

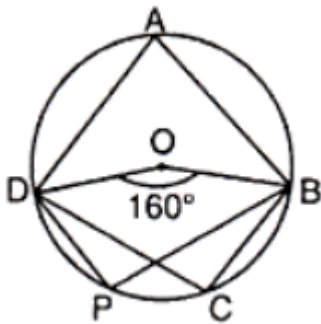
ICSE Class 10 Maths Mock Sample Paper 3 with Solutions

SECTION A

Attempt all questions from this section.

Question 1

(a) In the figure, ABCD is a cyclic quadrilateral, O is the centre of the circle. If $\angle BOD = 160^\circ$, find $\angle BPD$ and $\angle BCD$. [3]



Solution:

Given,

ABCD is a cyclic quadrilateral.

$$\angle BOD = 160^\circ$$

We know that,

$$\angle BAD = \left(\frac{1}{2}\right) \angle BOD$$

$$= \left(\frac{1}{2}\right) \times 160^\circ$$

$$= 80^\circ$$

ABPD is a cyclic quadrilateral. (from the given figure)

The sum of the opposite angles of a cyclic quadrilateral is supplementary.

$$\angle BAD + \angle BPD = 180^\circ$$

$$80^\circ + \angle BPD = 180^\circ$$

$$\angle BPD = 180^\circ - 80^\circ$$

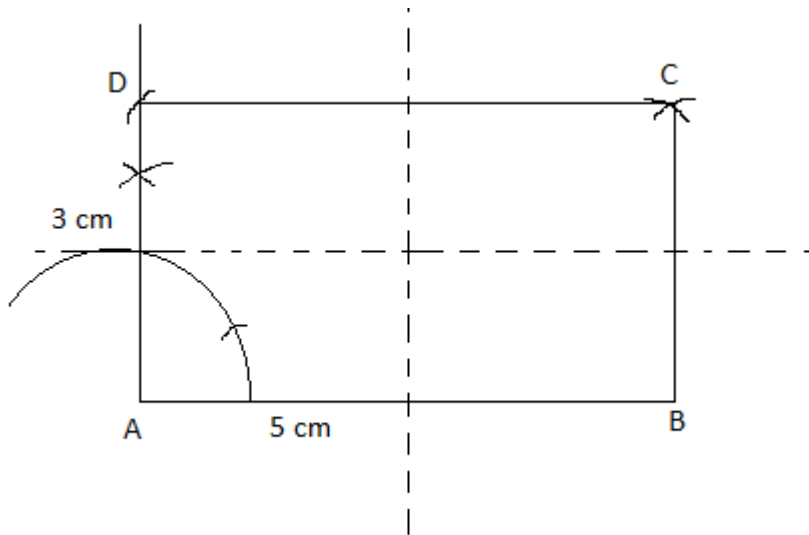
$$\angle BPD = 100^\circ$$

Also, angles in the same segment are equal.

$$\text{Therefore, } \angle BPD = \angle BCD = 100^\circ$$

(b) Using a ruler and a pair of compasses, construct a rectangle ABCD in which $AB = 5$ cm and $AD = 3$ cm. Construct its line of symmetry. [3]

Solution:



(c)

If \bar{x} is the mean of $x_1, x_2, x_3, \dots, x_n$, then prove that the mean of $x_1 + a, x_2 + a, \dots, x_n + a$ is $\bar{x} + a$.

[4]

Solution:

Given,

$$\text{Mean of } x_1, x_2, x_3, \dots, x_n = \bar{x}$$

Mean = Sum of the observations / Number of observations

$$\bar{x} = (x_1 + x_2 + x_3 + \dots + x_n) / n$$

$$n \cdot \bar{x} = x_1 + x_2 + x_3 + \dots + x_n \dots (i)$$

Now, the observations are $x_1 + a, x_2 + a, x_3 + a, \dots, x_n + a$

$$\text{Mean} = (x_1 + a + x_2 + a + x_3 + a + \dots + x_n + a) / n$$

$$= [(x_1 + x_2 + x_3 + \dots + x_n) + (a + a + a + \dots \text{ n times})] / n$$

$$= [n \cdot \bar{x} + na] / n$$

$$= n(\bar{x} + a) / n$$

$$= \bar{x} + a$$

Hence proved.

Question 2

(a) Without actual division, find the remainder obtained on dividing $(3x^2 + 5x - 9)$ by $3x + 2$.

[3]

Solution:

$$\text{Let } p(x) = 3x^2 + 5x - 9$$

Now,

$$3x + 2 = 0$$

$$3x = -2$$

$$x = -2/3$$

Substituting $x = -2/3$ in $p(x)$,

$$\begin{aligned}
 p(-\frac{2}{3}) &= 3(-\frac{2}{3})^2 + 5(-\frac{2}{3}) - 9 \\
 &= 3(\frac{4}{9}) - (\frac{10}{3}) - 9 \\
 &= (\frac{4}{3}) - (\frac{10}{3}) - 9 \\
 &= (4 - 10 - 27)/3 \\
 &= -33/3 \\
 &= -11
 \end{aligned}$$

Therefore, the remainder is -11.

(b)

Given $P = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -3 & 2 \\ 1 & 4 \end{bmatrix}$, calculate $PQ + Q^2$.

3] [

Solution:

Given,

$$\begin{aligned}
 P &= \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}, Q = \begin{bmatrix} -3 & 2 \\ 1 & 4 \end{bmatrix} \\
 PQ &= \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 1 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 9+1 & -6+4 \\ -6+0 & 4+0 \end{bmatrix} \\
 &= \begin{bmatrix} 10 & -2 \\ -6 & 4 \end{bmatrix} \\
 Q^2 &= \begin{bmatrix} -3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ 1 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 9+2 & -6+8 \\ -3+4 & 2+4 \end{bmatrix} \\
 &= \begin{bmatrix} 11 & 2 \\ 1 & 6 \end{bmatrix} \\
 PQ + Q^2 &= \begin{bmatrix} 10 & -2 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 11 & 2 \\ 1 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} 21 & 0 \\ -5 & 10 \end{bmatrix}
 \end{aligned}$$

(c) Find the range of values of x, which satisfy $-\frac{1}{3} < x/2 - 1 \frac{1}{3} < \frac{1}{6}$.

[4]

Solution:

$$\begin{aligned}
 -\frac{1}{3} < \frac{x}{2} - 1 \quad \frac{1}{3} < \frac{1}{6} \\
 -\frac{1}{3} < \frac{x}{2} - \frac{4}{3} < \frac{1}{6} \\
 (-\frac{1}{3}) + (\frac{4}{3}) < \frac{x}{2} < (\frac{1}{6}) + (\frac{4}{3}) \\
 \frac{3}{3} < \frac{x}{2} < \frac{9}{6} \\
 1 < \frac{x}{2} < \frac{3}{2} \\
 2(1) < x < 2(\frac{3}{2}) \\
 2 < x < 3
 \end{aligned}$$

Therefore, the range of values of x is $2 < x < 3$.

Question 3

(a) If $(3a + 5b) : (3a - 5b) = (3c + 5d) : (3c - 5d)$, show that $a : b = c : d$ [3]

Solution:

Given,

$$\begin{aligned}
 (3a + 5b) : (3a - 5b) &= (3c + 5d) : (3c - 5d) \\
 \text{i.e } (3a + 5b) / (3a - 5b) &= (3c + 5d) / (3c - 5d)
 \end{aligned}$$

Using componendo and dividendo rule,

$$\begin{aligned}
 (3a + 5b + 3a - 5b) / (3a + 5b - 3a + 5b) &= (3c + 5d + 3c - 5d) / (3c + 5d - 3c + 5d) \\
 6a / 10b &= 6c / 10d \\
 \Rightarrow a/b &= c/d
 \end{aligned}$$

Therefore, $a : b = c : d$

(b) A spherical ball of iron has been melted and recast into smaller ones. If the radius of each of the smaller balls is one-fourth of the original one, how many such balls can be made? How does the total surface area of the smaller balls compare with that of the original one? [3]

Solution:

Let R be the radius of the original ball and r be the radius of smaller spherical balls.

Given,

$$r = R/4$$

$$\Rightarrow R = 4r$$

$$\begin{aligned}
 \text{The volume of original ball} &= (4/3)\pi R^3 \\
 &= (4/3)\pi(4r)^3
 \end{aligned}$$

$$\text{The volume of smaller ball} = (4/3)\pi r^3$$

Number of balls = Volume of original ball / Volume of one small ball

$$\begin{aligned}
 &= [(4/3)\pi(4r)^3] / [(4/3)\pi r^3] \\
 &= 64r^3 / r^3 \\
 &= 64
 \end{aligned}$$

Now, S_1 be the surface area of the original spherical ball and S_2 be the volume of 64 smaller balls.

$$\begin{aligned}
 S_1/S_2 &= (4\pi R^2) / (64 \times 4\pi r^2) \\
 &= [4\pi(4r)^2] / [64 \times 4\pi r^2] \\
 &= 16/64
 \end{aligned}$$

$$S_1/S_2 = 1/4$$

$$\Rightarrow S_2 = 4S_1$$

Therefore, the surface area of smaller balls is four times the surface area of the original ball.

(c) Entries in a savings account passbook are as follows:

Date	Particulars	Withdrawn (Rs)	Deposited (Rs)	Balance (Rs)
4-2-2003	B/F	–	–	2150
10-2-2003	By cash	–	350	2500
1-5-2003	To cheque	400	–	2100
21-5-2003	By cash	–	400	2500
2-7-2003	To cheque	1500	–	1000

Calculate the interest for the six months (February to July) at 4.5% p.a. on minimum balance on or after the 10th day of each month. [4]

Solution:

Month	Minimum balance on or after 10th day (i.e. between 10th and the last day of the month)
Feb	2500
March	2500
Apr	2500
May	2100
June	2500
July	1000

Total = Rs. 13100

Rate of interest = $R = 4.5\%$

Time = 6 months

$I = \frac{PTR}{100}$

$= \frac{(13100 \times 4.5 \times 6)}{(12 \times 100)}$

$= 294.75$

Hence, the interest for six months is Rs. 294.75.

Question 4

(a) Solve for θ : $2 \cos 3\theta = 1$, $0^\circ < \theta < 90^\circ$. [3]

Solution:

Given,

$$2 \cos 3\theta = 1$$

$$\cos 3\theta = \frac{1}{2}$$

$$\cos 3\theta = \cos \frac{\pi}{3} \text{ or } \cos 3\theta = \cos \left(\frac{5\pi}{3}\right)$$

$$\cos 3\theta = \cos 60^\circ \text{ or } \cos 3\theta = \cos 300^\circ$$

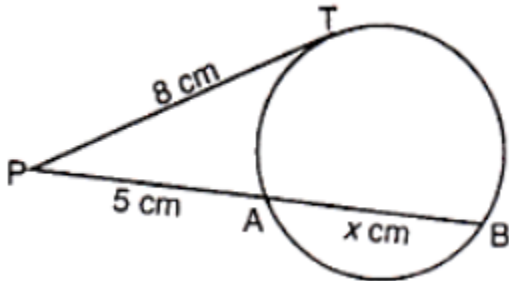
$$3\theta = 60^\circ \text{ or } 3\theta = 300^\circ$$

$$\theta = 20^\circ \text{ or } \theta = 100^\circ$$

$\theta = 100^\circ$ is not possible since $0^\circ < \theta < 90^\circ$.

Therefore, $\theta = 20^\circ$

(b) Find the unknown length x in the figure. [3]



Solution:

From the given,

PT = tangent

PAB = secant

We know that,

$$PT^2 = PA \times PB$$

$$(8)^2 = 5 \times (PA + AB)$$

$$64/5 = (5 + x)$$

$$x = (64/5) - 5$$

$$x = (64 - 25)/5$$

$$x = 39/5$$

$$x = 7.8 \text{ cm}$$

(c) An article is available for Rs. 6048 inclusive of sales tax at the rate of 8%. Find its list price. What will be its new selling price if the rate of sales tax changes to 12%? [4]

Solution:

Given,

Price of an article inclusive of sales tax = Rs. 6048

Sales tax = 8%

List price = $(100/108) \times \text{Rs. } 6048$

= Rs. 5600

New sales tax = 12%

Selling price = $(112/100) \times \text{Rs. } 5600$

= Rs. 6272

Therefore, the new selling price of the article is Rs. 6272.

SECTION - B

Attempt any four questions from this section.

Question 5

(a) A part of Rs. 3020 is invested in 6% Rs. 100 shares at Rs. 97 and the rest in 12% Rs. 100 shares at Rs. 108. If both bring the same dividend, find:

- the sum invested in the shares selling at a discount
- the sum invested in the shares selling above par

(iii) total dividend.

[3]

Solution:

Let x be the number of shares invested at 6% and y be the number of shares invested at 12%.

According to the given,

$$97x + 108y = 3020 \dots (i)$$

Also, the dividend is the same in both cases.

$$6x = 12y$$

$$x = 2y \dots (ii)$$

From (i) and (ii),

$$97(2y) + 108y = 3020$$

$$194y + 108y = 3020$$

$$302y = 3020$$

$$y = 3020 / 302$$

$$y = 10$$

Therefore, $x = 20$

$$(i) \text{ Amount invested in 6\%} = 97x = 97 \times 20 = \text{Rs. } 1940$$

Thus, the sum invested in the shares selling at a discount is Rs. 1940.

$$(ii) \text{ Amount invested in 12\%} = 108y = 108 \times 10 = \text{Rs. } 1080$$

Thus, the sum invested in the shares selling above par is Rs. 1080.

$$(iii) \text{ Total dividend} = 6x + 12y$$

$$= 6(20) + 12(10)$$

$$= 120 + 120$$

$$= \text{Rs. } 240$$

(b)

Find a matrix X such that $X + \begin{bmatrix} 4 & 6 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -5 & -2 \end{bmatrix}$

[3]

Solution:

Given,

$$X + \begin{bmatrix} 4 & 6 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -5 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & 1 \\ -5 & -2 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -3 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 - 4 & 1 - 6 \\ -5 + 3 & -2 - 7 \end{bmatrix}$$

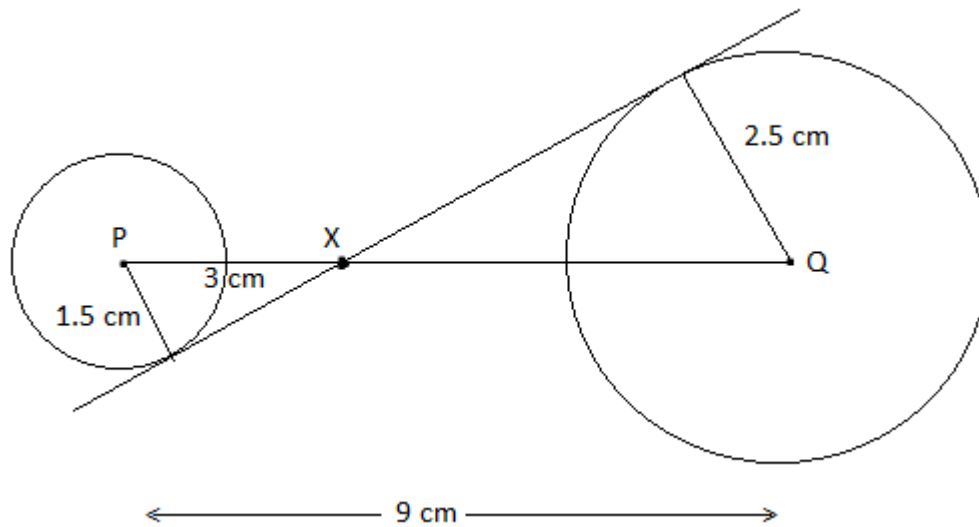
$$= \begin{bmatrix} -1 & -5 \\ -2 & -9 \end{bmatrix}$$

(c) Take points P and Q at a distance of 9 cm from each other. At P , draw a circle of radius 1.5 cm and at Q , draw

a circle of radius 2.5 cm.

Locate point X in PQ such that $PX = 3$ cm. Through X, draw a tangent to the circle with centre P and a tangent to the circle with centre Q. Use ruler and compasses only. [4]

Solution:



Question 6

(a) Write down the coordinates of the image of the point (4, -5) when:

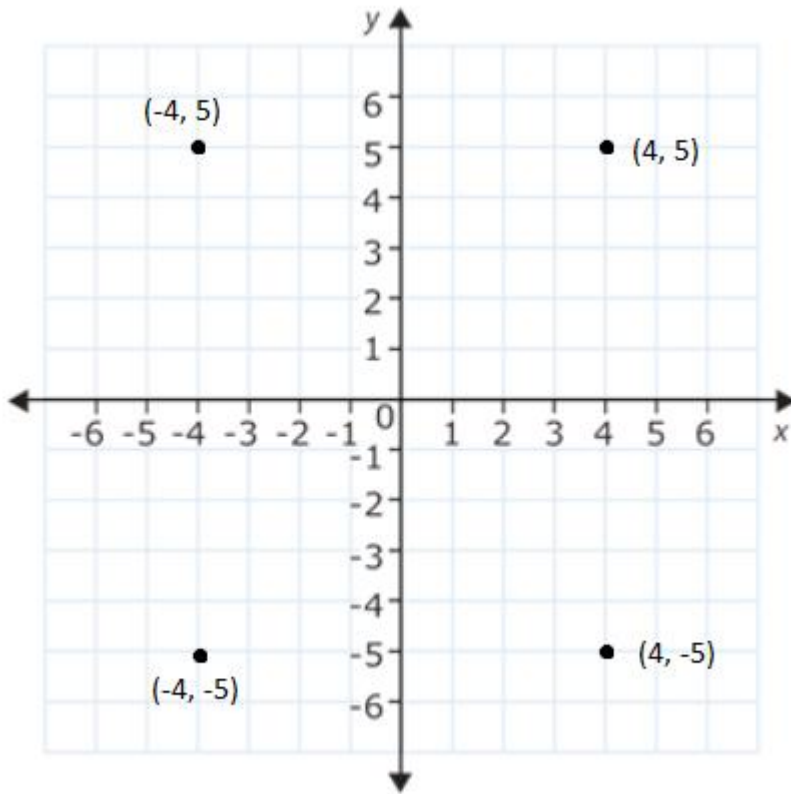
- (i) reflected in x-axis
- (ii) reflected in y-axis
- (iii) reflected in x-axis followed by a reflection in the y-axis
- (iv) reflected in the origin.

[3]

Solution:

The given point is (4, -5).

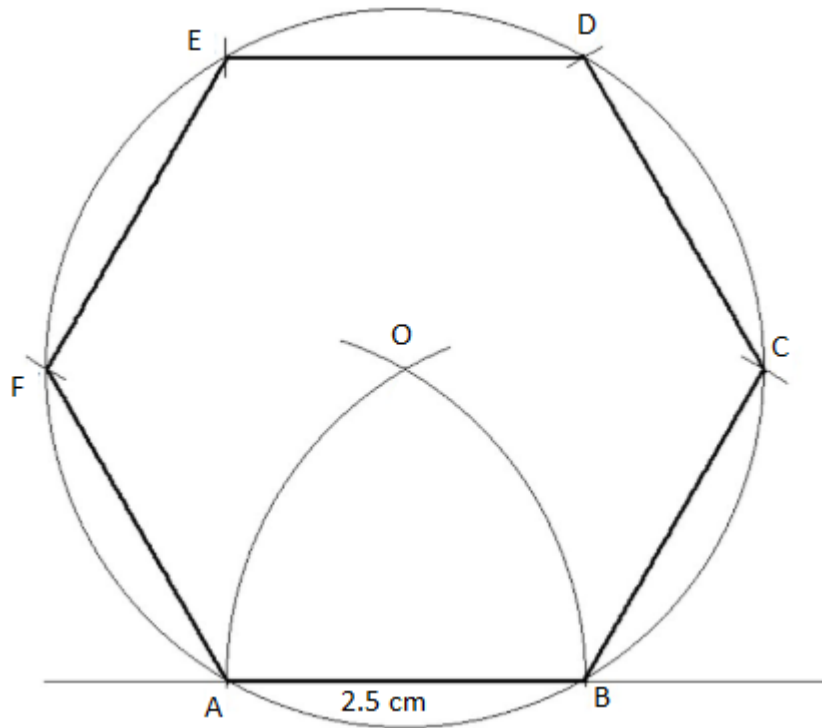
- (i) Coordinates of the images of (4, -5) when reflected in the x-axis = (4, 5)
- (ii) Coordinates of the images of (4, -5) when reflected in the y-axis = (-4, -5)
- (iii) Coordinates of the images of (4, -5) when reflected in x-axis followed by a reflection in the y-axis = (-4, 5)
- (iv) Coordinates of the image of (4, -5) when reflected in the origin = (-4, 5)



(b) Draw a regular hexagon of side 2.5 cm. Circumscribe a circle to it.

[3]

Solution:



(c) One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to the mountain and the remaining 15 camels were on the bank of the river. Find the total number of camels. [4]

Solution:

Let x be the total numbers of camels.

Number of camels were seen in the forest = $x/4$

Number of camels gone to mountains = $2\sqrt{x}$

Number of camels on the bank of river (i.e. remaining camels) = 15

Total number of camels = $x/4 + 2\sqrt{x} + 15$

$$x = (x + 8\sqrt{x} + 60)/4$$

$$\Rightarrow x + 8\sqrt{x} + 60 = 4x$$

$$\Rightarrow 3x - 8\sqrt{x} - 60 = 0$$

$$\Rightarrow 3x - 60 = 8\sqrt{x}$$

Squaring on both sides,

$$(3x - 60)^2 = (8\sqrt{x})^2$$

$$9x^2 + 3600 - 360x = 64x$$

$$9x^2 - 424x + 3600 = 0$$

$$9x^2 - 324x - 100x + 3600 = 0$$

$$9x(x - 36) - 100(x - 36) = 0$$

$$(9x - 100)(x - 36) = 0$$

$$9x - 100 = 0, x - 36 = 0$$

$$x = 100/9, x = 36$$

x cannot be a fraction.

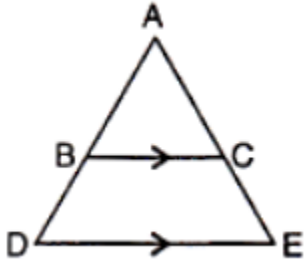
Therefore, the total number of camels is 36.

Question 7

(a) In the figure, BC is parallel to DE. Area of $\Delta ABC = 25 \text{ cm}^2$, area of trapezium BCED = 24 cm^2 , DE = 14 cm.

Calculate the length of BC.

[3]



Solution:

Given,

$$\text{Area of triangle ABC} = 25 \text{ cm}^2$$

$$\text{Area of trapezium BCED} = 24 \text{ cm}^2$$

$$\text{Area of triangle ADE} = \text{Area of triangle ABC} + \text{Area of trapezium}$$

$$= 25 + 24$$

$$= 49 \text{ cm}^2$$

In $\triangle ABC$ and $\triangle ADE$,

$BC \parallel DE$,

$$\angle ABC = \angle ADE \text{ (corresponding angles)}$$

$$\angle ACB = \angle AED \text{ (corresponding angles)}$$

$$\angle BAC = \angle DAE \text{ (common)}$$

By AAA similarity.

$$\triangle ABC \sim \triangle ADE$$

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle ADE)} = \frac{BC^2}{DE^2}$$

$$\frac{25}{49} = \frac{BC^2}{(14)^2}$$

$$BC^2 = \frac{(25 \times 196)}{49}$$

$$BC^2 = \frac{4900}{49}$$

$$BC^2 = 100$$

$$\Rightarrow BC = 10 \text{ cm}$$

(b) David received Rs. 7875 as the maturity amount of a monthly recurring deposit of 2 years at 9% p.a. Find the monthly installment. [3]

Solution:

Given,

$$\text{Maturity amount} = \text{Rs. } 7875$$

$$\text{Rate of interest} = r = 9\%$$

$$\text{Time} = n = 2 \text{ years} = 24 \text{ months}$$

$$\text{Monthly installment} = P \text{ (say)}$$

$$\text{Maturity value} = (P \times n) + P \times \left[\frac{n(n+1)}{2 \times 12} \right] \times \left(\frac{r}{100} \right)$$

$$7875 = (P \times 24) + P \times \left[\frac{(24 \times 25)}{(2 \times 12)} \right] \times \left(\frac{9}{100} \right)$$

$$7875 = 24P + P\left(\frac{9}{4}\right)$$

$$7875 \times 4 = 96P + 9P$$

$$105P = 7875 \times 4$$

$$P = \frac{(7875 \times 4)}{105}$$

$$P = 300$$

Hence, the monthly installment is Rs. 300.

(c) If the roots of the equation $l(m - n)x^2 + m(n - 1)x + n(1 - m) = 0$ are equal, show that $m = 2ln / (l + n)$. [4]

Solution:

Given,

$$l(m - n)x^2 + m(n - 1)x + n(1 - m) = 0$$

Comparing with the standard form $ax^2 + bx + c = 0$,

$$a = l(m - n), b = m(n - 1), c = n(1 - m)$$

Also, given that the roots are equal.

Therefore, the discriminant is 0.

$$b^2 - 4ac = 0$$

$$[m(n - 1)]^2 - 4 [l(m - n)] [n(1 - m)] = 0$$

$$m^2(n - 1)^2 - 4nl(m - n)(1 - m) = 0$$

$$m^2(n^2 + 1^2 - 2nl) - 4nl(ml - m^2 - nl + nm) = 0$$

$$m^2n^2 + m^2l^2 - 2m^2nl - 4mnl^2 + 4m^2nl + 4n^2l^2 - 4mn^2l = 0$$

$$m^2n^2 + m^2l^2 + 2m^2nl - 4mnl^2 + 4n^2l^2 - 4mn^2l = 0$$

$$(n^2 + l^2 + 2nl)m^2 + (-4nl^2 - 4n^2l)m + 4n^2l^2 = 0$$

$$(n + l)^2m^2 + (-4nl^2 - 4n^2l)m + 4n^2l^2 = 0$$

This is a quadratic equation in m .

Using quadratic formula,

$$m = \frac{4ln(l + n) \pm \sqrt{(4ln(l + n))^2 - 4((l + n)^2)(4l^2n^2)}}{2(l + n)^2}$$

$$= \frac{4ln(l + n) \pm \sqrt{(4ln(l + n))^2 - (4ln(l + n))^2}}{2(l + n)^2}$$

$$= \frac{4ln(l + n) \pm 0}{2(l + n)^2}$$

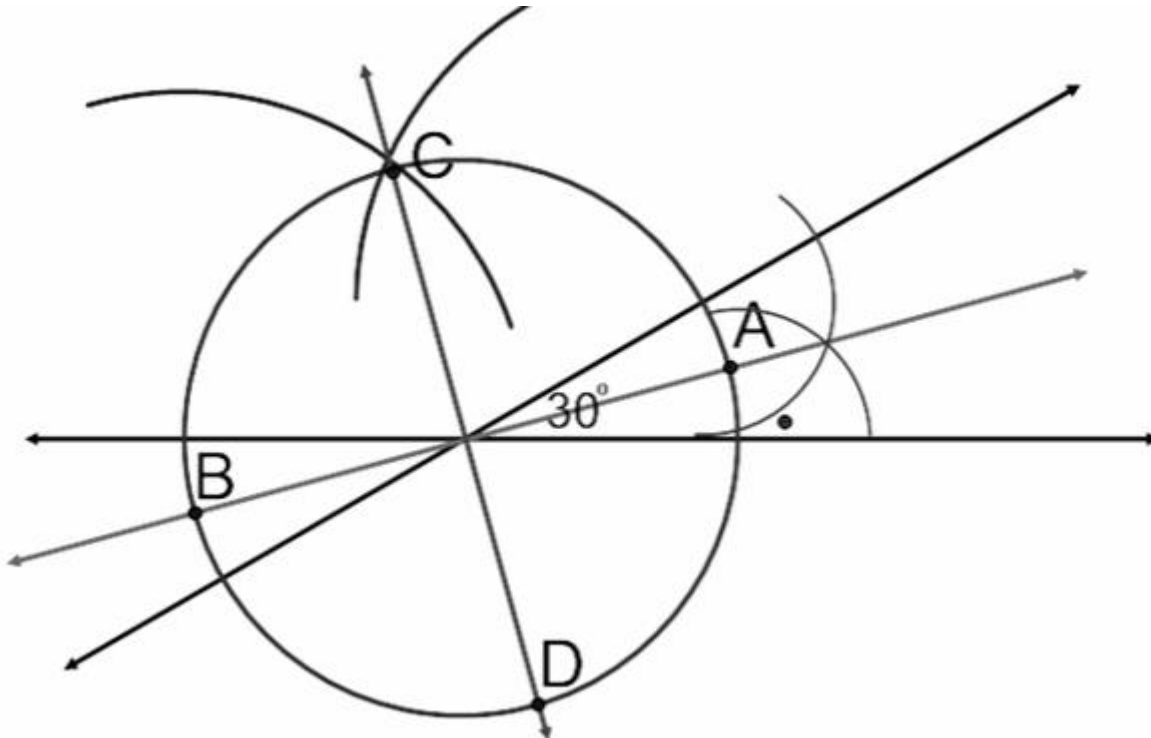
$$= \frac{4ln(l + n)}{2(l + n)^2}$$

$$= \frac{2ln}{l + n}$$

Question 8

(a) Draw two intersecting lines to include an angle of 30° . Locate points which are equidistant from these lines and also 2 cm away from their point of intersection. How many such points exist? [3]

Solution:



Hence, there exist 4 points that are equidistant from these lines and also 2 cm away from their point of intersection.

(b) The value of a building decreases every year at a rate of 5%. If its value at the end of 3 years was Rs. 411540, what was its original value at the beginning of these three years? [3]

Solution:

Rate of depreciation = 5%

Value of the building at the end of 3 years = Rs. 411540

$$411540 = P[1 - (5/100)]^3$$

$$411540 = P[(100 - 5)/100]^3$$

$$411540 = P(95/100)^3$$

$$411540 = P(19/20)^3$$

$$P = (411540 \times 20 \times 20 \times 20) / (19 \times 19 \times 19)$$

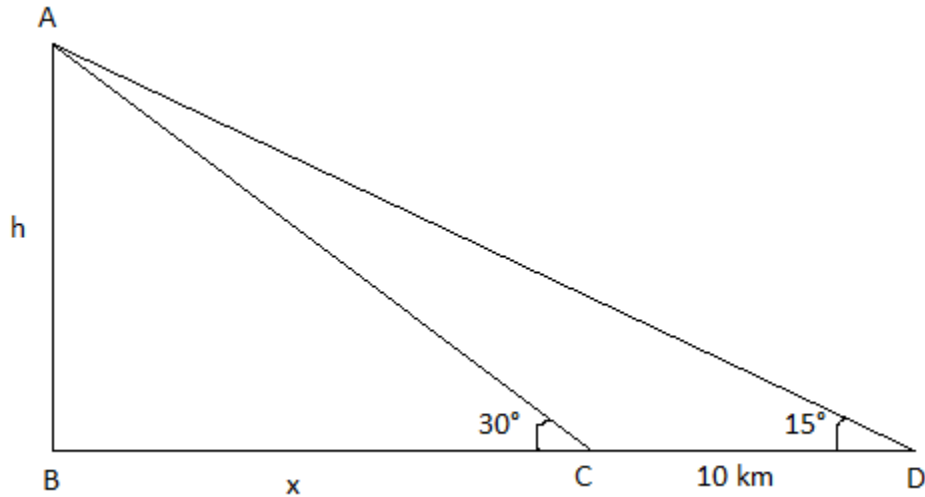
$$P = 480000$$

Hence, the original value of the building is Rs. 480000.

(c) Find the height of a mountain, if the elevation of its top at an unknown distance from the base is 30° and at a distance, 10 km further off from the mountain, along the same line, the angle of elevation is 15° . [4]

Solution:

Let AB be the height of the mountain.



CD = 10 km

AB = h and BC = x

In right triangle ABC,

$$\tan 30^\circ = AB/BC$$

$$1/\sqrt{3} = h/x$$

$$x = h\sqrt{3} \dots (i)$$

In right triangle ABD,

$$\tan 15^\circ = AB/BD$$

$$(\sqrt{3} - 1)/(\sqrt{3} + 1) = h/(x + 10)$$

$$(x + 10)(\sqrt{3} - 1) = h(\sqrt{3} + 1)$$

$$(h\sqrt{3} + 10)(\sqrt{3} - 1) = h\sqrt{3} + h$$

$$h(\sqrt{3})(\sqrt{3}) - h\sqrt{3} + 10\sqrt{3} - 10 = h\sqrt{3} + h$$

$$3h - h\sqrt{3} + 10\sqrt{3} - 10 = h\sqrt{3} + h$$

$$h\sqrt{3} + h - 3h + h\sqrt{3} - 10\sqrt{3} + 10 = 0$$

$$2h\sqrt{3} - 2h - 10(\sqrt{3} - 1) = 0$$

$$2h(\sqrt{3} - 1) = 10(\sqrt{3} - 1)$$

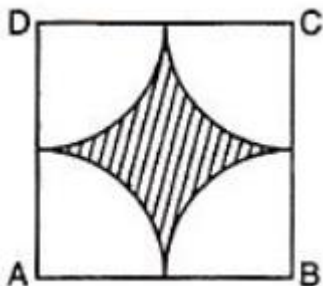
$$2h = 10$$

$$h = 5$$

Hence, the height of the mountain is 5 km.

Question 9

(a) In the figure, ABCD is a square of side 14 cm, and A, B, C, D are corners of circular arcs, each of radius 7 cm. Find the area of the shaded region. [3]



Solution:

Given,

Side of square ABCD = 14 cm

Radius of quadrants at A, B, C, and D = $r = 7$ cm

Area of the shaded region = Area of square - Area of 4 quadrants at A, B, C, and D

$$= (\text{side})^2 - 4 \times \left(\frac{1}{4}\right) \pi r^2$$

$$= (14)^2 - \left(\frac{22}{7}\right) \times 7 \times 7$$

$$= 196 - 154$$

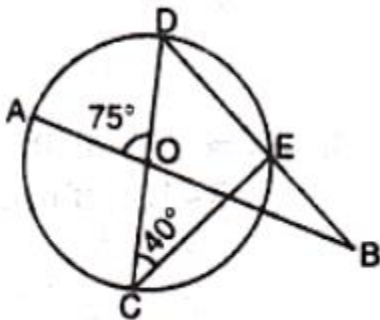
$$= 42 \text{ cm}^2$$

(b) In the figure, the straight lines AB and CD pass through the centre O of the circle. If $\angle AOD = 75^\circ$ and $\angle OCE = 40^\circ$, find:

(i) $\angle CDE$

(ii) $\angle OBE$.

[3]



Solution:

Given,

$\angle AOD = 75^\circ$ and $\angle OCE = 40^\circ$

(i) CD is the diameter of the circle.

We know that the angle in a semicircle is a right angle.

i.e. $\angle CED = 90^\circ$

In triangle CDE,

$$\angle CDE + \angle DCE + \angle CED = 180^\circ$$

$$\angle CDE + 40^\circ + 90^\circ = 180^\circ \text{ (from the figure, } \angle OCE = \angle DCE \text{)}$$

$$\angle CDE = 180^\circ - 40^\circ - 90^\circ$$

$$\angle CDE = 50^\circ$$

(ii) $\angle AOD + \angle AOC = 180^\circ$ (linear pair)

$$75^\circ + \angle AOC = 180^\circ$$

$$\angle AOC = 180^\circ - 75^\circ = 105^\circ$$

$$\angle AOC = \angle BOD = 105^\circ \text{ (vertically opposite angles)}$$

Now, in triangle BOD,

$$\angle BOD + \angle OBD + \angle ODE = 180^\circ$$

$$105^\circ + \angle OBD + 50^\circ = 180^\circ \text{ (} \angle ODE = \angle CDE \text{)}$$

$$\angle OBD = 180^\circ - 105^\circ - 50^\circ$$

$$\angle OBD = 25^\circ$$

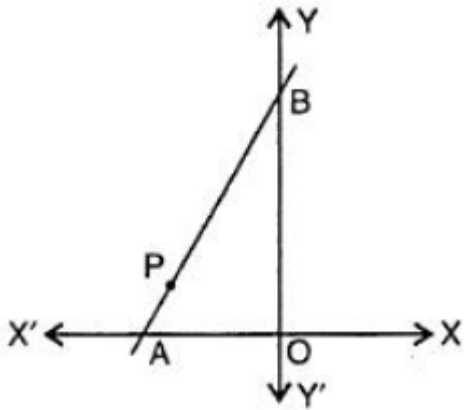
We know that,

$$\angle OBD = \angle OBE$$

Therefore, $\angle OBE = 25^\circ$

(c) In the figure, line APB meets the x-axis at A, y-axis at B. P is the point (-4, 2) and $AP : PB = 1 : 2$. Write down the coordinates of A and B.

[4]



Solution:

Given,

P is the point $(-4, 2)$ and $AP : PB = 1 : 2$.

Let $(x, 0)$ and $(0, y)$ be the coordinates of A and B, respectively.

Using section formula,

$$-4 = (1 \times 0 + 2 \times x) / (1 + 2)$$

$$-4 = 2x/3$$

$$2x = -12$$

$$x = -6$$

And

$$2 = (1 \times y + 2 \times 0) / (1 + 2)$$

$$2 = y/3$$

$$y = 6$$

Hence, the coordinates of A and B are $(-6, 0)$ and $(0, 6)$, respectively.

Question 10

(a) Show that the points $A(1, 1)$, $B(-1, -1)$ and $C(-\sqrt{3}, \sqrt{3})$ are the vertices of an equilateral triangle. [4]

Solution:

Given,

$A(1, 1)$, $B(-1, -1)$ and $C(-\sqrt{3}, \sqrt{3})$

Using distance formula,

$$AB = \sqrt{[(-1 - 1)^2 + (-1 - 1)^2]}$$

$$= \sqrt{(4 + 4)}$$

$$= \sqrt{8}$$

$$BC = \sqrt{[(-\sqrt{3} + 1)^2 + (\sqrt{3} + 1)^2]}$$

$$= \sqrt{(3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3})}$$

$$= \sqrt{(4 + 4)}$$

$$= \sqrt{8}$$

$$CA = \sqrt{(1 + \sqrt{3})^2 + (1 - \sqrt{3})^2]}$$

$$= \sqrt{(1 + 3 + 2\sqrt{3} + 1 + 3 - 2\sqrt{3})}$$

$$= \sqrt{(4 + 4)}$$

$$= \sqrt{8}$$

$$AB = BC = CA$$

Therefore, the given points are the vertices of an equilateral triangle.

(b) Attempt this question on a graph paper.

Marks less than	10	20	30	40	50	60	70	80	90	100
No. of students	5	10	30	60	105	180	270	355	390	400

Use 2 cm = 10 units on both the axes and plot these values and draw a smooth curve through the points. From the graph estimate:

- (i) the median marks
- (ii) the quartile marks.

[6]

Solution:

From the given,

$$N = 400$$

Cumulative frequency = Number of students.

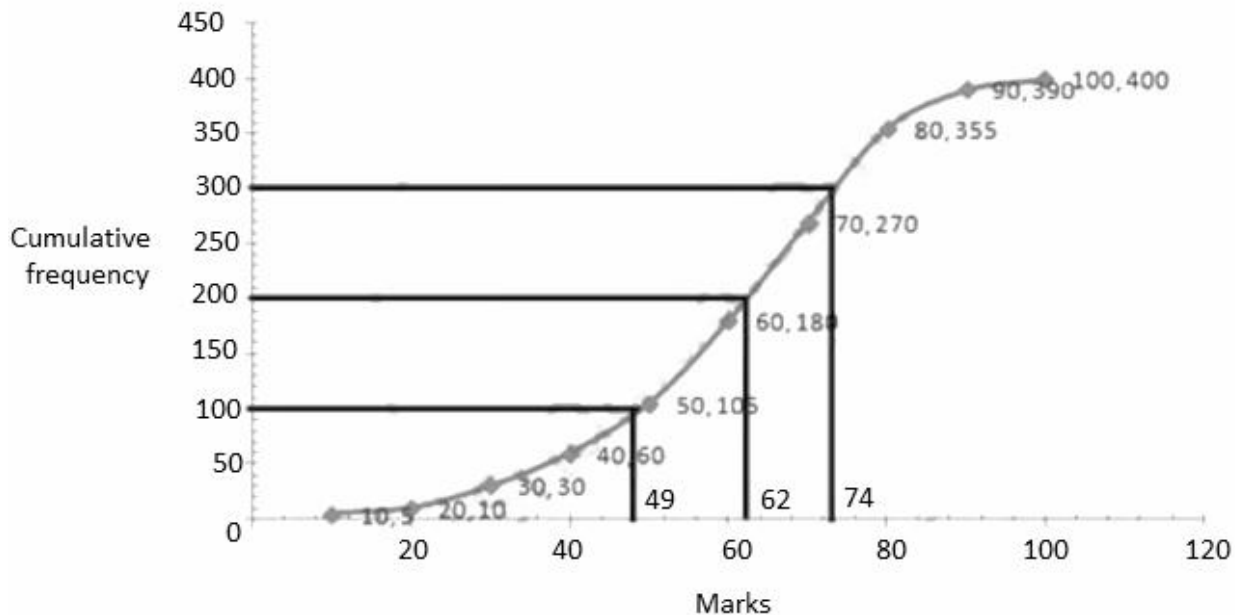
$$(i) N/2 = 400/2 = 200$$

$$200\text{th observation} = 62$$

$$\text{Median} = 62$$

$$(ii) \text{Lower quartile} = Q_1 = (N/4)\text{th observation} = (400/4) = 100\text{th observation} = 49 \text{ marks}$$

$$\text{Upper quartile} = Q_3 = (3N/4)\text{th observation} = (1200/4) = 300\text{th observation} = 74 \text{ marks}$$



Question 11

(a) In a lottery, there are 15 prizes and 85 blanks. Find the probability of getting a prize.

[3]

Solution:

Given,

A lottery has 15 prizes and 85 blanks.

$$\text{Total number of outcomes} = 15 + 85 = 100$$

$$\text{Number of favourable outcomes (prizes)} = 15$$

$$P(\text{getting a prize}) = \text{Number of favourable outcomes} / \text{Total number of outcomes}$$

$$= 15/100$$

$$= 3/20$$

(b) If the mean of the following observations is 54, find the value of p.

Variates	10	30	50	70	90
Frequency	17	8	p	24	19

[3]

Solution:

Given,

Mean = 54

Variates (x)	Frequency (f)	fx
10	17	170
30	8	240
50	p	50p
70	24	1680
90	19	1710
	$\Sigma f = 68 + p$	$\Sigma fx = 3800 + 50p$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$54 = \frac{(3800 + 50p)}{(68 + p)}$$

$$54(68 + p) = 3800 + 50p$$

$$3672 + 54p = 3800 + 50p$$

$$54p - 50p = 3800 - 3672$$

$$4p = 128$$

$$p = \frac{128}{4}$$

$$p = 32$$

(c) Prove that: $\sin A(1 + \tan A) + \cos A(1 + \cot A) = \sec A + \operatorname{cosec} A$

[4]

Solution:

$$\text{LHS} = \sin A(1 + \tan A) + \cos A(1 + \cot A)$$

$$= \sin A[1 + (\sin A / \cos A)] + \cos A[1 + (\cos A / \sin A)]$$

$$= \sin A[(\cos A + \sin A) / \cos A] + \cos A[(\sin A + \cos A) / \sin A]$$

$$= \tan A(\cos A + \sin A) + \cot A(\cos A + \sin A)$$

$$= (\sin A + \cos A)(\tan A + \cot A)$$

$$= (\sin A + \cos A)[(\sin A / \cos A) + (\cos A / \sin A)]$$

$$= (\sin A + \cos A)[(\sin^2 A + \cos^2 A) / (\sin A \cos A)]$$

$$= (\sin A + \cos A)(1) / (\sin A \cos A)$$

$$= (\sin A / \sin A \cos A) + (\cos A / \sin A \cos A)$$

$$= (1/\cos A) + (1/\sin A)$$

$$= \sec A + \operatorname{cosec} A$$

$$= \text{RHS}$$

Hence proved.

