KBPE Class 12th Maths Question Paper With Solution 2017

QUESTION PAPER CODE 5018

Question 1[a]: Let R be a relation defined on A = {1, 2, 3} by R = [{1, 3} , {3, 1}, {2, 2}]. R is

(a) Reflexive (b) Symmetric
(c) Transitive (d) Reflexive but not transitive

Answer: [b]

[b] Find f o g and g o f if f(x) = |x + 1| and g (x) = 2x - 1.

Solution:
f o g (x) = f (g (x)) = f (2x - 1)
= |2x - 1 + 1|
= |2x|
g o f (x) = g (f (x)) = g (x + 1)
= 2 |x + 1| - 1

[c] Let "*" be a binary operation defined on N x N by (a, b) * (c, d) = (a + c, b + d). Find the identity element for * if it exists.

Solution:
If (c, d) is the identity element,
(a, b) * (c, d) = (a, b)
(a, b) * (c, d) = (a + c, b + d)
(a + c, b + d) = (a, b)
(c, d) = (0, 0) is not an element of N x N.
So, the identity element doesn't exist.

Question 2[a]: The principal value of $\cot^{-1}(1/\sqrt{3})$ is

(a) $\pi / 3$ (b) $-\pi / 3$ (c) $\pi / 6$ (d) $2\pi / 3$

Answer: [d]

[b] Solve $\tan^{-1}(x - 1)/(x - 2) + \tan^{-1}(x + 1)/(x + 2) = \pi / 4$.

Solution:

$\tan^{-1}(x - 1)/(x - 2) + \tan^{-1}(x + 1)/(x + 2)$

$\pi / 4 = \tan^{-1}[\{(x - 1)/(x - 2)\} + \{(x + 1)/(x + 2)\}] / [1 - \{(x - 1)/(x - 2)\} * \{(x + 1)/(x + 2)\}]$

$\pi / 4 = [2x^2 - 4] / -3$

$[2x^2 - 4] / -3 = 1$

$x = \pm 1/\sqrt{2}$

Question 3[a]: The value of k such that the matrix $\begin{pmatrix} 1 & k \\ -k & 1 \end{pmatrix}$ is symmetric is

(a) 0 (b) 1 (c) -1 (d) 2

Answer: k = 0

[b] If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then prove that A^2 = $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$.

Solution:

$$A^2 = A * A$$
$$= \begin{bmatrix} cos\Theta & sin\Theta \\ -sin\Theta & cos\Theta \end{bmatrix} * \begin{bmatrix} cos\Theta & sin\Theta \\ -sin\Theta & cos\Theta \end{bmatrix}$$
$$= \begin{bmatrix} cos^2\Theta - sin^2\Theta & sin\Theta cos\Theta + cos\Theta sin\Theta \\ -sin\Theta cos\Theta - cos\Theta sin\Theta & cos^2\Theta - sin^2\Theta \end{bmatrix}$$
$$= \begin{bmatrix} cos2\Theta & sin2\Theta \\ -sin2\Theta & cos2\Theta \end{bmatrix}$$

[c] If A = $\begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix}$, then find |3A'|.

Solution:

$$A' = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}$$
$$3A' = \begin{bmatrix} 3 & 12 \\ 9 & 3 \end{bmatrix}$$
$$|3A'| = -99$$

Question 4[a]: If A = $\begin{bmatrix} a & 1 \\ 1 & 0 \end{bmatrix}$ such that A^2 = I then a equals to

(a) 1 (b) -1 (c) 0 (d) 2

Answer: [c]

[b] Solve the system of equations:
x - y + z = 4
2x + y - 3z = 0
x + y + z = 2 by matrix method.

Solution:

$$AX = B$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$|A| = 10$$

$$A^{-1} = adjA/|A|$$

$$= \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$X = A^{-1}(B)$$

$$= \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Question 5[a]: Find the value of a and b such that the function

$$f(x) = \begin{cases} 5a & x \leq 0 \\ a \sin x + \cos x & 0 < x < \frac{\pi}{2} \\ b - \frac{\pi}{2} & x \geq \frac{\pi}{2} \end{cases}$$

is continuous.

Solution:

5a = 1

a = 1 / 5

a = b - (**π** / 2)

b = (1 / 5) + (**π** / 2)

[b] Find dy / dx if (sin x)$^{\cos y}$ = (cos y)$^{\sin x}$.

Solution:

log ((sin x)$^{\cos y}$) = log (cos y)$^{\sin x}$

cos y log sin x = sin x log cos y

cos y * (1 / sin x) * cos x + log sin x * (- sin y) (dy / dx) = sin x * (1 / cos y) * (- sin y) (dy / dx) log cos y cos x

dy / dx = [cos x * log cos y - cos y cot x] / [sin x tany - sin y log sin x]

Question 6[a]: Slope of the normal curve $y^2 = 4x$ at (1, 2) is

(a) 1 (b) 1 / 2 (c) 2 (d) -1

[b] Find the interval in which $2x^3 + 9x^2 + 12x - 1$ is strictly increasing.

OR

[a] The rate of change of volume of a sphere with respect to its radius when its radius is 1 unit

(a) 4π (b) 2π (c) π (d) π / 2

[b] Find two positive numbers whose sum is 16 and the sum of whose cubes are minimum.

Solution:

[a] Answer: [d]

[b] $f'(x) = 6x^2 + 18x + 12$

$f'(x) = 0$

$x = -1, -2$

The intervals are $(-\infty, -2)$, $(-2, -1)$ and $(-1, \infty)$.

$f'(x) > 0$ is increasing in $(-\infty, -2)$ and $(-1, \infty)$.

OR

[a] Answer: [a]

[b] Let the two numbers be x and y.

$x + y = 16$

$y = 16 - x$

Sum $S = x^2 + y^2$

$= x^2 + (16 - x)^2$

$= x^2 + 256 + x^2 - 32x$

$= 2x^2 - 32x + 256$

$dS / dx = 4x - 32$

For the sum to be minimum, $dS / dx = 0$

$4x - 32 = 0$
$4x = 32$
$x = 8$
$y = 16 - x$
Substituting the x value,
$y = 16 - 8 = 8$
So, the two numbers are 8 and 8.

Question 7: Find the following.
[a] $\int 1 / [x (x^7 + 1)]\, dx$
[b] $\int_1^4 |x - 2|\, dx$

Solution:
[a] $\int 1 / [x (x^7 + 1)]\, dx$
$= \int x^6 / [x^7 (x^7 + 1)]\, dx$
Put $t = x^7$
$dt = 7x^6\, dx$
$= (1 / 7) \int dt / t (t + 1)$
$= (1 / 7) [\int([1 / t] + [-1 / t + 1])]\, dt$
$= (1 / 7) [\log x^7 - \log (1 + x^7)] + c$

[b] $\int_1^4 |x - 2|\, dx$
$= \int_1^2 (2 - x)\, dx + \int_2^4 (x - 2)\, dx$
$= [(2 - x)^2 / -2]_1^2 + [(x - 2)^2 / 2]_2^4$
$= (0 - (-1 / 2)) + ((4 / 2) + 0)$
$= 5 / 2$

Question 8[a]: Evaluate $\int_0^{\pi/2} \log \sin x\, dx$

OR

[b] Evaluate $\int_0^4 x^2\, dx$ as the limit of a sum.

Solution:

[a] $I = \int_0^{\pi/2} \log \sin x \, dx$

$= \int_0^{\pi/2} \log \sin (\pi / 2 - x) \, dx$

$I = \int_0^{\pi/2} \log \cos x \, dx$

$2I = \int_0^{\pi/2} [\log \sin x + \log \cos x] \, dx$

$= \int_0^{\pi/2} \log \sin 2x - \int_0^{\pi/2} \log 2 \, dx$

$= I - (\pi / 2) \log 2$

$I = - (\pi / 2) \log 2$

[b] $\int_0^4 x^2 \, dx = \lim_{h \to 0} h [h^2 + (2h)^2 + \ldots.. + ((n - 1)h)^2]$

$= \lim_{h \to 0} h^3 [1^2 + 2^2 + \ldots\ldots (n - 1)^2]$

$= \lim_{h \to 0} h^3 [n (n - 1) (2n - 1) / 6]$

$= \lim_{h \to 0} h^3 n^3 * (1 - [1 / n]) (2 - [1 / n]) / 6$

$= 4 * 4 * 8 / 6$

$= 64 / 3$

Question 9[a]: Area bounded by the curves $y = \cos x$, $x = \pi / 2$, $x = 0$, $y = 0$ is

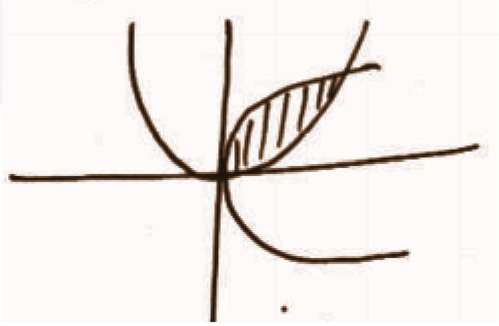
(a) 1 / 2 (b) 2 / π (c) 1 (d) π / 2

[b] Find the area between the curve $y^2 = 4ax$ and $x^2 = 4ay$, $a > 0$.

Solution:

[a] Answer: (c)

[b]



The point of intersection is (4a, 4a).

$\text{Area} = \int_0^{4a} \sqrt{4ax} \, dx - \int_0^{4a} (x^2 / 4a) \, dx$

$= 2\sqrt{a} \, [(x^{3/2} / (3 / 2)]_0^{4a} - (1 / 4a) \, (x^3 / 3)_0^{4a}$

= [16 / 3] a^2

Question 10[a]: The order of the differential equation $x^4\ d^2y / dx^2 = 1 + (dy / dx)^3$ is

(a) 1 (b) 3 (c) 4 (d) 2

[b] Find the particular solution of the differential equation $(1 + x^2)\ dy / dx + 2xy = 1 / 1 + x^2$, y = 0 when x = 1.

Solution:

[a] Answer: [d]

[b] $(1 + x^2)\ dy / dx + 2xy = 1 / 1 + x^2$

Divide both sides by $(1 + x^2)$

$dy / dx + 2xy / 1 + x^2 = 1 / (1 + x^2)^2$

$dy / dx + (2x / 1 + x^2)y = 1 / (1 + x^2)^2$

Comparing with dy / dx + Py = Q

IF = $e^{\int P\ dx}$

Let $1 + x^2 = t$

2x dx = dt

dx = dt / 2

IF = $e^{\int (2x / t)\ (dt / 2x)}$

= $e^{\int dt / t}$

= $e^{\log |t|}$

= t

IF = $1 + x^2$

Solution of the differential equation is y * IF = $\int$Q * IF dx

$y * (1 + x^2) = \int [1 / (1 + x^2)^2] * (1 + x^2)\ dx$

$y * (1 + x^2) = \tan^{-1} x + c$

C = - **π** / 4

$y * (1 + x^2) = \tan^{-1} x$ - **π** / 4

Question 11[a]: The projection of the vector 2i + 3j + 2k on the vector i + j + k is

(a) 3 / √3 (b) 7 / √3 (c) 3 / √17 (d) 7 / √17

[b] Find the area of a parallelogram whose adjacent sides are the vectors 2i + j + k and i - j.

Solution:

[a] Answer: [b]

[b] A = |a x b|

|a x b| = i + j - 3k

|a x b| = $\sqrt{11}$

Question 12[a]: The angle between the vectors i + j and j + k is

(a) 60º (b) 30º (c) 45º (d) 90º

[b] If a, b, c are unit vectors such that a + b + c = 0, find the value of a . b + b . c + c . a.

Solution:

[a] Answer: [a]

[b] |a x b x c| = 0

$(a + b + c)^2 = 0$

$|a|^2 + |b|^2 + |c|^2 + 2ab + 2bc + 2ca = 0$

3 + 2 (a . b + a . c + b . c) = 0

(a . b + a . c + b . c) = - 3 / 2

Question 13[a]: The line x - 1 = y = z is perpendicular to the line

(a) x - 2 / 1 = y - 1 / 2 = z / -3 (b) x - 2 = y - 2 = z

(c) x - 2 / 1 = y - 1 / 2 = z / -3 (d) x = y = 2 / 3

[b] Find the shortest distance between the lines r = i + 2j + 3k + λ (i + j + k) and r = i + j + k + μ (i + j + k).

Solution:

[a] Answer: [a]

[b] a_1 = i + 2j + 3k

a_2 = i + j + k

b_1 = i + j + k

b_2 = i + j + k

$a_2 - a_1 = - j - 2k$

Shortest distance = $|b \times (a_2 - a_1) / |b|| = \sqrt{2}$

Question 14[a]: Distance of the point (0, 0, 1) from the plane x + y + z = 3.

(a) $1 / \sqrt{3}$ (b) $2 / \sqrt{3}$ (c) $\sqrt{3}$ (d) $\sqrt{3} / 2$

[b] Find the equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to x - y + z = 0.

Solution:

[a] Answer: [b]

[b] $(x + y + z - 1) + \lambda (2x + 3y + 4z - 5) = 0$

$(2\lambda + 1) x + (3\lambda + 1) y + (4\lambda + 1) z - 5\lambda - 1 = 0$

$(2\lambda + 1) (1) + (3\lambda + 1) (-1) + (4\lambda + 1) (1) = 0$

$\lambda = - 1 / 3$

$(x + y + z - 1) - (1 / 3) (2x + 3y + 4z - 5) = 0$

$x - z + 2 = 0$

Question 15[a]: Consider the linear programming problem:

Maximise Z = 50x + 40y

Subject to constraints

$x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0, y \geq 0$

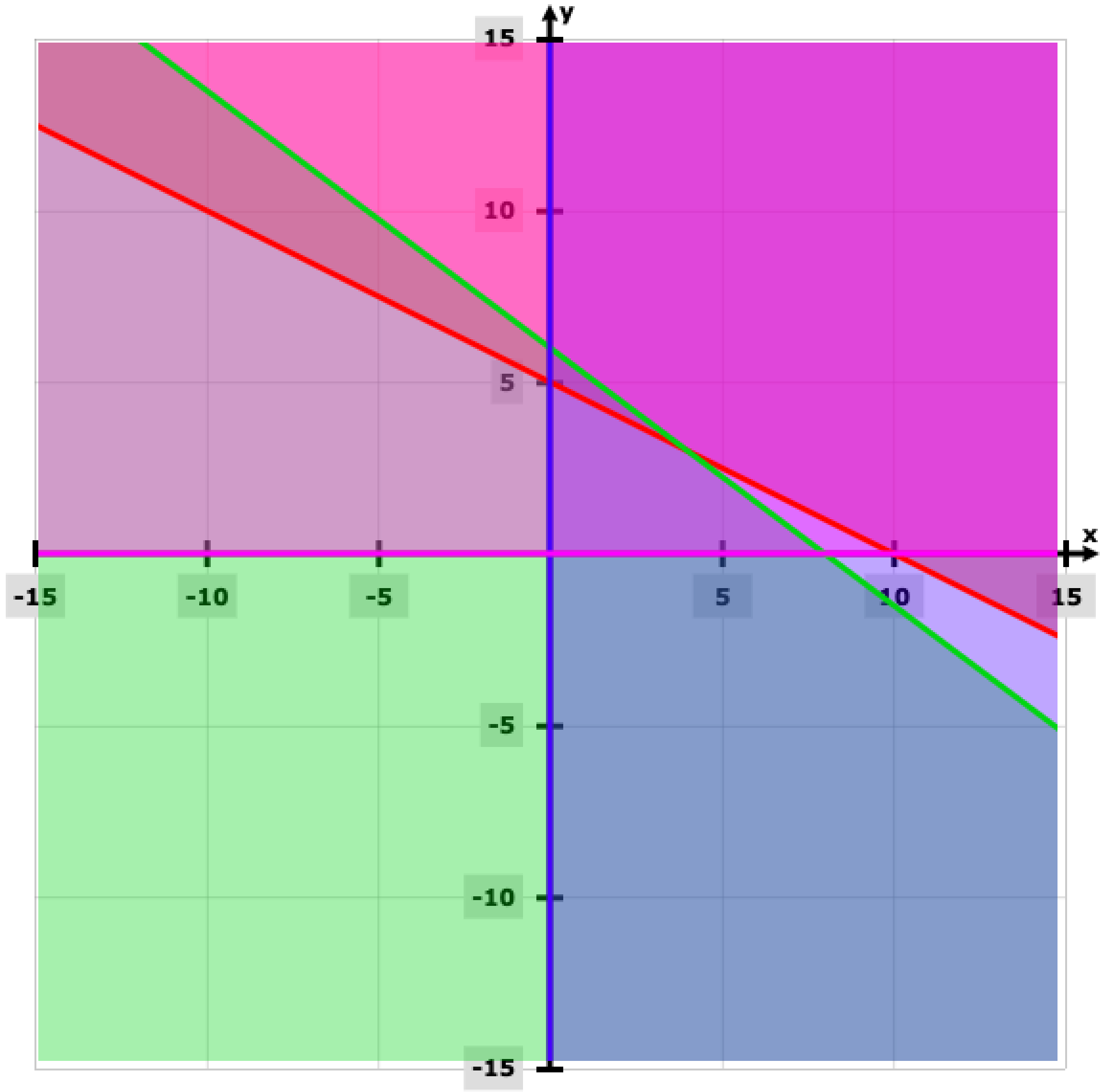
[a] Find the feasible region.

[b] Find the corner points of the feasible region.

[c] Find the maximum value of Z.

Solution:

[a]



[b] The corner points are (0, 5), (0, 6), (4, 3).

[c] At (0, 5), Z = 200

(0, 6), Z = 240

(4, 3), Z = 320

Z is maximum at (4, 3) and it is 320.

Question 16[a]: If A and B are two events such that $A \subset B$ and $P(A) \neq 0$, then $P(A / B)$ is

(a) $P(A) / P(B)$ (b) $P(B) / P(A)$ (c) $1 / P(A)$ (d) $1 / P(B)$

[b] There are two identical bags. The bag I contain 3 red and 4 black balls while Bag II contains 5 red and 4 black balls. One ball is drawn at random from one of the bags.
[i] Find the probability that the ball drawn is red.
[ii] If the ball drawn is red what is the probability that it was drawn from Bag I?

OR

Consider the following probability distribution of a random variable X.

X	**0**	**1**	**2**	**3**	**4**
P (X)	**1 / 16**	**2 / 16**	**K**	**5 / 16**	**1 / 16**

[i] Find the value of K.
[ii] Determine the mean and variance of K.

Solution:
[a] Answer: [a]
[b] [i] P (E1) = P (E2) = 0.5
P (A / E1) = 5 / 9
P (A / E2) = 3 / 7
P (the ball drawn is red) = (1 / 2) (5 / 9) + (1 / 2) (3 / 7)
= 62 / 126
= 31 / 63
[ii] P (the ball was drawn from Bag I)
= (1 / 2) * (3 / 7) / (1 / 2) (5 / 9) + (1 / 2) (3 / 7)
= 27 / 62

OR

[i] ∑P (x) = 1
K = 7 / 16
[ii] E (X) = ∑X * P (X)

$= 0 + (2 / 16) + (14 / 16) + (15 / 16) + (4 / 16)$

$= 35 / 16$

$E(X^2) = \sum X^2 * P(X)$

$= 0 + (2 / 16) + (28 / 16) + (45 / 16) + (16 / 16)$

$= 91 / 16$

$\text{Variance} = \sum X^2 * P(X) - [\sum X * P(X)]^2$

$= 91 / 16 - [35 / 16]^2$

$= 231 / 256$

$= 0.902$