = $18 - 4 = 14$

$P(B) = n(B)/n(S) = 14/18 = 7/9$

(iii) Let C be the event of getting neither white nor black balls.

Number of outcomes favourable to C = $n(C) = 8$

i.e. only red balls = 8

$P(C) = n(C)/n(S) = 8/18 = 4/9$

(b) A can do a piece of work in x days and B can do it in (x + 16) days. If both working together can do it in 15 days, find x. [3]

Solution:

Given,

A can do a piece of work in x days and B can do it in (x + 16) days.

A and B can finish the work in 15 days.

i.e. $x(x + 16)/(x + x + 16) = 15$

$(x^2 + 16x)/(2x + 16) = 15$

$x^2 + 16x = 15(2x + 16)$

$x^2 + 16x = 30x + 240$

$x^2 + 16x - 30x - 240 = 0$

$x^2 - 14x - 240 = 0$

$x^2 - 24x + 10x - 240 = 0$

$x(x - 24) + 10(x - 24) = 0$

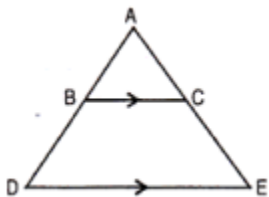
$(x + 10)(x - 24) = 0$

$x = -10, x = 24$

The number of days cannot be negative.

Therefore, x = 24

(c) In the figure, BC is parallel to DE. Area of triangle ABC = 25 cm², area of trapezium BCED = 24 cm², DE = 14 cm. Calculate the length of BC. [4]



Solution:

Given,

Area of triangle ABC = 25 cm²

Area of trapezium BCED = 24 cm²

Area of triangle ADE = Area of triangle ABC + Area of trapezium

= 25 + 24

= 49 cm²

In ΔABC and ΔADE ,

$BC \parallel DE$,

$\angle ABC = \angle ADE$ (corresponding angles)
 $\angle ACB = \angle AED$ (corresponding angles)
 $\angle BAC = \angle DAE$ (common)

By AAA similarity.

$\Delta ABC \sim \Delta ADE$

$\text{ar}(\Delta ABC) / \text{ar}(\Delta ADE) = BC^2 / DE^2$

$25/49 = BC^2 / (14)^2$

$BC^2 = (25 \times 196) / 49$

$BC^2 = 4900 / 49$

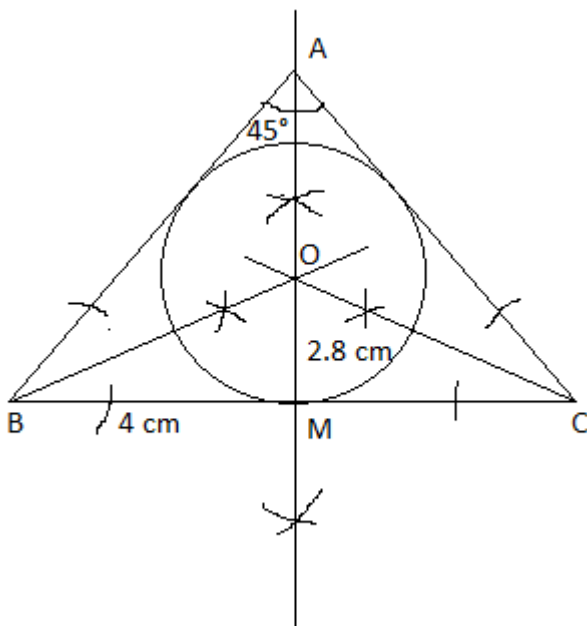
$BC^2 = 100$

$\Rightarrow BC = 10 \text{ cm}$

Question 8

(a) Using a ruler and compasses only, construct an isosceles ΔABC having base = 4 cm, vertical angle = 45° , and median through vertex equal to 2.8 cm. Draw the incircle of the triangle. [3]

Solution:



(b) Using the properties of proportion, solve for x:

$$\frac{[\sqrt{(a+x)} + \sqrt{(a-x)}]}{[\sqrt{(a+x)} - \sqrt{(a-x)}]} = b$$

[3]

Solution:

Given,

$$\frac{[\sqrt{(a+x)} + \sqrt{(a-x)}]}{[\sqrt{(a+x)} - \sqrt{(a-x)}]} = b$$

Using componendo and dividendo rule,

$$\frac{[\sqrt{(a+x)} + \sqrt{(a-x)} + \sqrt{(a+x)} - \sqrt{(a-x)}]}{[\sqrt{(a+x)} + \sqrt{(a-x)} - \sqrt{(a+x)} + \sqrt{(a-x)}]} = \frac{(b+1)}{(b-1)}$$

$$2\sqrt{(a+x)} / 2\sqrt{(a-x)} = \frac{(b+1)}{(b-1)}$$

$$\sqrt{(a+x)} / \sqrt{(a-x)} = \frac{(b+1)}{(b-1)}$$

Squaring on both sides,

$$(a+x) / (a-x) = \frac{(b+1)^2}{(b-1)^2}$$

Again, using componendo and dividendo rule,

$$(a + x + a - x) / (a + x - a + x) = [(b + 1)^2 + (b - 1)^2] / [(b + 1)^2 - (b - 1)^2]$$

$$2a/2x = 2(b^2 + 12) / 2(2b)$$

$$a/x = (b^2 + 1)/2b$$

$$\Rightarrow x/a = 2b / (b^2 + 1)$$

$$\Rightarrow x = 2ab / (b^2 + 1)$$

(c) A man invests Rs. 8800 on buying shares of the face value of Rs. 100 each at a premium of 10% in a company. If he earns Rs. 1200 at the end of the year as a dividend, find:

- (i) the number of shares he has in the company
(ii) the dividend percentage per share.

[4]

Solution:

Given,

Total investment = Rs. 8800

Nominal value = Rs. 100

Market value = 10% of Rs. 100 + Rs. 100

= Rs. 10 + Rs. 100

= Rs. 110

(i) Number of shares purchased = $8800/110 = 80$

(ii) Let d be the dividend percentage.

Nominal value of 80 shares = $80 \times \text{Rs. } 100 = \text{Rs. } 8000$

According to the given,

$d\%$ of Rs. 8000 = Rs. 1200

$(d/100) \times 8000 = 1200$

$d \times 80 = 1200$

$d = 1200/80$

$d = 15\%$

Hence, the dividend percentage per share is 15%.

Question 9

(a) Show that the points A(1, 0), B(5, 3), C(2, 7), and D(-2, 4) are the vertices of a square.

[3]

Solution:

Given,

A(1, 0), B(5, 3), C(2, 7), and D(-2, 4)

Using distance formula,

$AB = \sqrt{[(5 - 1)^2 + (3 - 0)^2]}$

$= \sqrt{(16 + 9)}$

$= \sqrt{25}$

$AB = 5$

$BC = \sqrt{[(2 - 5)^2 + (7 - 3)^2]}$

$= \sqrt{(9 + 16)}$

$= \sqrt{25}$

$BC = 5$

$CD = \sqrt{[(-2 - 2)^2 + (4 - 7)^2]}$

$= \sqrt{(16 + 9)}$

$= \sqrt{25}$

$CD = 5$

$DA = \sqrt{[(1 + 2)^2 + (0 - 4)^2]}$

$= \sqrt{(9 + 16)}$

$$= \sqrt{25}$$

$$DA = 5$$

All the sides are equal.

AC and BD are the diagonals of quadrilateral ABCD.

$$\text{Midpoint of AC} = [(1 + 2)/2, (0 + 7)/2] = (3/2, 7/2)$$

$$\text{Midpoint of BD} = [(5 - 2)/2, (3 + 4)/2] = (3/2, 7/2)$$

Midpoint of AC = Midpoint of BD

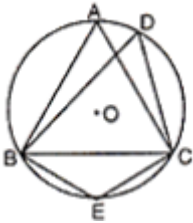
Therefore, the given points are the vertices of a square.

(b) In the figure, O is the centre of the circle and ΔABC is equilateral. Find

(i) $\angle BDC$

(ii) $\angle BEC$.

[3]



Solution:

Given,

ΔABC is an equilateral triangle.

$$\angle ABC = \angle BCA = \angle CAB = 60^\circ$$

(i) We know that the angles subtended by an arc on the circumference on the same segment are equal.

$$\angle BDC = \angle CAB = 60^\circ$$

(ii) ABEC is a cyclic quadrilateral.

We know that the sum of opposite angles of a cyclic quadrilateral is supplementary.

$$\angle BAC + \angle BEC = 180^\circ$$

$$60^\circ + \angle BEC = 180^\circ$$

$$\angle BEC = 180^\circ - 60^\circ$$

$$\angle BEC = 120^\circ$$

(c) A bucket is raised from a well by means of a rope that is wound around a wheel of diameter 77 cm. Given that the bucket ascends in 1 min 28 sec with a uniform speed of 1.1 m/sec, calculate the number of complete revolutions the wheel makes in raising the bucket. Take $\pi = 22/7$

[4]

Solution:

Given,

$$\text{Diameter of wheel} = 77 \text{ cm}$$

$$\text{Radius of the wheel} = r = 77/2 \text{ cm} = 77/200 \text{ m}$$

$$\text{Time} = 1 \text{ min } 28 \text{ sec} = (60 + 28) \text{ sec} = 88 \text{ sec}$$

$$\text{Speed} = 1.1 \text{ m/s}$$

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$= 1.1 \times 88$$

$$= 96.8 \text{ m}$$

$$\text{Number of revolutions} = \text{Distance} / \text{Circumference of wheel}$$

$$= 96.8 / 2\pi r$$

$$= 96.8 / [2 \times (22/7) \times (77/200)]$$

$$= 96.8 / 2.42$$

= 40

Therefore, the number of complete revolutions the wheel makes in raising the bucket is 40.

Question 10

(a) Prove the following identity:

$$[1/(\sin \theta + \cos \theta)] + [1/(\sin \theta - \cos \theta)] = 2 \sin \theta / (1 - 2 \cos^2 \theta) \quad [4]$$

Solution:

$$\begin{aligned} \text{LHS} &= [1/(\sin \theta + \cos \theta)] + [1/(\sin \theta - \cos \theta)] \\ &= [(\sin \theta + \cos \theta + \sin \theta - \cos \theta) / (\sin \theta + \cos \theta)(\sin \theta - \cos \theta)] \\ &= 2 \sin \theta / (\sin^2 \theta - \cos^2 \theta) \\ &= 2 \sin \theta / [(1 - \cos^2 \theta) - \cos^2 \theta] \\ &= 2 \sin \theta / (1 - \cos^2 \theta - \cos^2 \theta) \\ &= 2 \sin \theta / (1 - 2 \cos^2 \theta) \\ &= \text{RHS} \end{aligned}$$

(b) The following table shows the distribution of the heights of a group of factory workers:

Height (in cm)	150 - 155	155 - 160	160 - 165	165 - 170	170 - 175	175 - 180	180 - 185
No. of workers	6	12	18	20	13	8	6

(i) Determine the cumulative frequencies.

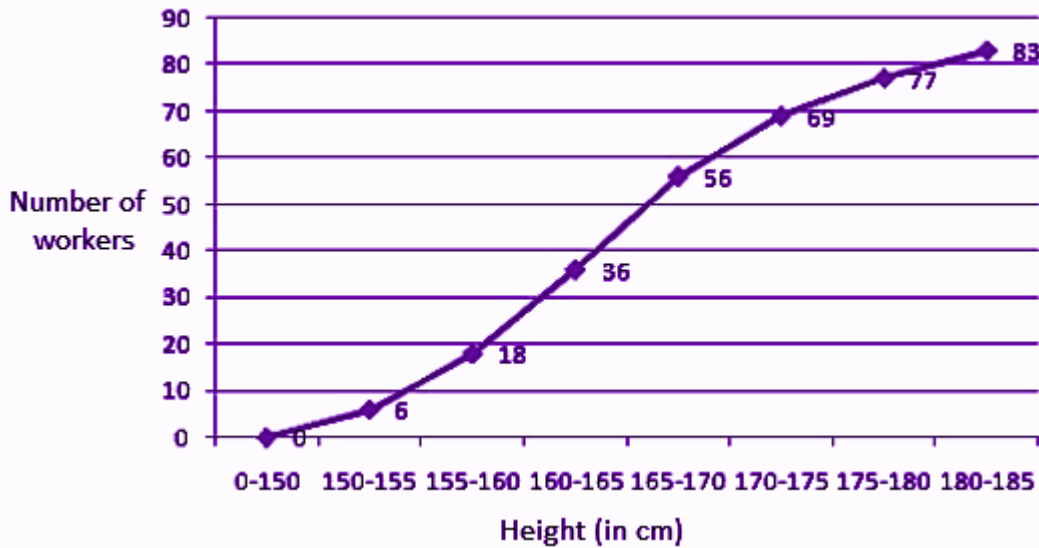
(ii) Draw the cumulative frequency curve on a graph paper. [6]

Solution:

(i) Cumulative frequency table:

Height (in cm)	No. of workers (f)	Cumulative frequency (cf)
150 - 155	6	6
155 - 160	12	18
160 - 165	18	36
165 - 170	20	56
170 - 175	13	69
175 - 180	8	77
180 - 185	6	83

(ii) Plot the points (155, 6), (160, 18), (165, 36), (170, 56), (175, 69), (180, 77) and (185, 83) on the graph and then join them with a free hand to obtain a cumulative frequency curve.



Question 11

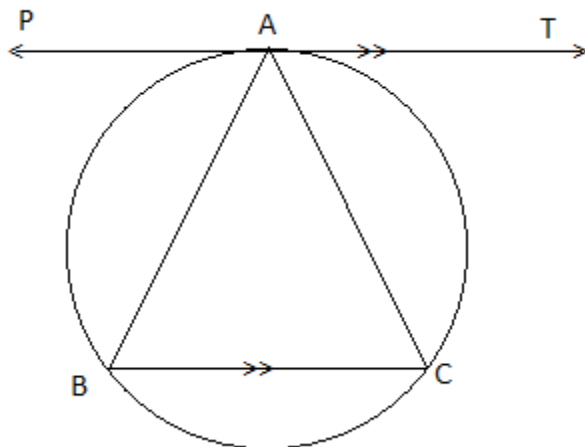
(a) PAT is a tangent to the circumcircle of ΔABC such that $PT \parallel BC$. Show that $AB = AC$.

[3]

Solution:

Given,

PAT is a tangent to the circumcircle of ΔABC such that $PT \parallel BC$.



$\angle PAC = \angle ABC$(i) (angles in the alternate segment)

$\angle PAC = \angle ACB$(ii) (alternate interior angles)

From (i) and (ii),

$\angle ABC = \angle ACB$

We know that the sides opposite to equal angles are equal.

$AB = AC$

(b) A die is thrown once. What is the probability that the number obtained is:

(i) even?

(ii) other than 4?

[3]

Solution:

Given,

A die is thrown once.

Sample space = $S = \{1, 2, 3, 4, 5, 6\}$

$$n(S) = 6$$

(i) Let A be the event of getting an even number.

$$A = \{2, 4, 6\}$$

$$n(A) = 3$$

$$P(A) = n(A)/n(S)$$

$$= 3/6$$

$$= 1/2$$

(ii) Let B be the event of getting a number other than 4.

$$B = \{1, 2, 3, 5, 6\}$$

$$n(B) = 5$$

$$P(B) = n(B)/n(S)$$

$$= 5/6$$

(c) Find the equation of the line passing through $(-2, -4)$ and perpendicular to the line $3x - y + 5 = 0$. [4]

Solution:

Given,

$$3x - y + 5 = 0 \dots (i)$$

$$y = 3x + 5$$

Gradient of the line = 3

The slope of the which is perpendicular to (i) is $m = -1/3$

The equation of line passing through $(-2, -4)$ and perpendicular to (i) is:

$$y - (-4) = m[x - (-2)]$$

$$y + 4 = (-1/3)(x + 2)$$

$$3(y + 4) = -x - 2$$

$$3y + 12 + x + 2 = 0$$

$$x + 3y + 14 = 0$$