KBPE Class 12th Maths Question Paper With Solution 2016

QUESTION PAPER CODE 1018

Question 1[a]: The function f:N → N, given by f (x) = 2x is

(a) one-one but onto

(b) one-one but not onto

(c) not one-one not onto

(d) onto but not one-one

Answer: [b]

[b]: Find g o f (x), if f (x) = $8x^3$ and g (x) = $x^{1/3}$.

Solution:

g o f (x) = g (f (x))

= g ($8x^3$)

= $(8x^3)^{1/3}$

= 2x

[c]: Let * be an operation such that a * b = LCM of a and b defined on the set A = {1, 2, 3, 4, 5}. Is * a binary operation? Justify your answer.

Solution:

a * b = LCM of a and b

a, b ∈ {1, 2, 3, 4, 5}

Let a = 2, b = 3

a * b = 2 * 3 = LCM of 2 and 3 is 6

Since the element 6 doesn't exist in the set {1, 2, 3, 4, 5}, * is not a binary operation.

Question 2[a]: If xy < 1, $\tan^{-1} x + \tan^{-1} y$ is ______.

Solution:
$\tan^{-1} [(x + y) / (1 - xy)]$

[b] Prove that $2 \tan^{-1} (1 / 2) + \tan^{-1} (1 / 7) = \tan^{-1} (31 / 17)$

Solution:
$2 \tan^{-1} (1 / 2) + \tan^{-1} (1 / 7)$
$= \tan^{-1} [(2 * (1 / 2)) / (1 - (1 / 2)^2] + \tan^{-1} (1 / 7)$
$= \tan^{-1} (4 / 3) + \tan^{-1} (1 / 7)$
$= \tan^{-1} [((4 / 3) + (1 / 7)) / (1 - (4 / 3) (1 / 7)]$
$= \tan^{-1} [(31 * 21) / (21 * 17)]$
$= \tan^{-1} (31 / 17)$

Question 3[a]: If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$ **then BA =**

(i) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (iv) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Solution: [iii]

[b] Write $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ **as the sum of a symmetric and a skew-symmetric matrix.**

Solution:

$$A^T = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$A = \frac{1}{2}[A + A^T] + \frac{1}{2}[A - A^T]$$

$$= (\frac{1}{2}) \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} + (\frac{1}{2}) \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

[c] Find the inverse of $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$.

Solution:

Write the augmented matrix

	A_1	A_2	B_1	B_2
1	2	-6	1	0
2	1	-2	0	1

Find the pivot in the 1st column and swap the 2nd and the 1st rows

	A_1	A_2	B_1	B_2
1	1	-2	0	1
2	2	-6	1	0

Eliminate the 1st column

	A_1	A_2	B_1	B_2
1	1	-2	0	1
2	0	-2	1	-2

Make the pivot in the 2nd column by dividing the 2nd row by -2

	A_1	A_2	B_1	B_2
1	1	-2	0	1
2	0	1	-1/2	1

Eliminate the 2nd column

	A_1	A_2	B_1	B_2
1	1	0	-1	3
2	0	1	-1/2	1

There is the inverse matrix on the right

	A_1	A_2	B_1	B_2
1	1	0	-1	3
2	0	1	-1/2	1

Question 4[a]: The value of $\begin{vmatrix} x & x-1 \\ x+1 & x \end{vmatrix}$ is

(i) 1 (ii) x (iii) x^2 (iv) 0

Answer: [i]

[b] Using the properties of determinants, show that

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

Solution:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$$R_1 \to R_1 + R_2 + R_3 = (1+x+x^2)\begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$$= (1+x+x^2)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1-x^2 & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix}$$

$$C_2 \to C_2 - C_1; C_3 \to C_3 - C_1 = (1+x+x^2)(1-x)(1-x)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix}$$

$$= (1+x+x^2)(1-x)(1-x)(1+x+x^2) = (1-x^3)^2$$

Question 5[a]: Find all the points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} 2x+3, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$$

Solution:

LHL = $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 2x + 3 = 7$

RHL = $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 2x - 3 = 1$

$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

LHL ≠ RHL

2 is a point of discontinuity.

[b] If $e^{x-y} = x^y$, then prove that dy / dx = log x / $(\log ex)^2$.

Solution:

(x - y) log e = y log x

x - y = y log x

x = y log x + y

x = y (log x + 1)

y = x / (log x + 1)

dy / dx = (1 + log x - 1) / $(1 + \log x)^2$ = log x / $(1 + \log x)^2$

= log x / $(\log ex)^2$

Question 6[a]: The slope of the tangent to the curve given by x = 1 - cos θ, y = θ - sin θ at θ = π / 2 is

(i) 0 (ii) -1 (iii) 1 (iv) Not defined

Solution: (ii)

[b] Find the intervals in which the function f (x) = x^2 - 4x + 6 is strictly decreasing.

Solution:

f ‘ (x) = 2x - 4

f ‘ (x) = 0

2x - 4 = 0

x = 2

The intervals are $(-\infty, 2)$ and $(2, \infty)$.

f (x) is decreasing at $(-\infty, 2)$.

[c] Find the minimum and maximum value, if any, of the function $f(x) = (2x - 1)^2 + 3$.

Solution:

$f(x) = (2x - 1)^2 + 3$

$f'(x) = 4(2x - 1)$

$f''(x) = 8$

$f'(x) = 0$

$2x - 1 = 0$

$x = 1 / 2$

f (x) has minimum value at $x = 1 / 2$.

OR

[a] Which of the following functions has neither local maxima nor local minima?

(i) $f(x) = x^2 + x$ (ii) $f(x) = \log x$

(iii) $f(x) = x^3 - 3x + 3$ (iv) $f(x) = 3 + |x|$

Solution: [ii]

[b] Find the equation of the tangent to the curve $y = 3x^2$ at (1, 1).

Solution:

$y = 3x^2$

$dy / dx = 6x$

Slope at (1, 1) = 6

Equation of the tangent is $y - y_1 = (dy / dx)_{(1, 1)} (x - x_1)$

$y - y_1 = 6(x - 1)$

$y = 6x + 5$

[c] Use differential to approximate $\sqrt{36.6}$.

Solution:

$y = \sqrt{x}$

$x = 36$

$\Delta x = 0.6$

$dy = (dy / dx) \Delta x = 1 / 2 * \sqrt{x}$

$= 1 / 2 * (0.6)$

$= 0.05$

$\sqrt{36.6} = 6 + 0.05 = 6.05$

Question 7[a]: The angle between the vectors a and b such that |a| = |b| = √2 and a . b = 1 is

(i) **π** / 2 (ii) **π** / 3 (iii) **π** / 4 (iv) 0

Solution: [ii]

[b] Find the vector along a - b where a = i + 3j - k and b = 3i + 2j + k.

Solution:

Unit vector along a - b = (a - b) / |a - b|

a - b = -2i + j - 2k

$|a - b| = \sqrt{9} = 3$

Unit vector = (1 / 3) (-2i + j - 2k)

Question 8[a]: If the points A and B are (1, 2, -1) and (2, 1, -1) respectively, then AB is

(i) i + j (ii) i - j (iii) 2i + j - k (iv) i + j + k

Solution: [ii]

[b] Find the value of λ for which the vectors 2i - 4j + 5k, i - λj + k and 3i + 2j - 5k are coplanar.

Solution:

$$\begin{vmatrix} 2 & -4 & 5 \\ 1 & -\lambda & 1 \\ 3 & 2 & -5 \end{vmatrix} = 0$$

$2 (5\lambda - 2) + 4 (- 5 - 3) + 5 (2 + 3\lambda) = 0$

$\lambda = 26 / 25$

[c] Find the angle between the vectors a = i + j - k and b = i - j + k.

Solution:

$\cos \theta = (a . b) / |a| |b|$

$(a . b) = -1$

$|a| = \sqrt{3}$

$|b| = \sqrt{3}$

$\cos \theta = - 1 / 3$

$\theta = \cos^{-1} (- 1 / 3)$

Question 9[a]: Prove that $\int \cos^2 x \, dx = (x / 2) + (\sin 2x / 4) + c$.

Solution:

$\cos^2 x = (1 + \cos 2x) / 2$

$\int \cos^2 x \, dx = (1 / 2) \int (1 + \cos 2x) \, dx$

$= (1 / 2) [x + (\sin 2x / 2)] \, dx$

[b] Find $\int dx / \sqrt{2x - x^2}$.

Solution:

$2x - x^2 = - (x^2 - 2x)$

$= - (x^2 - 2x + 1 - 1)$

$= - [(x - 1)^2 - 1^2]$

$= 1 - (x - 1)^2$

$\int dx / \sqrt{2x - x^2} = \int dx / \sqrt{1 - (x - 1)^2}$

$= \sin^{-1} (x - 1) + c$

[c] Find $\int x \cos x \, dx$.

Solution:

$\int x \cos x \, dx$

$= x \sin x - \int \sin x \, dx$

$= x \sin x + \cos x + c$

Question 10[a]: Evaluate $\int_0^{\pi} \log (1 + \cos x) \, dx$.

Solution:

$I = \int_0^{\pi} \log (1 + \cos x) \, dx$

$= \int_0^{\pi} \log (1 + \cos [\pi - x] \, dx)$

$= \int_0^{\pi} \log (1 - \cos x) \, dx$

$= \int_0^{\pi} \log 2 \sin^2 [x / 2] \, dx$

$= \int_0^{\pi} (\log 2 + 2 \log \sin (x / 2)) \, dx$

$= \int_0^{\pi} \log 2 \, dx + 2 \int_0^{\pi} \log \sin (x / 2) \, dx$

Put $x / 2 = t$

$dx = 2 \, dt$

When $x = 0$, $t = 0$ and $x = \pi$, $t = \pi / 2$

$I = \log 2 \, (x)_0^{\pi} + 2 \int_0^{\pi/2} \log \sin t * 2 \, dt$

$= \log 2 \, (\pi - 0) + 4 (- \pi / 2 \log 2)$

$= - \pi \log 2$

OR

[b] Find $\int_0^5 (x + 1) \, dx$ as limit of sum.

Solution:

$\int_a^b f(x) \, dx = \lim_{h \to 0} h \, [f(a) + f(a + h) + \ldots.. + f(a + (n - 1)h]$

$\int_0^5 (x + 1) \, dx = \lim_{h \to 0} h \, [1 + (1 + h) + (1 + 2h) + \ldots.. 1 + (n - 1)h]$

$= \lim_{h \to 0} h \, [n + h (1 + 2 + 3 \ldots.. + (n - 1))$

$= \lim_{h \to 0} h \, [n + h * n (n - 1) / 2]$

$= \lim_{h \to 0} [nh + (n^2h^2 - h^2n) / 2]$

$= \lim_{h \to 0} [nh + nh (nh - h) / 2]$

$= \lim_{h \to 0} (5 + [5 - h]5 / 2]$

$= 35 / 2$

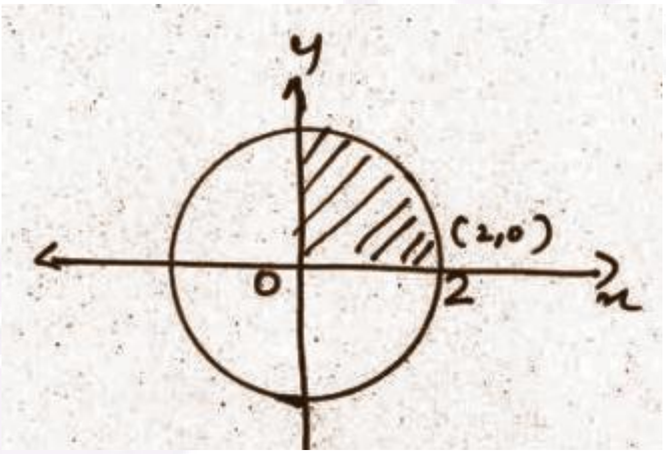
Question 11[a]: The area bounded by the curve y = f (x), above the x-axis, between x = a and x = b is

(i) $\int_{f(a)}^{b} y\, dy$ (ii) $\int_{a}^{f(b)} x\, dx$ (iii) $\int_{a}^{b} x\, dy$ (iv) $\int_{a}^{b} y\, dx$

Solution: [iv]

[b] Find the area of the circle $x^2 + y^2 = 4$ using integration.

Solution:



Area of the shaded region $= \int_0^2 y\, dx$

$= \int_0^2 \sqrt{4 - x^2}\, dx$

$= [(x / 2) \sqrt{4 - x^2} + (4 / 2) \sin^{-1} (x / 2)\, dx$

$= (4 / 2) \sin^{-1} (1)$

$= 2 * (\pi / 2)$

$= 4\pi$

Question 12[a]: y = cos x + b sin x is the solution of the differential equation.

(i) $d^2y / dx^2 + y = 0$ (ii) $d^2y / dx^2 - y = 0$

(iii) $dy / dx + y = 0$ (iv) $dy / dx + x (dy / dx) = 0$

Solution: [i]

[b] Find the solution of the differential equation $x (dy / dx) + 2y = x^2$ $(x \neq 0)$ given that $y = 0$ when $x = 1$.

Solution:

$(dy / dx) + (2y / x) = x$

$P = (2 / x)$, $Q = x$

$IF = e^{\int P\, dx}$

$= e^{(2 \log x)}$

$= x^2$

The solution is $y = x^2 = \int x * x^2\, dx = \int x^3\, dx$

$yx^2 = x^4 / 4 + c$

When $x = 1$, $y = 0$,

$0 = (1 / 4) + c$

$c = (- 1 / 4)$

The particular solution is $yx^2 = (x^4 / 4) + (- 1 / 4)$

Question 13[a]: Find the shortest distance between the lines $r = i + j + \lambda (2i - j + k)$ and $r = 2i + j - k + \mu (3i - 5j + 2k)$.

Solution:

Shortest distance = $|[(a_2 - a_1) . (b_1 - b_2)] / |[b_1 \times b_2]||$

$a_1 = i + j$

$a_2 = 2i + j - k$

$b_1 = 2i - j + k$

$b_2 = 3i - 5j + 2k$

$(a_2 - a_1) = i - k$

$[b_1 \times b_2] = 3i - j + 7k$

$|[b_1 \times b_2]| = \sqrt{59}$

Shortest distance = $10 / \sqrt{59}$

Question 14[a]: Equation of the plane with intercepts 2, 3, 4 on the x, y and z-axis respectively is

(i) $2x + 3y + 4z = 1$ (ii) $2x + 3y + 4z = 12$

(iii) $6x + 4y + 3z = 1$ (iv) $6x + 4y + 3z = 12$

Solution: [iv]

[b] Find the cartesian equation of the plane passing through the points A (2, 5, -3), B (- 2, - 3, 5) and C (5, 3, - 3).

Solution:

The equation is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - 2 & y - 5 & z + 3 \\ -2 - 2 & -3 - 5 & 5 + 3 \\ 5 - 2 & 3 - 5 & -3 + 3 \end{vmatrix} = 0$$

$(x - 2)\, 16 + (y - 5)\, 24 + (z + 3)\, 32 = 0$

$2x + 3y + 4z = 7$

Question 15[a]: Consider the following LPP:

Maximise Z = 3x + 2y

Subject to constraints

$x + 2y \leq 10$

$3x + y \leq 15$

$x, y \geq 0$

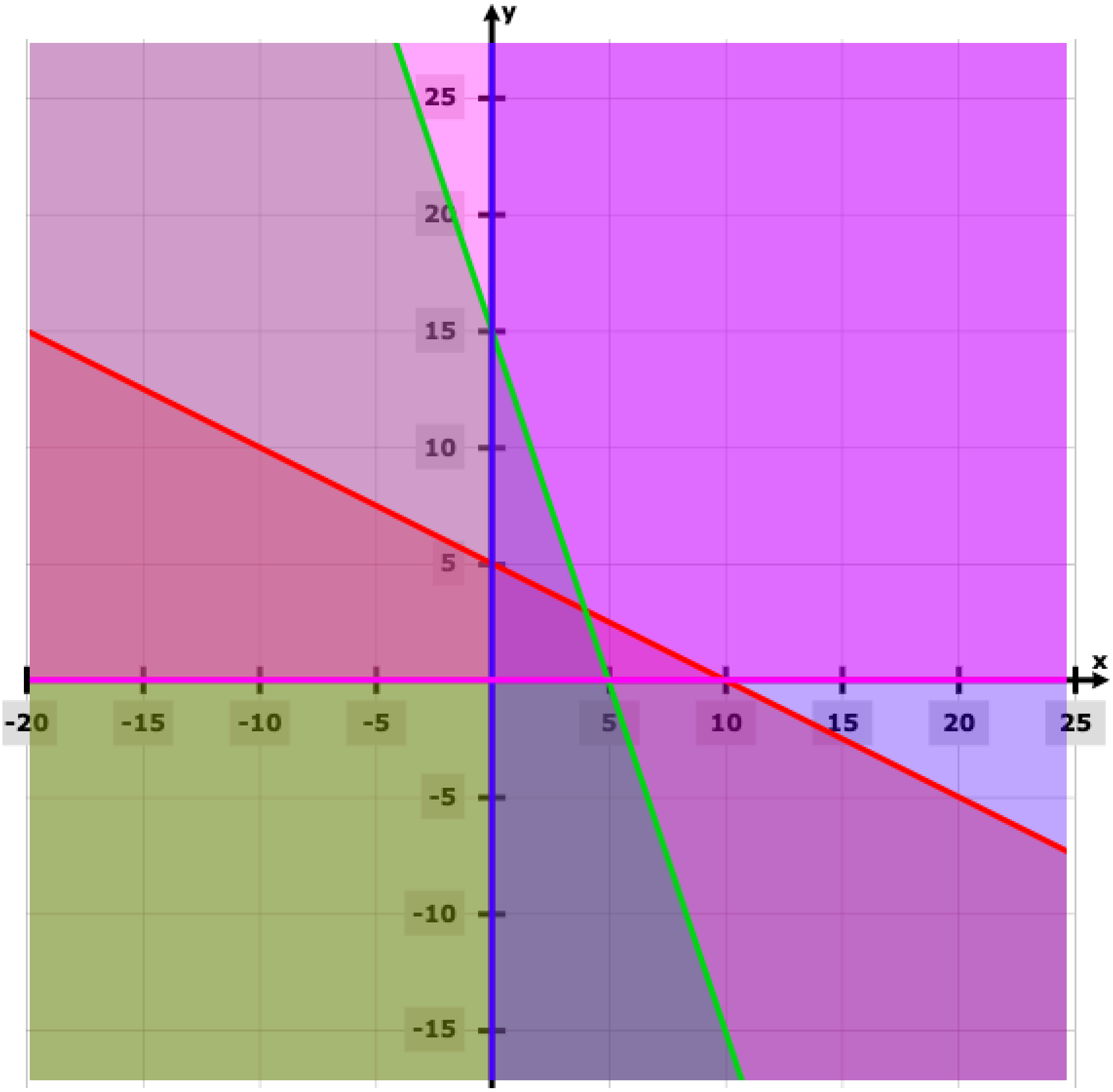
[a] Draw the feasible region.

[b] Find the corner points of the feasible region.

[c] Find the maximum value of Z.

Solution:

[a]



[b] The corner points are (0, 0), (4, 3), (0, 5).
[c] Z is maximum at (4, 3) = 18

Question 16[a]: If P (A) = 0.3, P (B) = 0.4, then the value of P (A ∪ B) where A and B are independent events is

(i) 0.48 (ii) 0.51 (iii) 0.52 (iv) 0.58

Solution: [iv]

[b] A card from a pack of 52 cards is lost. From the remaining cards of the pack, 2 cards are drawn and are found to be diamonds. Find the probability of the lost card being a diamond.

Solution:

E_1: The lost card is a diamond.

E_2: The lost card is not a diamond.

A: Selecting 2 diamonds from the remaining cards

$P(E_1) = 13 / 52 = 1 / 4$

$P(E_2) = 39 / 52 = 3 / 4$

$P(A / E_1) = {}^{12}C_2 / {}^{51}C_2 = 12 * 11 / 51 * 50$

$P(A / E_2) = {}^{13}C_2 / {}^{51}C_2 = 13 * 12 / 51 * 50$

$P(E_1 / A) = [P(E_1) * P(A / E_1)] / [P(E_1) * P(A / E_1)] + [P(E_2) * P(A / E_2)]$

$= [(1 / 4) * [12 * 11 / 51 * 50]] / [(1 / 4) * (12 * 11 / 51 * 50) + (3 / 4) * (13 * 12 / 51 * 50)]$

$= 132 / 600$

$= 11 / 50$

OR

A pair of dice is thrown 4 times. If getting a doublet is considered as a success,
[i] Find the probability of getting a doublet.
[ii] Find the probability of two successes.

Solution:

[i] 1 / 6

[ii] n = 4

$q = 1 - p$

$= 1 - (1 / 6)$

$= 5 / 6$

$P(X = 2) = {}^{4}C_2 (5 / 6)^2 (1 / 6)^2$

$= 25 / 216$