

ICSE Class 10 Maths Mock Sample Paper 5 with Solutions

SECTION A

Attempt all questions from this section.

Question 1

(a) If $2 \tan \theta = 5$, find the value of $(3 \sin \theta - 4 \cos \theta) / (\sin \theta + 4 \cos \theta)$. [3]

Solution:

Given,

$$2 \tan \theta = 5$$

$$\tan \theta = 5/2$$

$$(3 \sin \theta - 4 \cos \theta) / (\sin \theta + 4 \cos \theta)$$

Dividing the numerator and denominator by $\cos \theta$,

$$= (3 \tan \theta - 4) / (\tan \theta + 4)$$

$$= [3(5/2) - 4] / [(5/2) + 4]$$

$$= [(15 - 8) / 2] / [(5 + 8) / 2]$$

$$= 7/13$$

(b) Using factor theorem, show that $(x - 2)$ is a factor of $2x^3 + 5x^2 - 4x - 3$. [3]

Solution:

$$\text{Let } p(x) = 2x^3 + 5x^2 - 4x - 3$$

For checking $(x - 2)$ is a factor of $p(x)$ or not, substitute $x = 2$ in $p(x)$,

$$p(2) = 2(2)^3 + 5(2)^2 - 4(2) - 3$$

$$= 2(8) + 5(4) - 8 - 3$$

$$= 16 + 20 - 11$$

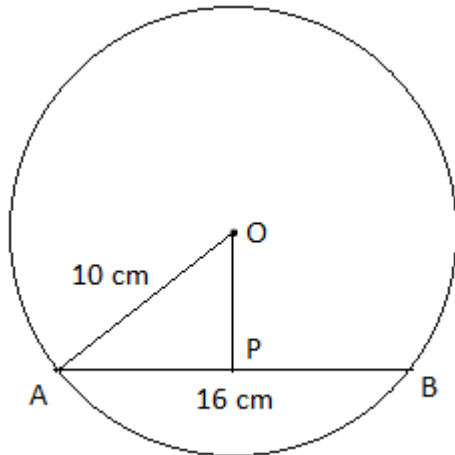
$$= 25 \neq 0$$

Therefore, $(x - 2)$ is not a factor of the given polynomial $2x^3 + 5x^2 - 4x - 3$.

(c) A chord of length 16 cm is drawn in a circle of radius 10 cm. Calculate the distance of the chord from the centre of the circle. [4]

Solution:

Let OA be the radius and AB be the chord of a circle.



$$OA = 10 \text{ cm}$$

$$AB = 16 \text{ cm}$$

We know that the perpendicular drawn from the centre of the circle to the chord bisects the chord.

$$AP = PB = 16/2 = 8 \text{ cm}$$

In right triangle OPA,

By Pythagoras theorem,

$$OA^2 = OP^2 + AP^2$$

$$(10)^2 = OP^2 + (8)^2$$

$$OP^2 = 100 - 64$$

$$OP^2 = 36$$

$$OP = 6 \text{ cm}$$

Therefore, the distance of the chord from the centre of the circle is 6 cm.

Question 2

(a) Given below are the entries in a savings bank account passbook.

Date	Particulars	Withdrawal (Rs)	Deposit (Rs)	Balance (Rs)
Jan 9, 2010	B/F	–	–	4000
Feb 20, 2010	To self	1500	–	2500
April 15, 2010	By cash	–	1200	3700
June 15, 2010	To self	3000	–	700
July 10, 2010	By cash	–	5000	5700

Calculate interest from Jan. to July at 4.5% per annum on minimum balance on or after the 10th day of each month. [3]

Solution:

Month	Minimum balance on or after 10th day (i.e.
-------	--

	between 10th and the last day of the month)
Jan	4000
Feb	4000
March	2500
April	2500
May	3700
June	3700
July	5700

Total = Rs. 26100

Rate of interest = R = 4.5%

Time = 7 months

$I = \frac{PTR}{100}$

$$= \frac{(26100 \times 4.5 \times 7)}{(12 \times 100)}$$

$$= 685.125$$

Hence, the interest for seven months, i.e. from Jan to July is Rs. 685.125.

(b) The circumference of the edge of a hemispherical bowl is 132 cm. Find the capacity of the bowl. [3]

Solution:

Let r be the radius of the hemispherical bowl.

Given,

Circumference = 132 cm

$$2\pi r = 132$$

$$2 \times \frac{22}{7} \times r = 132$$

$$r = \frac{(132 \times 7)}{(2 \times 22)}$$

$$r = 21 \text{ cm}$$

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 44 \times 21 \times 21$$

$$= 19404 \text{ cm}^3$$

Therefore, the capacity of the hemispherical bowl is 19404 cm³.

(c) A cylindrical can whose base is horizontal and of radius 3.5 cm contains sufficient water so that when a sphere is placed in the can, the water just covers the sphere. Given that the sphere just fits into the can, calculate:

(i) the total surface area of the can in contact with water when the sphere is in it.

(ii) the depth of water in the can before the sphere was put into the can.

[4]

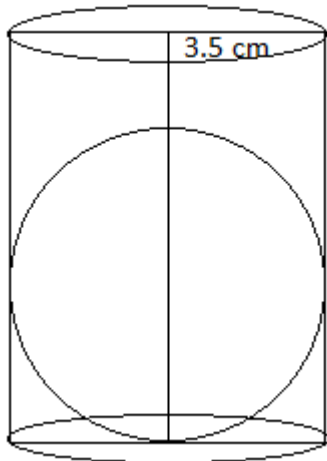
Solution:

Given,

Radius of cylindrical can = r = 3.5 cm

Given that the sphere just fits into the can.

Thus, Radius of sphere = Radius of can = r = 3.5 cm



Volume of cylinder up to the height $2r$ (diameter of sphere) of cylinder = $\pi r^2 (2r)$
 $= 2\pi r^3$

Volume of water in the gap between sphere and cylinder = Volume of cylinder up to height $2r$ - Volume of sphere
 $= 2\pi r^3 - (4/3)\pi r^3$
 $= (2/3)\pi r^3$

(i) Surface area of can which is in contact with the water when sphere is in it = Area of the base + Curved surface area of cylinder up to height $2r$

$$\begin{aligned} &= \pi r^2 + 2\pi r h \\ &= \pi r^2 + 2\pi r(2r) \\ &= 5\pi r^2 \\ &= 5 \times (22/7) \times 3.5 \times 3.5 \\ &= 192.5 \text{ cm}^2 \end{aligned}$$

(ii) Depth of the water in the can before the sphere is put in it = Volume of water / Area of base of the can

$$\begin{aligned} &= [(2/3)\pi r^3] / \pi r^2 \\ &= (2/3)r \\ &= (2/3) \times 3.5 \\ &= 7/3 \text{ cm} \end{aligned}$$

Question 3

(a) The numbers 13, 15, 17, 18, and n are arranged in ascending order. If the mean is equal to the median, find the value of n . [3]

Solution:

Given,

The ascending order of numbers is:

13, 15, 17, 18, n

Mean = Sum of observations / Number of observations

$$= (13 + 15 + 17 + 18 + n) / 5$$

$$= (63 + n) / 5$$

Number of observations = $n = 5$ (odd)

Median = $(n + 1) / 2$ th observation

$$= (5 + 1) / 2$$

$$= 6 / 2$$

3rd observation

$$= 17$$

According to the given,

Mean = Median

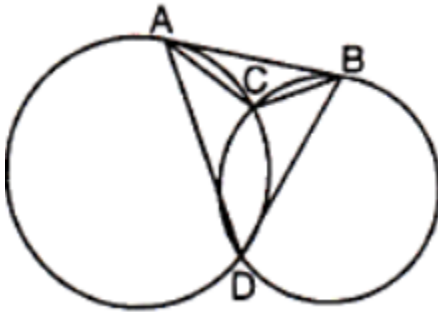
$$(63 + n)/5 = 17$$

$$63 + n = 85$$

$$n = 85 - 63$$

$$n = 22$$

(b) In the figure, AB is a common tangent to two circles intersecting at C and D. Write down the measure of $(\angle ACB + \angle ADB)$. Justify your answer. [3]



Solution:

Given,

AB is a common tangent to two circles intersecting at C and D.

We know that the angle made by the chord and tangent is equal to the angles in the alternate segment.

$$\angle CBA = \angle CDB \dots (i)$$

and

$$\angle CAB = \angle CDA \dots (ii)$$

Adding (i) and (ii),

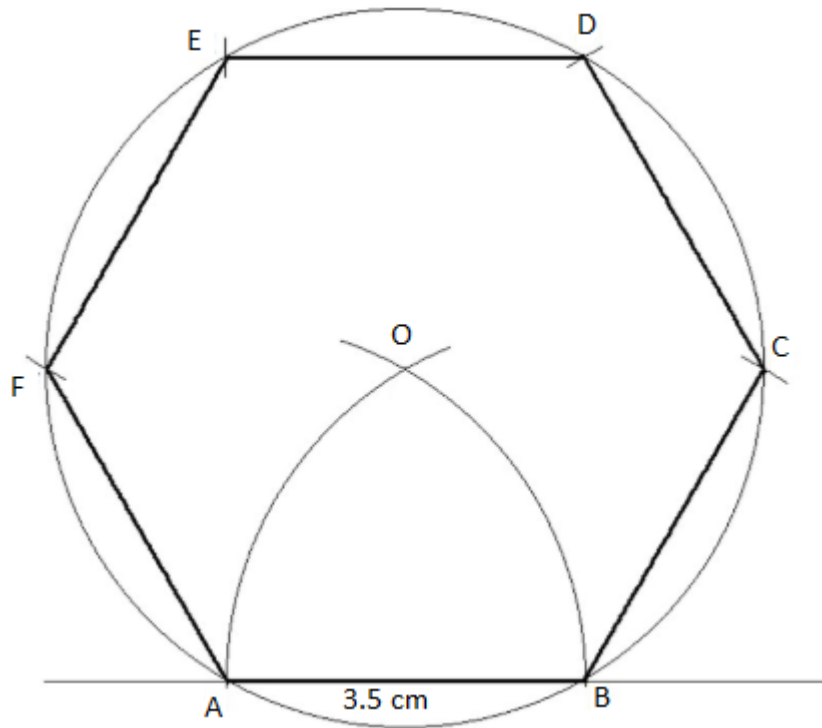
$$\angle CBA + \angle CAB = \angle CDB + \angle CDA$$

$$180^\circ - \angle ACB = \angle ADB \text{ (in triangle ABC, } \angle CBA + \angle CAB + \angle ACB = 180^\circ)$$

$$\angle ADB + \angle ACB = 180^\circ$$

(c) Draw a regular hexagon of side 3.5 cm. Circumscribe a circle to it. [4]

Solution:



Question 4

(a)

Given $\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$

Write down

- (i) the order of the matrix X
- (ii) the matrix X.

[3]

Solution:

- (i) From the given,
 $[2 \times 2] \times (\text{Order of } X) = 2 \times 1$
Thus, the order of X is 2×1 .
- (ii)

Let $X = \begin{bmatrix} a \\ b \end{bmatrix}$

Given,

$$\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 8a - 2b \\ a + 4b \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$$

$$8a - 2b = 12 \dots (i)$$

$$a + 4b = 10$$

$$a = 10 - 4b \dots (ii)$$

Substituting (ii) in (i),

$$8(10 - 4b) - 2b = 12$$

$$80 - 32b - 2b = 12$$

$$34b = 80 - 12$$

$$34b = 68$$

$$b = 68/34$$

$$b = 2$$

Substituting $b = 2$ in (ii),

$$a = 10 - 4(2)$$

$$= 10 - 8$$

$$= 2$$

Therefore, $X = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(b) Find the value of x , which satisfies the inequation $-2 \leq (\frac{1}{2}) - (2x/3) \leq 1 \frac{5}{6}$, $x \in \mathbb{N}$
Graph the solution set on the number line.

[3]

Solution:

$$-2 \leq (\frac{1}{2}) - (2x/3) \leq 1 \frac{5}{6}$$

Now,

$$-2 \leq (\frac{1}{2}) - (2x/3)$$

$$(2x/3) \leq (\frac{1}{2}) + 2$$

$$(2x/3) \leq 5/2$$

$$x \leq (5/2)(3/2)$$

$$x \leq 15/4$$

And

$$(\frac{1}{2}) - (2x/3) \leq 1 \frac{5}{6}$$

$$(\frac{1}{2}) - (2x/3) \leq 11/6$$

$$-(2x/3) \leq (11/6) - (\frac{1}{2})$$

$$-2x/3 \leq (11 - 3)/6$$

$$-2x/3 \leq 8/6$$

$$-2x/3 \leq 4/3$$

$$2x/3 \geq -4/3$$

$$x \geq (-4/3)(3/2)$$

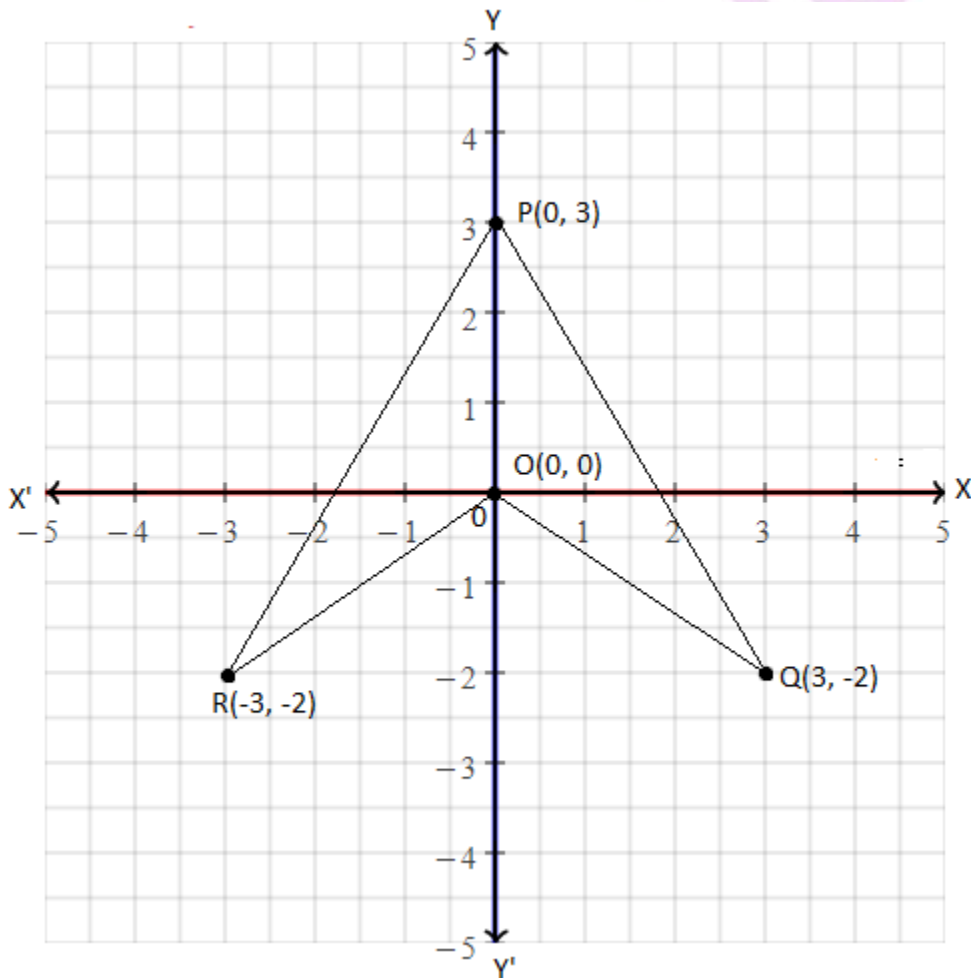
$$x \geq -2$$

Therefore, $-2 \leq x \leq 15/4$
And $1 \leq x \leq 3, x \in \mathbb{N}$



- (c) Use graph paper to solve this equation.
 (i) Plot the points $P(0, 3)$, $Q(3, -2)$ and $O(0, 0)$.
 (ii) Plot R , the image of Q , when reflected in the y -axis and write its coordinates.
 (iii) What is the geometrical name of the figure $PQOR$? Also, write the equation of the line of symmetry of $PQOR$? [4]

Solution:
Given,



- (ii) Coordinates of R = (-3, -2)
 - (iii) Geometrical name of PQOR is an arrowhead.
 - (iv) The y-axis is the line of symmetry of PQOR.
- Therefore, the equation of the line of symmetry is $x = 0$.

SECTION B

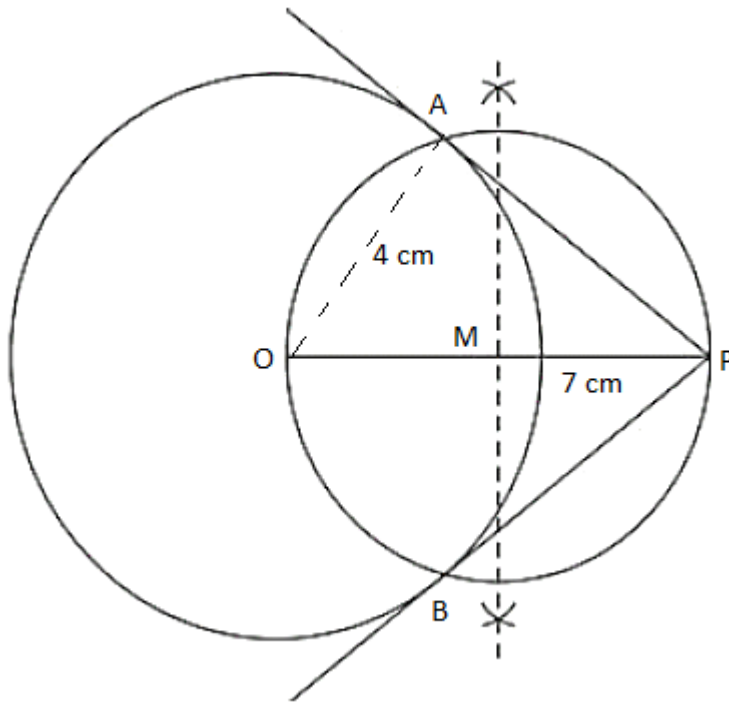
Attempt any four questions from this section

Question 5

(a) Use ruler and compasses for this question:

- (i) Draw a circle with centre O and radius 4 cm.
- (ii) Mark a point P such that $OP = 7$ cm. Construct the two tangents to the circle from P. Measure and record the length of one of the tangents. [3]

Solution:



PA and PB are the required tangents to the circle.
Length of the tangent = 5.8 cm (approximately by measure)

(b) Solve for x using the properties of proportion:

$$\frac{3x + \sqrt{9x^2 - 5}}{3x - \sqrt{9x^2 - 5}} = 5$$

[3]

Solution:

Given,

$$\frac{3x + \sqrt{9x^2 - 5}}{3x - \sqrt{9x^2 - 5}} = 5$$

Using componendo and dividendo rule,

$$\frac{3x + \sqrt{9x^2 - 5} + 3x - \sqrt{9x^2 - 5}}{3x + \sqrt{9x^2 - 5} - 3x + \sqrt{9x^2 - 5}} = \frac{5 + 1}{5 - 1}$$

$$\frac{6x}{2\sqrt{9x^2 - 5}} = \frac{6}{4}$$

$$\frac{x}{\sqrt{9x^2 - 5}} = \frac{1}{2}$$

Squaring on both sides,

$$x^2 / (9x^2 - 5) = 1/4$$

$$4x^2 = 9x^2 - 5$$

$$\Rightarrow 9x^2 - 4x^2 = 5$$

$$\Rightarrow 5x^2 = 5$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = 1$$

(c) Mr. Sharma has 60 shares of nominal value Rs. 100 and he decides to sell them when they are at a premium of 60%. He invests the proceeds in shares of the nominal value of Rs. 50, quoted at 4% discount paying 18% dividend annually. Calculate:

- (i) the sale proceeds
- (ii) the number of shares he buys
- (iii) his annual dividend from these shares.

[4]

Solution:

(i) Given,

Nominal value of 1 share = Rs. 100

Nominal value of 60 shares = 60 × Rs. 100

= Rs. 6,000

Market value of 1 share = Rs. 100 + 60% of Rs. 100

= Rs. 100 + Rs. 60

= Rs. 160

Market value of 60 shares = Rs. 60 × Rs. 160

= Rs. 9,600

Therefore, the sale proceeds at Rs. 9600.

(ii) Nominal value of 1 share = Rs. 50

Market value of 1 share = Rs. 50 - 4% of Rs. 50 (quoted at 4%)

= Rs. 50 - Rs.2

= Rs.48

No of shares purchased = $9600/48 = 200$

(iii) Nominal value of 200 shares = $200 \times \text{Rs. } 50 = \text{Rs. } 10000$

Dividend earned = 18% of Rs. 10000

= Rs. 1800

Question 6

(a) $A(4, -1)$, $B(0, 7)$ and $C(-2, 5)$ are the vertices of a triangle ABC . ΔABC is reflected in the y -axis and then reflected in the origin. Find the coordinates of the final images of the vertices. [3]

Solution:

Given,

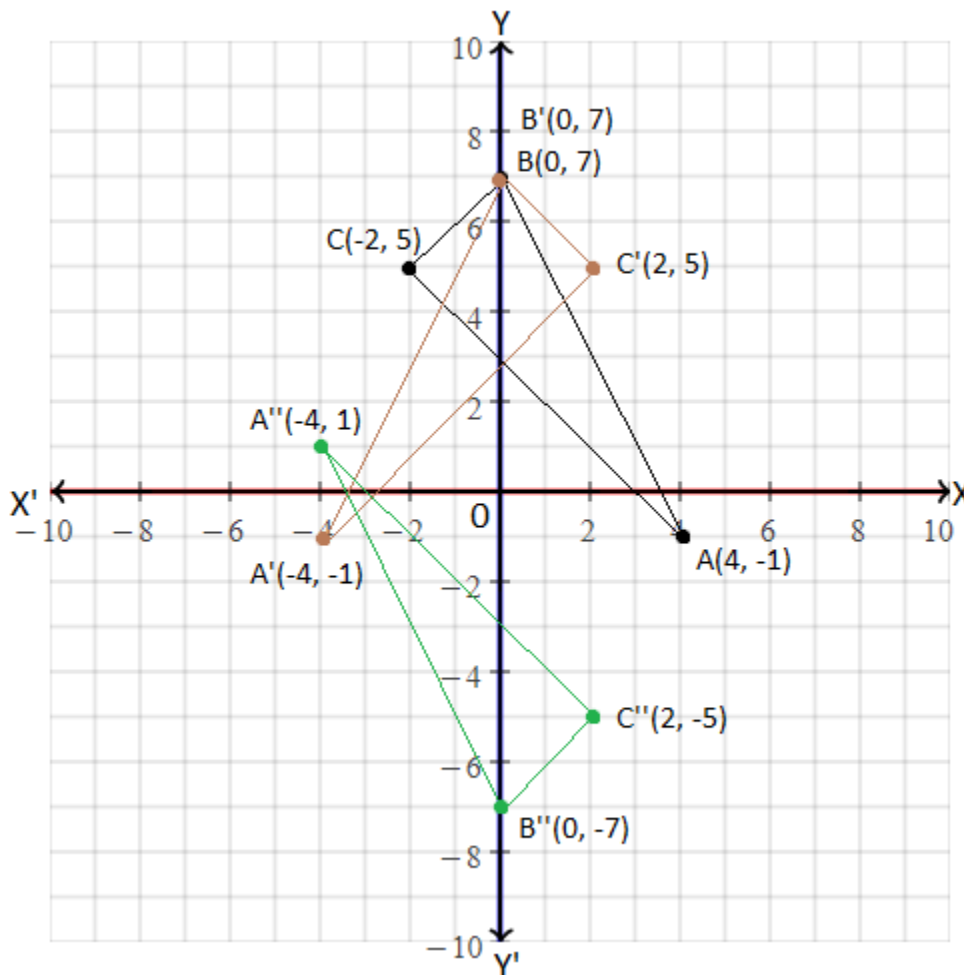
$A(4, -1)$, $B(0, 7)$ and $C(-2, 5)$

Coordinates of the triangle ABC , when reflected in the y -axis, are:

$A'(-4, -1)$, $B'(0, 7)$ and $C'(2, 5)$

Coordinates of the triangle ABC , when reflected in the origin, are:

$A''(-4, 1)$, $B''(0, -7)$ and $C''(2, -5)$



(b) A large firm employs 4250 employees. One person is chosen at random. What is the probability that the person's birthday is on Monday in the year 2008? [3]

Solution:

Given,

Number of employees = 4250

2008 is a leap year.

52 weeks + 2 odd days

These two odd days can be any of the following:

Sunday, Monday

Monday, Tuesday

Tuesday, Wednesday

Wednesday, Thursday,

Thursday, Friday

Friday, Saturday

Saturday, Sunday

The probability that a person's birthday is on Monday 2008 = $(2/7) \times (53/366) + (5/7) \times (52/366)$

$$= (106 + 260) / (366 \times 7)$$

$$= 366 / (366 \times 7)$$

$$= 1/7$$

Hence, the required probability = $(1/4250) \times (1/7)$

$$= 1/29750$$

(c) Lengths of the sides of a right triangle are $(5x + 2)$, $5x$ and $(3x - 1)$. Find the length of each side. [4]

Solution:

Given,

Lengths of the sides of a right triangle are $(5x + 2)$, $5x$ and $(3x - 1)$.

By Pythagoras theorem,

$$(5x + 2)^2 = (5x)^2 + (3x - 1)^2$$

$$25x^2 + 4 + 20x = 25x^2 + 9x^2 + 1 - 6x$$

$$9x^2 + 1 - 6x - 4 - 20x = 0$$

$$9x^2 - 26x - 3 = 0$$

$$9x^2 - 27x + x - 3 = 0$$

$$9x(x - 3) + 1(x - 3) = 0$$

$$(9x + 1)(x - 3) = 0$$

$$9x + 1 = 0, x - 3 = 0$$

$$x = -1/9, x = 3$$

$x = -1/9$ is not possible since the length cannot be negative.

Therefore, the length of each side is:

$$5x + 2 = 5(3) + 2 = 15 + 2 = 17$$

$$5x = 5(3) = 15$$

$$3x - 1 = 3(3) - 1 = 9 - 1 = 8$$

Question 7

(a) If the roots of the equation $2x^2 - 2cx + ab = 0$ be real and distinct, prove that the roots of $x^2 - 2(a + b)x + (a^2 + b^2 + c^2) = 0$ will be imaginary. [3]

Solution:

Given,

The roots of the equation $2x^2 - 2cx + ab = 0$ be real and distinct.

Therefore, discriminant > 0

$$(-2c)^2 - 4(2)(ab) > 0$$

$$4c^2 - 8ab > 0$$

$$4(c^2 - 2ab) > 0$$

$$c^2 - 2ab > 0 \dots (i)$$

Now,

$$x^2 - 2(a + b)x + (a^2 + b^2 + c^2) = 0$$

$$\text{Discriminant} = [-2(a + b)]^2 - 4(1)(a^2 + b^2 + c^2)$$

$$= 4(a + b)^2 - 4(a^2 + b^2 + c^2)$$

$$= 4(a^2 + b^2 + 2ab) - 4a^2 - 4b^2 - 4c^2$$

$$= 4a^2 + 4b^2 + 8ab - 4a^2 - 4b^2 - 4c^2$$

$$= 8ab - 4c^2$$

$$= -4(c^2 - 2ab)$$

$$= -4(+ve) < 0 \text{ [From (i)]}$$

Hence, the roots of the equation are imaginary.

(b) A shopkeeper bought a coat at a discount of 20% from the wholesaler. The printed price of the coat is Rs. 1600 and the rate of sales tax is 6%. The shopkeeper sold it to the customer at the printed price. Find the VAT paid by the shopkeeper to the government. Also, find the amount paid by the customer for the coat. [3]

Solution:

The printed price of coat = Rs. 1600

Discount got by the shopkeeper = 20%

Price of coat after discount = Rs. 1600 - 20% of Rs. 1600

$$= \text{Rs. } 1600 - (20/100) \times \text{Rs. } 1600$$

$$= \text{Rs. } 1600 - \text{Rs. } 320$$

$$= \text{Rs. } 1280$$

Rate of sales tax = 6% (given)

Tax paid by the shopkeeper = 6% of Rs. 1280

$$= (6/100) \times \text{Rs. } 1280$$

$$= \text{Rs. } 76.80$$

The selling price of the coat to the customer by the shopkeeper = Rs. 1600

Tax paid by the customer = 6% of Rs. 1600

$$= (6/100) \times \text{Rs. } 1600$$

$$= \text{Rs. } 96$$

(i) VAT paid by the shopkeeper = Tax paid by the customer - Tax paid by the shopkeeper

$$= \text{Rs. } 96 - \text{Rs. } 76.80$$

$$= \text{Rs. } 19.20$$

(ii) Amount paid by the customer for the coat = Rs. 1600 + Rs. 96

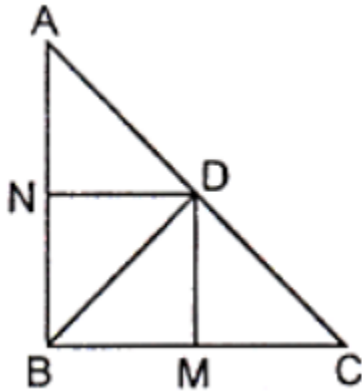
$$= \text{Rs. } 1696$$

(c) In the figure, ABC is a right triangle with $\angle ABC = 90^\circ$, $BD \perp AC$, $DM \perp BC$ and $DN \perp AB$. Prove that

(i) $DM^2 = DN \times MC$

(ii) $DN^2 = DM \times AN$

[4]



Solution:

(i) Given,

$BD \perp AC$, $DM \perp BC$ and $DN \perp AB$

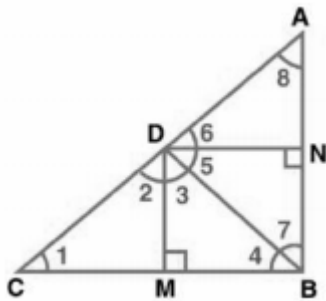
From the figure,

$DN \parallel CB$, $DM \parallel AB$ and $\angle B = 90^\circ$

Therefore, DMBN is a rectangle.

$DN = MB$ and $DM = NB$

Let us consider the angles as shown below:



The given condition which we have to prove is when D is the foot of the perpendicular drawn from B to AC.

$$\therefore \angle CDB = 90^\circ \Rightarrow \angle 2 + \angle 3 = 90^\circ \dots (i)$$

By the angle sum property of a triangle:

In $\triangle CDM$,

$$\angle 1 + \angle 2 + \angle DMC = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 90^\circ \dots (ii)$$

In $\triangle DMB$,

$$\angle 3 + \angle DMB + \angle 4 = 180^\circ$$

$$\Rightarrow \angle 3 + \angle 4 = 90^\circ \dots (iii)$$

From (i) and (ii),

$$\angle 1 = \angle 3$$

From (i) and (iii),

$$\angle 2 = \angle 4$$

In $\triangle DCM$ and $\triangle BDM$,

$$\angle 1 = \angle 3 \text{ (proved above)}$$

$$\angle 2 = \angle 4 \text{ (proved above)}$$

$\therefore \triangle DCM \sim \triangle BDM$ (by AA similarity criterion)

$$BM/DM = DM/MC$$

$$\Rightarrow DN/DM = DM/MC \quad (BM = DN)$$

$$\Rightarrow DM^2 = DN \times MC$$

(ii) In right triangle DBN,

$$\angle 5 + \angle 7 = 90^\circ \dots (iv)$$

In right triangle DAN,

$$\angle 6 + \angle 8 = 90^\circ \dots (v)$$

D is the point in triangle, which is foot of the perpendicular drawn from B to AC.

$$\therefore \angle ADB = 90^\circ \Rightarrow \angle 5 + \angle 6 = 90^\circ \dots (vi)$$

From (iv) and (vi),

$$\angle 6 = \angle 7$$

From (v) and (vi),

$$\angle 8 = \angle 5$$

In $\triangle DNA$ and $\triangle BND$,

$$\angle 6 = \angle 7 \text{ (proved above)}$$

$$\angle 8 = \angle 5 \text{ (proved above)}$$

$\therefore \triangle DNA \sim \triangle BND$ (by AA similarity criterion)

$$AN/DN = DN/NB$$

$$\Rightarrow DN^2 = AN \times NB$$

$$\Rightarrow DN^2 = AN \times DM \quad (NB = DM)$$

Hence proved.

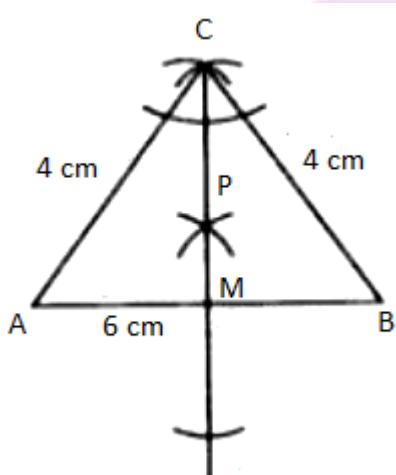
Question 8

(a) Construct an isosceles triangle ABC such that $AB = 6$ cm, $BC = AC = 4$ cm. Find a point P such that it is equidistant from A and B as well as from AC and BC. [3]

Solution:

Given,

$$AB = 6 \text{ cm}, BC = AC = 4 \text{ cm}$$



P is a point such that it is equidistant from A and B as well as from AC and BC.

Also, we can say that every point on CM will be equidistant from A and B as well as from AC and BC since ABC is an isosceles triangle.

(b) On a certain sum of money, the difference between the compound interest for a year, payable half-yearly and the simple interest for a year is Rs. 180. Find the sum lent out if the rate of interest in both the cases is 10% p.a.

[3]

Solution:

Given,

Rate of interest = 10%

Let P be the sum lent out for interest.

$$SI = PTR/100$$

$$SI = (P \times 1 \times 10)/100$$

$$SI = P/10 \dots (i)$$

$$CI = P[1 + (R/200)]^2 - P \text{ (half-yearly payable)}$$

$$= P[1 + (10/200)]^2 - P$$

$$= P(210/200)^2 - P$$

$$= P(21/20)^2 - P$$

$$= P(441/400) - P$$

$$CI = 41P/400 \dots (ii)$$

$$CI - SI = \text{Rs. } 180 \text{ (given)}$$

$$(41P/400) - (P/10) = 80$$

$$0.1025P - 0.1P = 180$$

$$0.0025P = 180$$

$$P = 180/0.0025$$

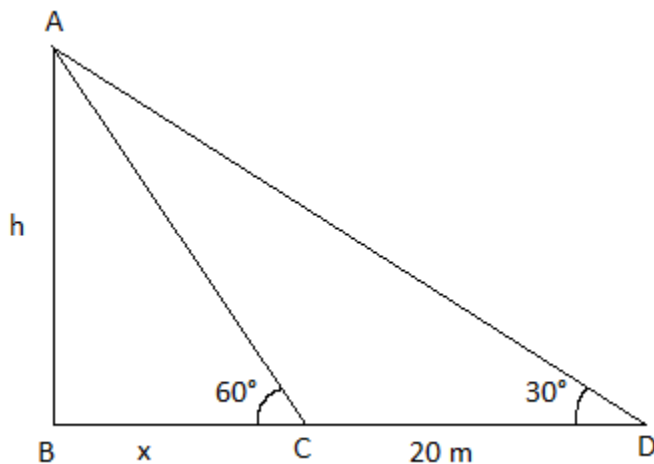
$$P = 72000$$

Therefore, the sum lent out is Rs. 72000.

(c) A boy standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60° . When he moves 20 m back from the bank, he finds the angle to be 30° . Find the height of the tree and the breadth of the river. [4]

Solution:

Let AB be the tree and C, D be the positions of a boy.



$$AB = h$$

BC = Breadth of the river.

In right triangle ABC,

$$\tan 60^\circ = AB/BC$$

$$\sqrt{3} = h/x$$

$$h = x\sqrt{3} \dots (i)$$

In right triangle ABD,

$$\begin{aligned}\tan 30^\circ &= AB/BD \\ 1/\sqrt{3} &= h/(x + 20) \\ x + 20 &= h\sqrt{3} \\ x + 20 &= x(\sqrt{3})(\sqrt{3}) \\ 3x - x &= 20 \\ 2x &= 20 \\ x &= 10 \text{ m} \\ h &= 10\sqrt{3} \text{ m}\end{aligned}$$

Therefore, the height of the tree is $10\sqrt{3}$ m and the breadth of the river is 10 m.

Question 9

(a) Write down the equation of the line whose gradient is $3/2$ and which passes through P, where P divides the line segment joining A(-2, 6) and B(3, -4) in the ratio 2 : 3. [3]

Solution:

Given,

P divides the line segment joining A(-2, 6) and B(3, -4) in the ratio 2 : 3.

Using section formula,

$$P = [(6 - 6)/(2 + 3), (-8 + 18)/(2 + 3)]$$

$$= (0/5, 10/5)$$

$$P = (0, 2)$$

Gradient (slope) of the line = $m = 3/2$

Equation of the line passing through (x_1, y_1) and having slope m is $y - y_1 = m(x - x_1)$.

$$y - 2 = (3/2)(x - 0)$$

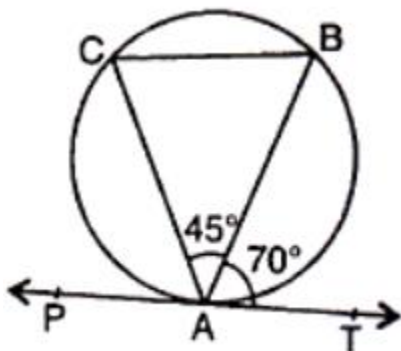
$$2(y - 2) = 3x$$

$$2y - 2 = 3x$$

$$3x - 2y + 2 = 0$$

Hence, the equation of required line is $3x - 2y + 2 = 0$.

(b) In the figure, PAT is a tangent at A. If $\angle TAB = 70^\circ$ and $\angle BAC = 45^\circ$, find $\angle ABC$. [3]



Solution:

Given,

PAT is a tangent at A. If $\angle TAB = 70^\circ$ and $\angle BAC = 45^\circ$.

$\angle TAB = \angle ACB = 70^\circ$ (angles in the same segment)

In triangle ABC,

$\angle ABC + \angle ACB + \angle BAC = 180^\circ$ (by the angle sum property)

$$\angle ABC + 70^\circ + 45^\circ = 180^\circ$$

$$\angle ABC = 180^\circ - 70^\circ - 45^\circ$$

$$\angle ABC = 65^\circ$$

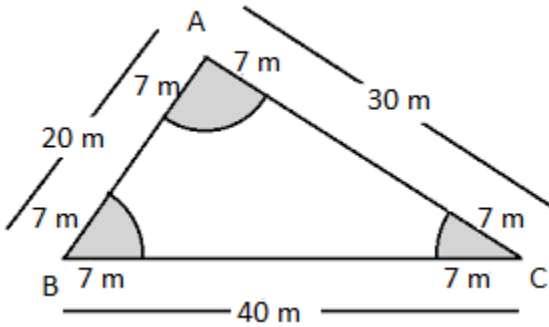
(c) Three horses are tethered at three corners of a triangular plot having sides 20 m, 30 m and 40 m with ropes of 7 m length each. Find the area of this plot which can be grazed by the horses. [4]

Solution:

Given,

Three horses are tethered at three corners of a triangular plot having sides 20 m, 30 m and 40 m with ropes of 7 m length each.

Let A, B and C be the corners at which three horses tied.



We know that the sum of all internal angles of a triangle is 180° .

$$\angle A + \angle B + \angle C = 180^\circ$$

Radius of sectors at A, B, and C = $r = 7$ m

Area of the grass grazed by three horses

$$\begin{aligned} &= (\angle A / 360^\circ) \times \pi r^2 + (\angle B / 360^\circ) \times \pi r^2 + (\angle C / 360^\circ) \times \pi r^2 \\ &= \pi(7)^2 [(\angle A + \angle B + \angle C) / 360^\circ] \\ &= (22/7) \times 49 \times (180^\circ / 360^\circ) \\ &= 22 \times 7 \times (1/2) \\ &= 77 \text{ m}^2 \end{aligned}$$

Question 10

(a) Prove that: $(1 - \cos \theta) / (1 + \cos \theta) = (\operatorname{cosec} \theta - \cot \theta)^2$ [4]

Solution:

$$\begin{aligned} \text{LHS} &= (1 - \cos \theta) / (1 + \cos \theta) \\ &= [(1 - \cos \theta) / (1 + \cos \theta)] \times [(1 - \cos \theta) / (1 - \cos \theta)] \\ &= (1 - \cos \theta)^2 / (1 - \cos^2 \theta) \\ &= (1 - \cos \theta)^2 / \sin^2 \theta \\ &= [(1 - \cos \theta) / \sin \theta]^2 \\ &= [(1 / \sin \theta) - (\cos \theta / \sin \theta)]^2 \\ &= (\operatorname{cosec} \theta - \cot \theta)^2 \\ &= \text{RHS} \end{aligned}$$

Hence proved.

(b) Draw an Ogive for the following frequency distribution:

Class	6500 - 7000	7000 - 7500	7500 - 8000	8000 - 8500	8500 - 9000	9000 - 9500	9500 - 10000
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Frequency	10	18	22	25	17	10	8
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From the Ogive, find the median.

[6]

Solution:

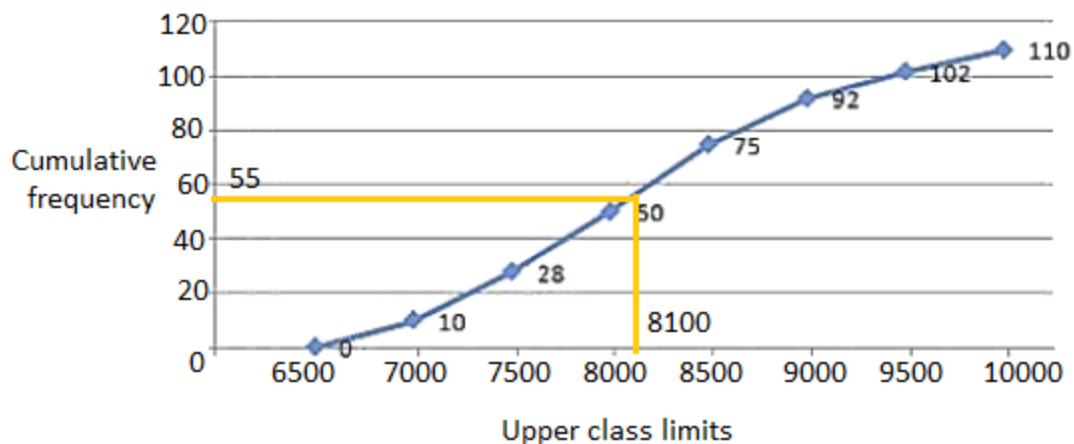
Cumulative frequency distribution table:

Class	Frequency	Cumulative frequency
6500 - 7000	10	10
7000 - 7500	18	28
7500 - 8000	22	50
8000 - 8500	25	75
8500 - 9000	17	92
9000 - 9500	10	102
9500 - 10000	8	110

$$N/2 = 110/2 = 55$$

Plot the points (7000, 10), (7500, 28), (8000, 50), (8500, 75), (9000, 92), (9500, 102) and (10000, 110).

Join these points to obtain an ogive.



$$\text{Median} = 8100$$

Question 11

(a) Determine whether the line through (-2, 3) and (4, 1) is perpendicular to the line $3x = y + 1$. Does the line $3x = y + 1$ bisect the line joining (-2, 3) and (4, 1)? [3]

Solution:

Slope of the line passing through the points (-2, 3) and (4, 1) = m_1
 $= (1 - 3) / (4 + 2)$

$$= -2/6$$

$$= -1/3$$

Given,

Equation of line is:

$$3x = y + 1$$

$$y = 3x - 1$$

Comparing with the slope-intercept form,

$$\text{Slope} = m_2 = 3$$

$$m_1 \times m_2 = (-1/3) \times 3 = -1$$

Therefore, the equation of the line passing through the points $(-2, 3)$ and $(4, 1)$ is perpendicular to the line $3x = y + 1$.

$$\text{Midpoint of } (-2, 3) \text{ and } (4, 1) = [(-2 + 4)/2, (3 + 1)/2]$$

$$= (2/2, 4/2)$$

$$= (1, 2)$$

Substituting $(1, 2)$ in $3x = y + 1$,

$$3(1) = 2 + 1$$

$$3 = 3$$

Hence, the line $3x = y + 1$ bisect the line joining $(-2, 3)$ and $(4, 1)$.

(b) Manish deposits Rs. 2000 per month in a recurring deposit account for $1\frac{1}{2}$ year at 8% p.a. Find the amount he will receive at the time of maturity. [3]

Solution:

Monthly installment = $P = \text{Rs. } 2000$

Number of months = $n = 18$ (i.e. $1\frac{1}{2}$ years)

Rate of interest = $r = 8\%$

$$SI = P \times [n(n + 1) / (2 \times 12)] \times (r/100)$$

$$= 2000 \times [8(8 + 1) / (2 \times 12)] \times (8/100)$$

$$= 2000 \times [(8 \times 9) / 24] \times (2/25)$$

$$= 2000 \times (6/25)$$

$$= 480$$

$$SI = \text{Rs. } 480$$

The amount the Manish will get at the time of maturity

$$= (P \times n) + SI$$

$$= \text{Rs. } 2000 \times 18 + \text{Rs. } 480$$

$$= \text{Rs. } 36000 + \text{Rs. } 480$$

$$= \text{Rs. } 36480$$

(c) Calculate the mean daily wage of a worker from the following table:

Daily wages (in Rs)	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65
No. of workers	2	3	7	12	6

[4]

Solution:

Daily wages (in Rs)	No. of workers (f_i)	Class mark (x_i)	$d_i = x_i - a$	$f_i d_i$
40 - 45	2	42.5	-10	-20
45 - 50	3	47.5	-5	-15
50 - 55	7	52.5 = A	0	0
55 - 60	12	57.5	5	60
60 - 65	6	62.5	10	60
	$\sum f_i = 30$			$\sum f_i d_i = 85$

Assumed mean = $a = 52.5$

Mean = $a + (\sum f_i d_i / \sum f_i)$

= $52.5 + (85 / 30)$

= $52.5 + 2.83$

= 55.33

Therefore, the mean daily wage is Rs. 55.33 (approx.)