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ISC Class 11 Maths Specimen Question Paper 2019 with Solutions

SECTION - A

Question 1
(i) If A = {1, 2, 3, 4, 5, 6} B = {2, 4, 5, 6, 8, 9, 10}, find A ∆ B.

Solution:
Given,
A = {1, 2, 3, 4, 5, 6}
B = {2, 4, 5, 6, 8, 9, 10}
A - B = {1, 2, 3, 4, 5, 6} - {2, 4, 5, 6, 8, 9, 10} = {1, 3}
B - A = {2, 4, 5, 6, 8, 9, 10} - {1, 2, 3, 4, 5, 6} = {8, 9, 10}
A ∆ B = (A - B) ∪ (B - A)
= {1, 3} ∪ {8, 9, 10}
= {1, 3, 8, 9, 10}

(ii) Let A = {2, 4, 6, 8}, B = {1, 2, 3, 4) and R = {(a, b) : a ∈ A, b ∈ B, a is divisible by b}. Write Relation R in set builder form.

Solution:
Given,
A = {2, 4, 6, 8}
B = {1, 2, 3, 4)
R = {(a, b) : a ∈ A, b ∈ B, a is divisible by b}
R = {(2, 1), (2, 2), (4, 1), (4, 2), (4, 4), (6, 1), (6, 2), (6, 3), (8, 1), (8, 2), (8, 4}

(iii) Prove that:
cos 2A/ (1 + sin 2A) = tan [(π/4) - A]

Solution:
LHS = cos 2A/ (1 + sin 2A)
= $(\cos^2 A - \sin^2 A)/ [\sin^2 A + \cos^2 A + 2 \sin A \cos A]$
= $(\cos^2 A - \sin^2 A) / (\sin A + \cos A)^2$
= $[(\cos A + \sin A) (\cos A - \sin A)]/ (\sin A + \cos A)^2$
= (cos A - sin A)/ (cos A + sin A)
= [cos A (1 - tan A)] / [cos A (1 + tan A)]
= (1 - tan A)/ (1 + tan A)
= tan [(π/4) - A]
= RHS
Hence proved.

(iv) In a ∆ABC, prove that sin A/ sin (A + B) = a/c.

Solution:

In a triangle ABC,
a/sin A = b/sin B = c/sin C
Let a/sin A = b/sin B = c/sin C = k
⇒ a = k sin A, b = k sin B, c = k sin C
RHS = a/c
= k sin A/k sin C
= sin A/ sin C
= sin A/ sin (180° - C) [since sin (180° - θ) = sin θ]
= sin A/ sin (A + B) [since A + B + C = 180°]
= LHS
Hence proved.

(v) If (2 + 3i)/ (3 - 4i) = a + ib, find the values of a and b.

Solution:
Given,
(2 + 3i)/ (3 - 4i) = a + ib
[(2 + 3i)/ (3 - 4i)] × [(3 + 4i)/ (3 + 4i)] = a + ib
[2(3) + 2(4i) + (3i)(3) + (3i)(4i)] / [$(3)^2$ - $(4i)^2$] = a + ib
[6 + 8i + 9i + $12i^2$] / [9 - $16i^2$] = a + ib
(6 + 17i - 12)/ (9 + 16) = a + ib
(17i - 6)/ 25 = a + ib
(-6/25) + i(17/25) = a + ib
Therefore, a = -6/25 and b = 17/25

(vi) If α and β are the roots of the equation $px^2 + qx + 1 = 0$, find $\alpha^2\beta + \beta^2\alpha$.

Solution:
Given,
α and β are the roots of the equation $px^2 + qx + 1 = 0$.
Sum of roots = -coefficient of x/ coefficient of x^2
α + β = -q/p
Product of roots = constant term/ coefficient of x^2
αβ = 1/p
$\alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta)$
= (1/p) (-q/p)
= $-q/p^2$

(vii) In how many ways can 12 books be arranged on a shelf if:
(a) 4 particular books must always be together.
(b) 2 particular books must occupy the first position and the last position.

Solution:
Given,
Total number of books = 12
(a) Consider 4 particular books like 1.
Now, the total number of books = 9
9 books can be arranged in 9! ways.
4 books (considered as 1) can be arranged in 4! ways.
Therefore, the number of ways in which 4 particular books must always be together = 9! × 4!

(b) 2 particular books can be arranged in 2! ways.
Remaining 10 books can be arranged in 10! ways.
Therefore, the number of ways in which 2 particular books must occupy the first position and the last position = 2! × 10! = 2 × 10!

(viii) Find the derivative of $(x^4 + 3x^3 + 4x^2 + 2)/ x^3$.

Solution:
$(x^4 + 3x^3 + 4x^2 + 2)/ x^3$
$= (x^4/x^3) + (3x^3/x^3) + (4x^2/x^3) + (2/x^3)$
$= x + 3 + (4/x) + (2/x^3)$
Now,
$d/dx\, [x + 3 + (4/x) + (2/x^3)] = 1 + 0 + 4(-1/x^2) + 2\,(-3/x^4)$
$= 1 - (4/x^2) - (6/x^4)$
$= (x^4 - 4x^2 - 6)/ x^4$

(ix) Evaluate:

$$\lim_{x\to 0} \frac{(1-x)^n-1}{x}$$

Solution:
By applying L'Hopital's rule,

$$\lim_{x\to 0} \frac{(1-x)^n-1}{x} = \lim_{x\to 0} \frac{\frac{d}{dx}\left((1-x)^n-1\right)}{\frac{d}{dx}(x)}$$

$$= \lim_{x\to 0} \frac{-n(1-x)^{n-1}\times 1-0}{1}$$

$$= \lim_{x\to 0} -n(1-x)^{n-1}$$

$$= -n \times 1^{n-1}$$

$$= -n$$

(x) An urn contains 60 blue pens and 40 red pens. Half of the pens of each one is defective. If one pen is chosen at random, what is the probability that it is a defective or a red pen?

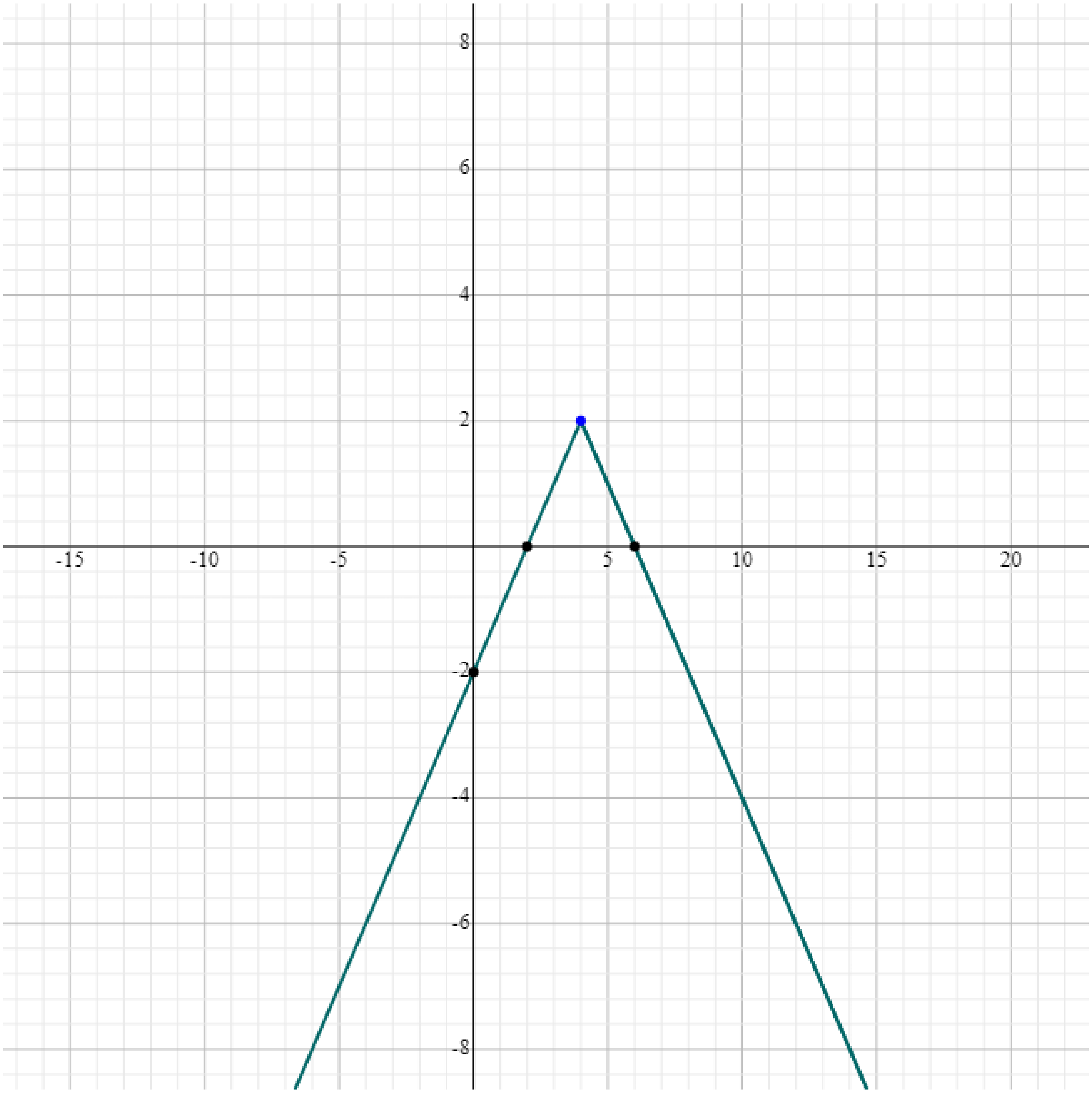
Solution:
Given,
An urn contains 60 blue pens and 40 red pens.
Half of the pens of each one is defective.
i.e. number defective blue pens = 30
And the number defective red pens = 20
Total number of defective pens = 30 + 20 = 50

P(getting a defective pen) = 50/100
P(getting a red pen) = 40/100
P(getting a red defective pen) = 20/100
P(getting a defective or a red pen) = (50/100) + (40/100) - (20/100)
= (50 + 40 - 20)/ 100
= 70/100
= 0.7
Hence, the probability that it is a defective or a red pen is 0.7.

Question 2
Find the domain and range of 2 - |x - 4|.

Solution:
Let f(x) = 2 - |x - 4|
The given function can be defined for all real values of x'
i.e. $x \in R$
Thus, the domain of the function is (-∞, ∞)

The range of an absolute functions of the form -c|ax + b| + k is f(x) ≤ k
Here, c = -1, a = 1, b = -4 and k = 2
Thus, f(x) ≤ 2
Range of f(x) is (-∞, 2]



Question 3

(a) Solve: sin 7x + sin 4x + sin x = 0 and 0 < x < π/2

Solution:

Given,

sin 7x + sin 4x + sin x = 0

(sin 7x + sin x) + sin 4x = 0

2 sin [(7x + x)/2] cos [(7x - x)/2] + sin 4x = 0

2 sin (8x/2) cos (6x/2) + sin 4x = 0

2 sin 4x cos 3x + sin 4x = 0

sin 4x(2 cos 3x + 1) = 0

sin 4x = 0, 2 cos 3x + 1 = 0

$2 \sin 2x \cos 2x = 0, 2 \cos 3x = -1$
$\sin 2x = 0$ or $\cos 2x, \cos 3x = -1/2$
$2 \sin x \cos x = 0$, or $2 \cos^2 x - 1 = 0, \cos 3x = -1/2$
$\sin x = 0$ or $\cos x = 0$ or $\cos^2 x = 1/2, \cos 3x = -1/2$
Now,
$\sin x = 0 \Rightarrow x = 0 \notin (0, \pi/2)$
$\cos x = 0 \Rightarrow x = \pi/2 \notin (0, \pi/2)$
$\cos^2 x = 1/2 \Rightarrow \cos x = \pm\sqrt{(1/2)}$
$\cos x = \sqrt{(1/2)} \Rightarrow x = \pi/4 \in (0, \pi/2)$
$\cos x = -\sqrt{(1/2)} < 0 \Rightarrow x \notin (0, \pi/2)$
And
$\cos 3x = 0$
$4 \cos^3 x - 3 \cos x = 0$
$\cos x(4 \cos^2 x - 3) = 0$
$\cos x = 0$ or $4 \cos^2 x - 3 = 0$
$\cos x = 0$ or $\cos^2 x = 3/4$
$\cos x = 0$ or $\cos x = \pm\sqrt{(3)}/2$
$\cos x = 0 \Rightarrow x = \pi/2 \notin (0, \pi/2)$
$\cos x = \sqrt{3}/2 \Rightarrow x = \pi/6 \in (0, \pi/2)$
$\cos x = -\sqrt{3}/2 < 0 \Rightarrow x \notin (0, \pi/2)$
Therefore, $x = \pi/4$ or $\pi/6$
But $x = \pi/6$ will satisfy the given equation.
Hence, the solution is $x = \pi/4$.

OR

(b) Prove that (cos A + cos 3A + cos 5A + cos 7A)/ (sin A + sin 3A + sin 5A + sin 7A) = cot 4A.

Solution:
LHS = (cos A + cos 3A + cos 5A + cos 7A)/ (sin A + sin 3A + sin 5A + sin 7A)
= [(cos 7A + cos A) + (cos 5A + cos 3A)] / [(sin 7A + sin A) + (sin 5A + sin 3A)]
Using:
cos x + cos y = 2 cos (x + y)/2 cos (x - y)/2
sin x + sin y = 2 sin (x + y)/2 cos (x - y)/2
= (2 cos 4A cos 3A + 2 cos 4A cos A)/ (2 sin 4A cos 3A + 2 sin 4A cos A)
= [2 cos 4A (cos 3A + cos A)] / [2 sin 4A (cos 3A + cos A)]
= cos 4A/ sin 4A
= cot 4A
= RHS
Hence proved.

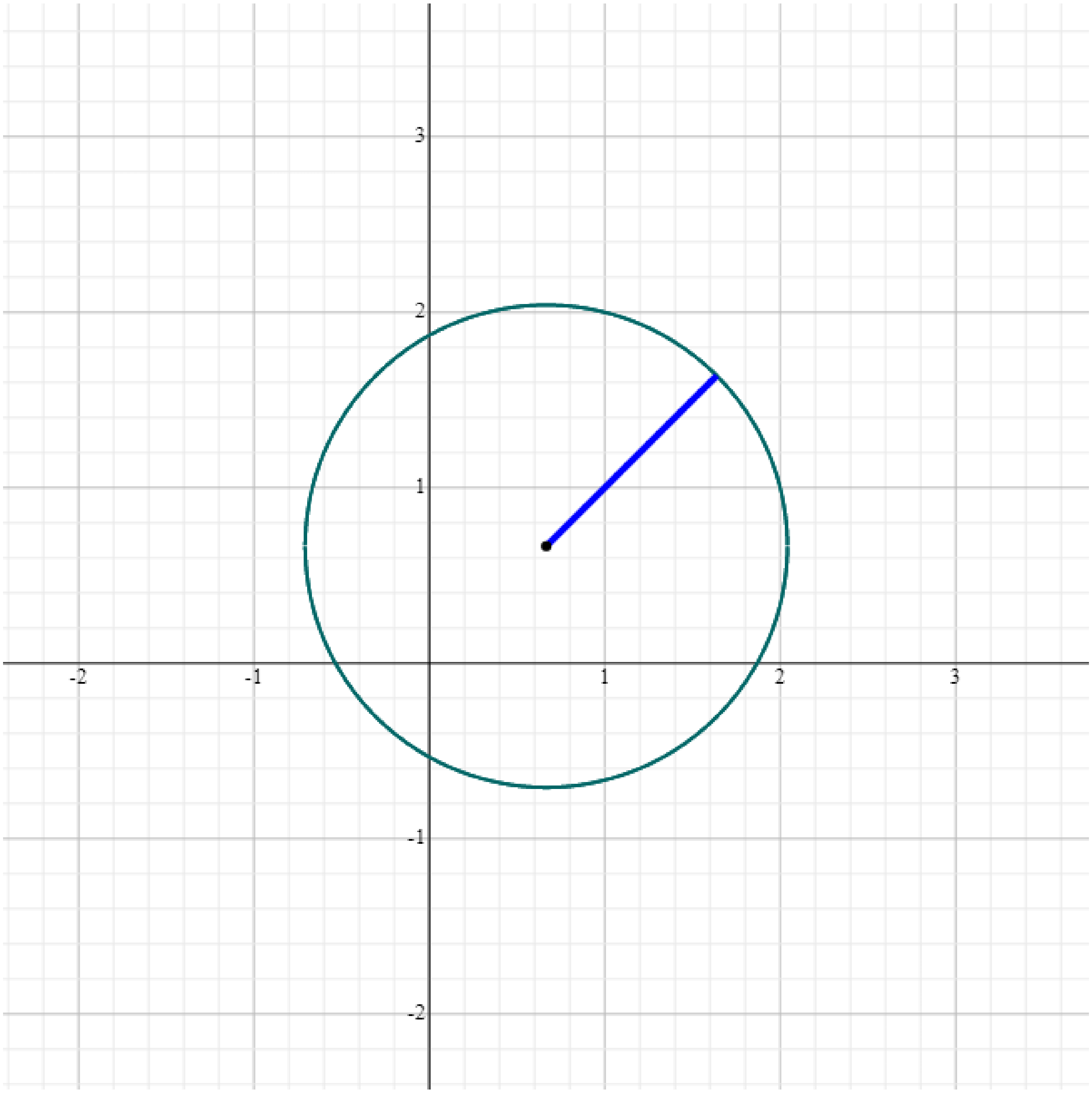
Question 4
Using Mathematical induction, prove that $10^n + 3.4^{n+2} + 5$ is divisible by 9 for an $n \in N$.

Solution:
Let $P(n) = 10^n + 3.4^{n+2} + 5$
When n = 1,
P(1) = 10 + 3.(4)3 + 5
= 10 + 3 × 64 + 5
= 15 + 192

= 207
= 9 × 23
Thus, P(1) is divisible by 9.
Let P(k) is true for n = k.
i.e. $P(k) = 10^k + 3.4^{k+2} + 5$ is divisible by 9.
$\Rightarrow 10^k + 3.4^{k+2} + 5 = 9m$
$\Rightarrow 10^k = 9m - 3.4^{k+2} - 5$….(i)
Now, take n = k + 1
$P(k + 1) = 10^{k+1} + 3.4^{k+1+2} + 5$
$= 10 \times 10^k + 3.4^{k+2} .4 + 5$
$= 10 (9m - 3.4^{k+2} - 5) + 3.4^{k+2} .4 + 5$ [From (i)]
$= 90m - 30. 4^{k+2} - 50 + 12.4^{k+2} + 5$
$= 90m - 18\ 4^{k+2} - 45$
$= 9(10m - 2.4^{k+2} - 5)$ which is divisible by 9 .
∴ P(k + 1) is divisible by 9.
Hence, by the Principle of mathematical induction, P(n) is true for all $n \in N$.

Question 5
If z = x + iy and |2z - 1| = |z + 2i|, find the locus of z and represent in the argand diagram.

Solution:
Given,
z = x + iy
|2z - 1| = |z + 2i|
|2(x + iy) - 1| = |x + iy + 2i|
|(2x - 1) + i2y| = |x + i(y + 2)|
$\sqrt{[(2x - 1)^2 + (2y)^2]} = \sqrt{[x^2 + (y + 2)^2]}$
Squaring on both sides,
$4x^2 + 1 - 4x + 4y^2 = x^2 + y^2 + 4 + 4y$
$4x^2 + 4y^2 - 4x + 1 - x^2 - y^2 - 4y - 4 = 0$
$3x^2 + 3y^2 - 4x - 4y - 3 = 0$
Comparing with $Ax^2 + By^2 + Cx + Dy + E = 0$,
A = 3, B = 3, C = -4, D = -4 and E = -3
A = B ≠ 0
Therefore, the equation represents a circle with centre (⅔, ⅔) and radius √17/ 3.
Thus, the locus of z is the circle.



Question 6
(a) A Committee of 6 members has to be formed from 8 boys and 5 girls. In how many ways can this be done if the Committee consists of:
(i) Exactly 3 girls
(ii) At least 3 girls

Solution:
Given,
8 boys and 5 girls
(i) A committee of 6 members consists of exactly 3 girls
$= 5C_3 \times 8C_3$

$= 5C_2 \times 8C_3$
$= [(5 \times 4)/ 2] \times (8 \times 7 \times 6)/ (3 \times 2)]$
$= 10 \times 56$
$= 560$
Therefore, it is possible to form a committee of 6 members consisting of exactly 3 girls in 560 ways.
(ii) A committee of 6 members consists of at least 3 girls
$= 5C_3 \times 8C_3 + 5C_4 \times 8C_2 + 5C_5 \times 8C_1$
$= 560 + 5C_1 \times 8C_2 + 1 \times 8$
$= 560 + 5 \times [(8 \times 7)/2] + 8$
$= 560 + 5 \times 28 + 8$
$= 568 + 140$
$= 708$
Hence, it is possible to form a committee of 6 members consisting of at least 3 girls in 560 ways.

OR

(b) How many different words can be formed of the letter of the word "GRANDMOTHER", so that:
(i) The word starts with G and ending with R.
(ii) The letters A, N, D are always together.
(iii) All vowels never come together

Solution:
Given,
The word "GRANDMOTHER"
Number of letters = 11
R - 2 times
(i) The word starts with G and ending with R.
G _ _ _ _ _ _ _ _ _ R
This can be done in 1 way.
The remaining 9 letters can be arranged in 9! Ways.
Therefore, the required number of words = 9! × 1 = 9!
(ii) The letters A, N, D are always together.
Consider, A, N, D as one letter.
(AND) _ _ _ _ _ _ _ _
Now, the total number of letters = 9
9 letters can be arranged in 9! ways.
A, N, D can be arranged in 3! ways.
Therefore, the total number of words = 9! × 3!
(iii) Number of vowels = 3
Total number of ways in which 11 letters can be arranged = 11!/ 2! (R- 2 times)
Now, consider 3 vowels together as one letter. (A, O, E)
In this case, the total number of ways of arranging the letters = (9! × 3!)/ 2!
Hence, the total number of words in which vowels never come together = (11!/ 2!) - [(9! × 3!)/ 2!]
$= [(11! - 9!) \times 6]/ 2$
$= 3 (11! - 9!)$
$= 3 \times [11 \times 10 \times 9! - 9!]$
$= 3 \times 9! \times (110 - 1)$
$= 3 \times 109 \times 9!$
$= 327 \times 9!$

Question 7

Find the term independent of x in the expression of :

$$\left(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{2x^2}\right)^{15}$$

Solution:

$[(\sqrt{x}/ \sqrt{3}) + (\sqrt{3}/ 2x^2)]^{15}$

We know that the general term of $(a + b)^n$ is:

$T_{r+1} = nCr\ (a)^{n-r}\ (a)^r$

$T_{r+1} = 15C_r\ (\sqrt{x}/ \sqrt{3})^{15-r} . (\sqrt{3}/ 2x^2)^r$

$= 15C_r\ (x/3)^{(15-r)/2} . (3)^{r/2}\ [1/ 2^r\ x^{2r}]$

$= 15C_r\ (3)^{[(r/2) - (15-r)/2]}\ 2^{-r}\ x^{[(15-r)/2 - 2r]}$

To get the term which is independent of x, assume the value of power of x as 0.

$[(15 - r)/2 - 2r] = 0$

$15 - r - 4r = 0$

$5r = 15$

$r = 3$

Thus, the third term will be the required term.

$T_{3+1} = 15C_3\ [3^{(3/2) - 6}]/ (2)^3$

$= 15C_3\ [3^{(-9/2)}]/ 8$

$= 455 \times (1/ 3^{9/2} \times 8)$

Question 8

Find the equation of acute angled bisector of lines:

$3x - 4y + 7 = 0$ and $12x - 5y - 8 = 0$

Solution:

Given,

$3x - 4y + 7 = 0$

$12x - 5y - 8 = 0$

Here,

$a_1 = 3, b_1 = -4, c_1 = 7$

$a_2 = 12, b_2 = -5, c_2 = -8$

$a_1a_2 + b_1b_2 = 3(12) + (-4)(-5)$

$= 36 + 20$

$= 56 > 0$

For acute angle, take a positive sign

$(a_1x + b_1y + c_1)/ \sqrt{(a_1^2 + b_1^2)} = -(a_2x + b_2y + c_2)/ \sqrt{(a_2^2 + b_2^2)}$

$(3x - 4y + 7)/ \sqrt{(9 + 16)} = -(12x - 5y - 8)/\sqrt{(144 + 25)}$

$(3x - 4y + 7)\sqrt{25} = -(12x - 5y - 8)/ \sqrt{169}$

$(3x - 4y + 7)/5 = -(12x - 5y - 8)/13$

$13(3x - 4y + 7) = -5(12x - 5y - 8)$

$39x - 52y + 91 = -60x + 25y + 40$

$39x - 52y + 91 + 60x - 25y - 40 = 0$

$99x - 77y + 51 = 0$

Question 9

(a) Find the equation of the tangent to the circle

$x^2 + y^2 - 2x - 2y - 23 = 0$ and parallel to line $2x + y + 3 = 0$

Solution:
$x^2 + y^2 - 2x - 2y - 23 = 0$ and parallel to line $2x + y + 3 = 0$
Given,
$x^2 + y^2 - 2x - 2y - 23 = 0$
Centre = (1, 1)
Radius = $\sqrt{[(1)^2 + (1)^2 + 23]} = \sqrt{(1 + 1 + 23)} = \sqrt{25} = 5$
Let $2x + y + k = 0$ be the equation of tangent parallel to $2x + y + 3 = 0$ (given).
$2x + y + k = 0$ is a tangent to the circle and the perpendicular distance from the centre of the circle (1, 1) to this line is equal to the radius 5.
$\Rightarrow |2(1) + 1 + k|/ \sqrt{(2^2 + 1^2)} = 5$
$|2 + 1 + k|/ \sqrt{(4 + 1)} = 5$
$|3 + k| = 5\sqrt{5}$
$3 + k = 5\sqrt{5}$ or $3 + k = -5\sqrt{5}$
$k = -3 + 5\sqrt{5}$ or $k = -(k + 5\sqrt{5})$
Hence, the equations of tangents are $2x + y - 3 + 5\sqrt{5} = 0$ and $2x + y - (3 + 5\sqrt{5}) = 0$.

OR

(b) Find the equation of the circle which passes through the points (2, 3), (4, 5), and the centre lies on the line $y - 4x + 3 = 0$.

Solution:
Let $x^2 + y^2 + 2gx + 2fy + c = 0$ be the equation of the circle where (-g, -f) is the centre of the circle.
Given,
The circle passes through the points (2, 3) and (4, 5).
$2^2 + 3^2 + 2g(2) + 2f(3) + c = 0$
$4g + 6f + c + 13 = 0$ ....(i)
And
$4^2 + 5^2 + 2g(4) + 2f(5) + c = 0$
$8g + 10f + c + 41 = 0$ ....(ii)
Solving (i) and (ii),
$4g + 4f + 28 = 0$
$\Rightarrow g + f + 7 = 0$ ....(iii)
Also, given that centre of circle lies on the line $y - 4x + 3 = 0$.
$-f + 4g + 3 = 0$.......(iv)
Solving (iii) and (iv),
$5g + 10 = 0$
$g = -2$
$\Rightarrow f = -7 - g = - 7 + 2 = -5$
Substituting f = -5 and g = -2 in (ii),
$8(-2) + 10(-5) + c + 41 = 0$
$-16 - 50 + c + 41 = 0$
$c = 25$
Therefore, the equation of the circle is:
$x^2 + y^2 + 2(-2)x + 2(-5)y + 25 = 0$
$x^2 + y^2 - 4x - 10y + 25 = 0$

Question 10
Differentiate the function sin (2x - 3) by the First Principle of differentiation.

Solution:
Let f(x) = sin (2x - 3)
And
f(x + h) = sin [2(x + h) - 3]
By the First Principle of differentiation,

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(2x-3+2h)-\sin(2x-3)}{h}$$

$$= \lim_{h\to 0} \frac{2}{h} \sin\left(\frac{2x-3+2h-2x+3}{2}\right) \cos\left(\frac{2x-3+2h+2x-3}{2}\right)$$

$$= \lim_{h\to 0} 2\left[\frac{\sin(h)}{h}\right] \cos(2x-3+h)$$

$$= 2\cos(2x-3)$$

Question 11
In a ΔABC, $(b^2 - c^2)/ (b^2 + c^2)$ = sin (B - C)/ sin (B + C), prove that it is either a right angled triangle or isosceles Δ.

Solution:
In a triangle ABC,
a/sin A = b/sin B = c/sin C
Let a/sin A = b/sin B = c/sin C = k
⇒ a = k sin A, b = k sin B, c = k sin C
Given,
$(b^2 - c^2)/ (b^2 + c^2)$ = sin (B - C)/ sin (B + C)
Using componendo and dividendo rule,
$(b^2 - c^2 + b^2 + c^2)/ (b^2 - c^2 - b^2 - c^2)$ = [sin (B - C) + sin (B + C)] / [sin (B - C) - sin (B + C)]
$2b^2/ (-2c^2)$ = (2 sin B cos C)/ (-2 cos B sin C)
b^2/c^2 = (sin B cos C)/ (cos B sin C)
$(k^2 \sin^2 B)/ (k^2 \sin^2 C)$ = (sin B cos C)/ (cos B sin C)
sin B/ sin C = cos C/ cos B
sin B cos B = sin C cos C
Multiplying by 2 on both sides,
2 sin B cos B = 2 sin C cos C
sin 2B = sin 2C
sin 2B - sin 2C = 0
2 cos (B + C) sin (B - C) = 0
cos (B + C) = 0 or sin (B - C) = 0
cos (B + C) = cos 90°
B + C = 90°....(i)
We know that,
A + B + C = 180°
A + 90° = 180°

A = 90°
And
sin (B - C) = sin 0
B - C = 0°
B = C....(ii)
From (i) and (ii),
B = C = 45°
Hence, ΔABC is an isosceles right angled triangle.

Question 12
(a) If 'x' be real, find the maximum and minimum value of $y = (x + 2)/ (2x^2 + 3x + 6)$.

Solution:
Let $y = f(x) = (x + 2)/ (2x^2 + 3x + 6)$
$f'(x) = [(2x^2 + 3x + 6)(1) - (x + 2)(4x + 3)]/ (2x^2 + 3x + 6)^2$
Let $f'(x) = 0$
$2x^2 + 3x + 6 - 4x^2 - 3x - 8x - 6 = 0$
$-2x^2 - 8x = 0$
$2x^2 + 8x = 0$
$2x(x + 4) = 0$
$x = 0, x = -4$
$f''(x) = d/dx [(-2x^2 - 8x)/ (2x^2 + 3x + 6)^2]$
$= [(2x^2 + 3x + 6)^2 (-4x - 8) - (-2x^2 - 8x) (2)(2x^2 + 3x + 6)(4x + 3)]/ (2x^2 + 3x + 6)^4$
At $x = 0$, $f''(x) = -(6^2 \times 8)/ 6^4 < 0$ Maxima
$x = -f$ $f''(x) > 0$ Minima
At x = 0 we will get the greatest value and at x = -4 we will get the least value.
$f(x) = (x + 2)/ (2x^2 + 3x + 6)$
Maximum value of $f(x) = (0 + 2)/ [2(0)^2 + 3(0) + 6] = 2/6 = 1/3$
Minimum value of $f(x) = (-4 + 2)/ [2(-4)^2 + 3(-4) + 6]$
$= -2/ (32 - 12 + 6)$
$= -2/26$
$= -1/13$

OR

(b) If α, β be the roots of $x^2 + lx + m = 0$, then form an equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$.

Solution:
Given,
α, β are the roots of $x^2 + lx + m = 0$.
Sum of roots = $\alpha + \beta = -l/1 = -l$
Product of roots = $\alpha\beta = m/1 = m$
Now,
$(\alpha + \beta)^2 = (-l)^2 = l^2$
$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = l^2 - 4m$
$(\alpha + \beta)^2 + (\alpha - \beta)^2 = l^2 + l^2 - 4m = 2l^2 - 4m$
$(\alpha + \beta)^2\ (\alpha - \beta)^2 = l^2(l^2 - 4m) = l^4 - 4l^2m$
The quadratic equation with roots $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ is:
x^2 - (sum of the roots)x + (product of the roots) = 0
$x^2 - (2l^2 - 4m)x + l^4 - 4l^2m = 0$

Question 13

(a) The sum of three consecutive numbers of a G.P is 56. If we subtract 1, 7 and 21 from these numbers in the order, the resulting numbers form an A.P., find the numbers.

Solution:

Let a , ar , ar^2 be the three numbers in GP.

Given,

$a + ar + ar^2 = 56$

$a(1 + r + r^2) = 56$….(i)

Also,

$(a - 1)$, $(ar - 7)$, $(ar^2 - 21)$ are in AP.

Thus, the common difference is always constant.

$(ar^2 - 21) - (ar - 7) = (ar - 7) - (a - 1)$

$ar^2 - ar - 21 + 7 = ar - a - 7 + 1$

$ar^2 - 2ar + a - 8 = 0$

$a(r^2 - 2r + 1) = 8$….(ii)

Dividing (i) by (ii),

$a(1 + r + r^2)/ [a(r^2 - 2r + 1)] = 56/8$

$(1 + r + r^2)/(1 - 2r + r^2) = 7$

$1 + r + r^2 = 7 - 14r + 7r^2$

$6r^2 - 15r + 6 = 0$

$2r^2 - 5r + 2 = 0$

$2r^2 - 4r - r + 2 = 0$

$2r(r - 2) -(r - 2) = 0$

$(2r - 1)(r - 2) = 0$

$r = 2, 1/2$

If r = 2 then, equ (i) becomes,

$a(1 + 2 + 4) = 56$

$7a = 56$

$a = 8$

Therefore, numbers are:

$a = 8$

$ar = 8 \times 2 = 16$

$ar^2 = 8 \times 2^2 = 32$

Similarly, if $r = ½$,

$a[1 + (½) + (¼)] = 56$

$7a/4 = 56$

$a = 8 \times 4$

$a = 32$

Therefore, the numbers are:

$a = 32$

$ar = 32 \times (½) = 16$

$ar^2 = 32 \times (1/2^2) = 32/4 = 8$

Hence, the numbers are 32, 16 , 8 or 8, 16, 32.

OR

(b) Find the sum of the series to n terms:

$$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + \ldots\ldots\ldots\ldots\ldots\ldots \text{ n terms.}$$

Solution:

$1^3/1 + (1^3 + 2^3)/(1 + 3) + (1^3 + 2^3 + 3^3)/(1 + 3 + 5) + \ldots..$

T_n = (nth term of the numerator) /(nth term of the denominator)

$= (1^3 + 2^3 + 3^3 + \ldots\ldots$ n terms) / $(1 + 3 + 5 + \ldots.$ n terms)

nth term of numerator $= 1^3 + 2^3 + 3^3 + \ldots..$ n terms

$= \sum n^3 = [n(n + 1)/2]^2$

nth term of denominator $= 1 + 3 + 5 + \ldots\ldots +$ n terms

$= n^2$ (sum of first n odd numbers)

$T_n = n^2(n + 1)^2/4n^2 = (n + 1)^2/4$

$S_n = \sum T_n$

$=1/4 \sum(n + 1)^2$

$= 1/4 (\sum n^2 + 2\sum n + \sum 1)$

$S_n = 1/4 [\{n(n + 1)(2n + 1)/6\} + \{2n(n + 1)/2\} + n]$

$= 1/4 [\{n(n + 1)(2n + 1)/6\} + n(n + 1) + n]$

$= n/24 [2n^2 + n + 2n + 1 + 6n + 6 + 6]$

$= n/24 [2n^2 + 9n + 13]$

Hence, sum of the given series to n terms $= n/24 [2n^2 + 9n + 13]$

Question 14

Find the mean, standard derivation for the following data:

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	2	3	5	10	3	5	2

Solution:

Class	Frequency (f_i)	Class mark (x_i)	f_ix_i	(x_i - mean) = (x_i - 35.67)	(x_i - mean)^2	f_i(x_i - mean)^2
0 - 10	2	5	10	-30.67	940.649	1881.298
10 - 20	3	15	45	-20.67	427.249	1281.747
20 - 30	5	25	125	-10.67	113.849	569.245
30 - 40	10	35	350	-0.67	0.4489	4.489
40 - 50	3	45	135	9.33	87.049	261.147
50 - 60	5	55	275	19.33	373.649	1868.245
60 - 70	2	65	130	29.33	860.249	1720.498

Total	$\sum f_i = 30$		$\sum f_i x_i = 1070$			7856.669

Mean = $\sum f_i x_i / \sum f_i$
= 1070/ 30
= 35.67
Variance = 1/N $[\sum f_i(x_i - \text{mean})^2]$
= 1/30 (7856.669)
= 261.89
Standard deviation = √261.89 = 16.18

SECTION B

Question 15
(a) Find the coordinates of a point on the parabola $y^2 = 8x$, whose focal distance is 4.

Solution:
Given,
Equation of parabola is $y^2 = 8x$
Comparing with $y^2 = 4ax$,
4a = 8
a = 2
Thus, the focus is S = (2, 0)
Let P(x, y) be any point on the parabola.
We know that,
Distance of any point on the parabola from the focus = Focal distance
$\sqrt{[(1 - 2)^2 + (y - 0)^2]} = 4$ (given)
$\sqrt{(x^2 + 4 - 4x + y^2)} = 4$
$\sqrt{(x^2 + 4 - 4x + 8x)} = 4$ [since $y^2 = 8x$]
$\sqrt{(x^2 + 4x + 4)} = 4$
$\sqrt{(x + 2)^2} = 4$
x + 2 = ±4
x + 2 = 4, x1 + 2 = -4
x = 2, x1 = -6
x = -6 is not possible.
Therefore, x = 2
Now,
$y^2 = 8(2)$
$y^2 = 16$
y = ±4
Hence, the coordinates of the required points are (2, 4) and (2, -4).

(b) Prove that: $\sim(p \Rightarrow q) = p \wedge (\sim q)$

Solution:

p	q	p ⇒ q	~(p ⇒ q)	~q	p^(~q)
T	T	T	F	F	F

T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

Therefore, $\sim(p \Rightarrow q) = p \wedge (\sim q)$

(c) Write Converse and inverse of the given conditional statement:
If a number n is even, then n^2 is even.

Solution:
Given,
If a number n is even, then n^2 is even.
Converse of the given conditional statement is:
If a number n^2 is even, then n is even.
Inverse of the given conditional statement is:
If a number n is not even, then n^2 is not even.

Question 16
(a) Find the equation of the ellipse whose focus (1, 2), directrix 3x + 4y - 5 = 0 and eccentricity is 1/2.

Solution:
Given,
Focus = S(1, 2)
Directrix is $3x + 4y - 5 = 0$
Eccentricity (e) = 1/2
Let P(x, y) be any point on the ellipse.
We know that,
The perpendicular distance from the point (x_1, y_1) to the line $ax + by + c = 0 = |ax1 + by1 + c|/ \sqrt{(a^2 + b^2)}$
Here,
$(x_1, y_1) = (x, y)$
a = 3, b = 4, c = -5
And
SP = e (perpendicular distance)
$SP^2 = e^2$ (perpendicular distance)^2
$(x - 1)^2 + (y - 2)^2 = (1/2)^2 [|3x + 4y - 5|/ \sqrt{(3^2 + 4^2)}]^2$
$x^2 - 2x + 1 + y^2 - 4y + 4 = (1/4) [|3x + 4y - 5|^2/ (9 + 16)]$
$x^2 + y^2 - 2x - 4y + 5 = (1/100) (9x^2 + 16y^2 + 25 + 24xy - 40y - 15x)$
$100x^2 + 100y^2 - 200x - 400y + 500 - 9x^2 - 16y^2 - 25 - 24xy + 40y + 15x = 0$
$91x^2 + 84y^2 - 185x - 360y - 24xy + 475 = 0$
Therefore, the equation of the ellipse is $91x^2 + 84y^2 - 24xy - 185x - 360y + 475 = 0$.

OR

(b) Find the centre, focus, eccentricity and latus rectum of the hyperbola $16x^2 - 9y^2 = 144$.

Solution:
Given,

Equation of hyperbola is:
$16x^2 - 9y^2 = 144$
$(16x^2/144) - (9y^2/144) = 1$
$(x^2/9) - (y^2/16) = 1$
This is of the form $(x^2/a^2) - (y^2/b^2) = 1$
Therefore, the centre is (0, 0).
Now,
$a^2 = 9$, $b^2 = 16$
$b^2 = a^2(e^2 - 1)$
$16 = 9(e^2 - 1)$
$e^2 - 1 = 16/9$
$e^2 = (16/9) + 1$
$e^2 = 25/9$
$e = 5/3$
Coordinates of foci = $(\pm ae, 0) = (\pm 5, 0)$
Eccentricity = e = 5/3
Latus rectum = $2b^2/a = 2(16)/ 3$
= 32/3

Question 17
(a) In what ratio the point P(-2, y, z) divides the line joining the points A(2, 4, 3) and B(-4, 5, -6). Also, find the coordinates of point P.

Solution:
Let P(-2, y, z) divide the line joining the points A(2, 4, 3) and B(-4, 5, -6) in the ratio m : n.
Here,
$(x_1, y_1, z_1) = (2, 4, 3)$
$(x_2, y_2, z_2) = (-4, 5, -6)$
Using section formula,
$P(x, y, z) = [(mx_2 + nx_1)/ (m + n), (my_2 + ny_1)/ (m + n), (mz_2 + nz_1)/ (m + n)]$
(-2, y, z) = [(-4m + 2n)/ (m + n), (5m + 4n)/ (m + n), (-6m + 3n)/ (m + n)]
By equating the corresponding coordinates,
-4m + 2n = -2(m + n)
-4m + 2n = -2m - 2n
2n + 2n = -2m + 4m
4n = 2m
⇒ 2m = 4n
⇒ m/n = 4/2
⇒ m : n = 2 : 1
And
y = (5m + 4n)/ (m + n)
= (10 + 4)/ (2 + 1)
= 14/3
z = (-6m + 3n)/ (m + n)
= (-12 + 3)/ (2 + 1)
= -9/3
= -3
Hence, the required ratio is 2 : 1 and the coordinates of P are (-2, 14/3, -3).

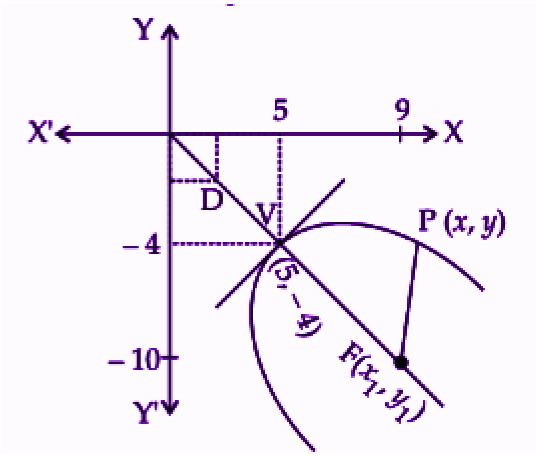
OR

(b) If the origin is the centroid of the triangle with vertices (-4, 2, 6) (2a, 3b, 2c) and (8, 14, -10) find the values of a, b and c.

Solution:
Let the given vertices of a triangle be:
$(-4, 2, 6) = (x_1, y_1, z_1)$
$(2a, 3b, 2c) = (x_2, y_2, z_2)$
$(8, 14, -10) = (x_3, y_3, z_3)$
Centroid = $[(x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3, (z_1 + z_2 + z_3)/3]$
= [(-4 + 2a + 8)/3, (2 + 3b + 14)/3, (6 + 2c - 10)/3]
= [(4 + 2a)/3, (16 + 3b)/3, (-4 + 2c)/3]
According to the given,
[(4 + 2a)/3, (16 + 3b)/3, (-4 + 2c)/3] = (0, 0, 0)
By equating the corresponding coordinates,
(4 + 2a)/3 = 0
4 + 2a = 0
2a = -4
a = -4/2 = -2
Now,
(16 + 3b)/3 = 0
16 + 3b = 0
3b = -16
b = -16/3
And
(-4 + 2c)/3 = 0
-4 + 2c = 0
2c = 4
c = 4/2 = 2
Therefore, a = -2, b = -16/3, and c = 2.

Question 18
Find the equation of Parabola whose directrix is 2x - 3y + 4 = 0 and vertex at (5 - 4).

Solution:
Given,
Vertex (V) = (5, -4)
Directrix is 2x - 3y + 4 = 0
Let P(x, y) be any point on the parabola and $F(x_1, y_1)$ be the focus.
Let us find the equation of the axis of parabola, which is perpendicular to the given directrix.
Thus, the equation which is perpendicular to the give directrix is $3x + 2y + \lambda = 0$
This line passes through the vertex (5, -4).
$3(5) + 2(-4) + \lambda = 0$
$15 - 8 + \lambda = 0$
$7 + \lambda = 0$
$\lambda = -7$
Therefore, the equation of the axis of the parabola is 3x + 2y - 7 = 0.



The point of intersection of the given directrix and the axis of parabola is D= (1, 2).
Vertex = Midpoint of directrix and focus
$(5, -4) = [(1 + x_1)/2, (2 + y_1)/2]$
$\Rightarrow 1 + x_1 = 10, 2 + y_1 = -8$
$\Rightarrow x_1 = 9, y_1 = -10$
$\Rightarrow F(x_1, y_1) = (9, -10)$
We know that,
$PF^2 = PM^2$
$(x - 9)^2 + (y + 10)^2 = [|2x - 3y + 4|/\sqrt{\{2^2 + (-3)^2\}}]^2$
$x^2 + 81 - 18x + y^2 + 100 + 20y = (4x^2 + 9y^2 + 14 - 12xy - 24y + 16x)/13$
$13x^2 + 1053 - 234x + 13y^2 + 1300 + 260y = 4x^2 + 9y^2 + 14 - 12xy - 24y + 16x$
$13x^2 + 1053 - 234x + 13y^2 + 1300 + 260y - 4x^2 - 9y^2 - 14 + 12xy + 24y - 16x$
$9x^2 + 4y^2 + 12xy - 250x + 284y + 2337 = 0$

SECTION C

Question 19
(a) The mean weight of 150 students in a certain class is 60 kg. The mean weight of boys is 70 kg and that of girls in the class is 55 kg. Find the number of boys and girls in the class.

Solution:
Given,
Mean weight of 150 students = 60 kg
Mean weight of boys = 70 kg
Mean weight of girls = 55 kg
Let x and y be the number of boys and girls in the class respectively.
Thus, x + y = 150
x = 150 - y….(i)
According to the given,
60 = (70x + 55y)/ 150
70x + 55y = 60 × 150
70(150 - y) + 55y = 9000 [From (i)]
10500 - 70y + 55y = 9000
10500 - 9000 = 15y
15y = 1500
y = 1500/ 15
y = 100

Substituting y = 100 in (i),
x = 150 - 100 = 50
Hence, the number of boys are 50 and the number of girls are 100.

(b) Compute D_3 and D_7 for the following distribution:

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
No. of students	3	10	17	7	6	4	2	1

Solution:

Marks	No. of students (frequency)	Cumulative frequency
0 - 10	3	3
10 - 20	10	13
20 - 30	17	30
30 - 40	7	37
40 - 50	6	43
50 - 60	4	47
60 - 70	2	49
70 - 80	1	50

Here,
N = 50
3N/10 = (3 × 50)/10 = 15
Cumulative frequency greater than and nearest to 15 is 30 which lies in the interval 20 - 30.
l = 20
h = 10
f = 17
cf = 13
D_3 = l + [{(3N/10) - cf}/ f] × h
= 20 + [(15 - 13)/ 17] × 10
= 20 + (20/17)
= 20 + 1.176
= 21.176

7N/10 = (7 × 50)/10 = 35
Cumulative frequency greater than and nearest to 35 is 37 which lies in the interval 30 - 40.
l = 30
h = 10

$f = 7$
$cf = 30$
$D_7 = l + [\{(7N/10) - cf\}/ f] \times h$
$= 30 + [(35 - 30)/ 7] \times 10$
$= 30 + (50/7)$
$= 30 + 7.14$
$= 37.14$

OR

Calculate the mode from the following data:

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of students	5	15	30	8	2

Solution:
From the given data,
Maximum frequency = 30
Modal class is 20 - 30
Frequency of the modal class = $f_1 = 30$
Frequency of the class preceding the modal class = $f_0 = 15$
Frequency of the class succeeding the modal class = $f_2 = 8$
Lower limit of the modal class = $l = 20$
Class height = $h = 10$
Mode = $l + [(f_1 - f_0)/ (2f_1 - f_0 - f_2)] \times h$
$= 20 + [(30 - 15)/ (2 \times 30 - 15 - 8)] \times 10$
$= 20 + [15/ (60 - 23)] \times 10$
$= 20 + (150/ 37)$
$= 20 + 4.054$
$= 24.054$

Question 20
(a) In a sample of 'n' observations given that $\sum d^2 = 55$ and rank correlation $r = 2/3$, then find the value of 'n'.

Solution:
Given,
$\sum d^2 = 55$
Rank correlation = $r = 2/3$
We know that,
Rank correlation = $1 - [(6 \times \sum d^2)/ (n(n^2 - 1)$
$2/3 = 1 - [(6 \times 55)/ (n(n^2 - 1)]$
$\Rightarrow 330/ n(n^2 - 1) = 1 - (2/3)$
$\Rightarrow 330/ n(n^2 - 1) = 1/3$
$\Rightarrow n(n^2 - 1) = 330 \times 3$
$\Rightarrow n(n^2 - 1) = 990$
$\Rightarrow n(n^2 - 1) = 10 \times 99$
$\Rightarrow n(n^2 - 1) = 10 \times (100 - 1)$

$\Rightarrow n(n^2 - 1) = 10 \times [(10)^2 - 1]$
$\Rightarrow n = 10$

(b) Find the correlation coefficient r(x, y) if:
$n = 10, \sum x = 60, \sum y = 60, \sum x^2 = 400, \sum y^2 = 580, \sum xy = 305$

Solution:
Given,
n = 10
$\sum x = 60$
$\sum y = 60$
$\sum x^2 = 400$
$\sum y^2 = 580$
$\sum xy = 305$
Correlation coefficient is:

$$r(x, y) = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{\left[n\Sigma x^2 - (\Sigma x)^2\right]}\sqrt{\left[n\Sigma y^2 - (\Sigma y)^2\right]}}$$

$$= \frac{10 \times 305 - 60 \times 60}{\sqrt{10 \times 400 - (60)^2}\sqrt{10 \times 580 - (60)^2}}$$

$$= \frac{3050 - 3600}{\sqrt{4000 - 3600}\sqrt{5800 - 3600}}$$

$= -550/ [\sqrt{400}\ \sqrt{2200}]$
$= -550/ [20 \times 10\sqrt{22}]$
$= -55/ (20 \times 4.69)$
$= -55/93.8$
$= -0.586$

OR

Ten students got the following percentages of marks in Mathematics and Physics:

Mathematics	56	64	75	85	85	87	91	95	97	98
Physics	89	90	86	74	78	66	56	74	86	90

Find the Spearman's rank correlation coefficient for the above data.

Solution:

Mathematics	Rank (R_1)	Physics	Rank (R_2)	$d = R_1 - R_2$	d^2
56	10	89	3	7	49
64	9	90	1.5	7.5	56.25
75	8	86	4.5	3.5	12.25
85	6.5	74	7.5	-1	1
85	6.5	78	6	0.5	0.25
87	5	66	9	-4	16
91	4	56	10	-6	36
95	3	74	7.5	-4.5	20.25
97	2	86	4.5	-2.5	6.25
98	1	90	1.5	-0.5	0.25

n = 10
$\sum d^2 = 197.5$
The spearman's rank correlation coefficient = $1 - [(6 \times \sum d^2)/ n(n^2 - 1)]$
$= 1 - [(6 \times 197.5)/ 10(100 - 1)]$
$= 1 - [1185/ 10 \times 99]$
$= 1 - (1185/990)$
$= (990 - 1185)/ 990$
$= -195/990$
$= -0.197$

Question 21
Calculate the index number for the year 1990 with respect to 1980 as base from the following data using weighted average of price relatives:

Commodity	Weights	Year 1990	Year 1980
A	22	320	200
B	48	120	100
C	17	20	28
D	13	60	40

Solution:

Commodity	Weights (w)	Year 1990 (p1)	Year 1980 (p0)	Price relative	Iw

				I = (p1/p0) × 100	
A	22	320	200	160	3520
B	48	120	100	120	5760
C	17	20	28	71.43	1214.31
D	13	60	40	150	1950
Total	∑w = 100				∑Iw = 12444.31

Index number of weighted average of price relatives = ∑Iw/ ∑w
= 12444.31/100
= 124.44

Question 22
The number of letters, in hundreds, posted in a certain city on each day for a week is given below:

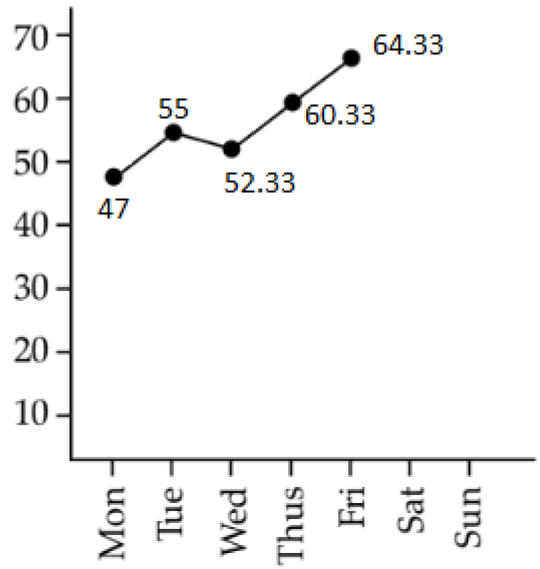
Mon.	Tue.	Wed.	Thur.	Fri.	Sat.	Sun.
35	70	36	59	62	60	71

Calculate 3 day moving averages and represent these graphically.

Solution:

Day	Number of letters (in 00's)	3 days moving average
Mon	35	
Tue	70	(35 + 70 + 36)/3 = 141/3 = 47
Wed	36	(70 + 36 + 59)/3 = 165/3 = 55
Thur	59	(36 + 59 + 62)/3 = 157/3 = 52.33
Fri	62	(59 + 62 + 60)/3 = 181/3 = 60.33
Sat	60	(62 + 60 + 71)/3 = 193/3 = 64.33
Sun	71	

Graph:


70
60
50
40
30
20
10
64.33
55
60.33
52.33
47
Mon
Tue
Wed
Thus
Fri
Sat
Sun