# KBPE Class 12th Maths Question Paper With Solution 2018

QUESTION PAPER CODE 9018

**Question 1 to 7 carries 3 scores each. Answer any six questions. [6 * 3 = 18]**

**Question 1: If f (x) = x / (x − 1), x ≠ 1,**
**[a] Find f o f (x)**
**[b] Find the inverse of f**

**Solution:**
[a] f o f (x) = f ( f (x))
= f (x / x - 1)
= (x / x - 1) / [(x / x - 1) - 1]
= x / [x - (x - 1)]
= x

[b] y = x / (x - 1)
xy - x = y
xy - y = x
y (x - 1) = x
$f^{-1}$ (y) = y / (y - 1)
$f^{-1}$ (x) = x / (x - 1)

**Question 2: Using elementary row operations, find the inverse of the matrix**
$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

**Solution:**

Write the augmented matrix

| | $A_1$ | $A_2$ | $B_1$ | $B_2$ |
|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 |
| 2 | 2 | -1 | 0 | 1 |

Find the pivot in the 1st column in the 1st row

| | $A_1$ | $A_2$ | $B_1$ | $B_2$ |
|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 |
| 2 | 2 | -1 | 0 | 1 |

Eliminate the 1st column

| | $A_1$ | $A_2$ | $B_1$ | $B_2$ |
|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 |
| 2 | 0 | -5 | -2 | 1 |

Make the pivot in the 2nd column by dividing the 2nd row by -5

| | $A_1$ | $A_2$ | $B_1$ | $B_2$ |
|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 0 |
| 2 | 0 | 1 | 2/5 | -1/5 |

Eliminate the 2nd column

| | $A_1$ | $A_2$ | $B_1$ | $B_2$ |
|---|---|---|---|---|
| 1 | 1 | 0 | 1/5 | 2/5 |
| 2 | 0 | 1 | 2/5 | -1/5 |

There is the inverse matrix on the right

| | $A_1$ | $A_2$ | $B_1$ | $B_2$ |
|---|---|---|---|---|
| 1 | 1 | 0 | 1/5 | 2/5 |
| 2 | 0 | 1 | 2/5 | -1/5 |

**Question 3[a]: f (x) is a strictly increasing function, if f ' (x) is _____.**

[i] positive [ii] negative [iii] 0 [iv] None of these

**Answer: positive**

**[b] Show that the function f given by $f(x) = x^3 - 3x^2 + 4$, $x \in R$ is strictly increasing.**

**Solution:**

$f(x) = x^3 - 3x^2 + 4$

$f'(x) = 3x^2 - 6x$

$= 3(x^2 - 2x + 1) > 0$

So, f (x) is strictly increasing in R.

**Question 4[a]: $\int_0^a f(a - x)\, dx$.**

[i] $\int_0^{2a} f(x)\, dx$ [ii] $\int_{-a}^{a} f(x)\, dx$ [iii] $\int_0^a f(x)\, dx$ [iv] $\int_a^0 f(x)\, dx$

**Answer:**

**$\int_0^a f(a - x)\, dx = \int_0^a f(x)\, dx$ [iii]**

**[b] Find the value of $\int_0^{\pi/2} \sin^4 x / [\sin^4 x + \cos^4 x]\, dx$.**

**Solution:**

$I = \int_0^{\pi/2} \sin^4 x / [\sin^4 x + \cos^4 x]\, dx$

$= \int_0^{\pi/2} \sin^4 (\pi / 2 - x)\, dx / [\sin^4 (\pi / 2 - x) + \cos^4 (\pi / 2 - x)]$

$= \int_0^{\pi/2} \cos^4 x\, dx / [\sin^4 x + \cos^4 x]\, dx$

$2I = \int_0^{\pi/2} [\sin^4 x + \cos^4 x] / [\sin^4 x + \cos^4 x]\, dx$

$= \int_0^{\pi/2} dx$

$= \pi / 2$

$I = \pi / 4$

**Question 5: Find the area of the region bounded by the curve $y^2 = x$, x-axis and the lines x = 1 and x = 4.**

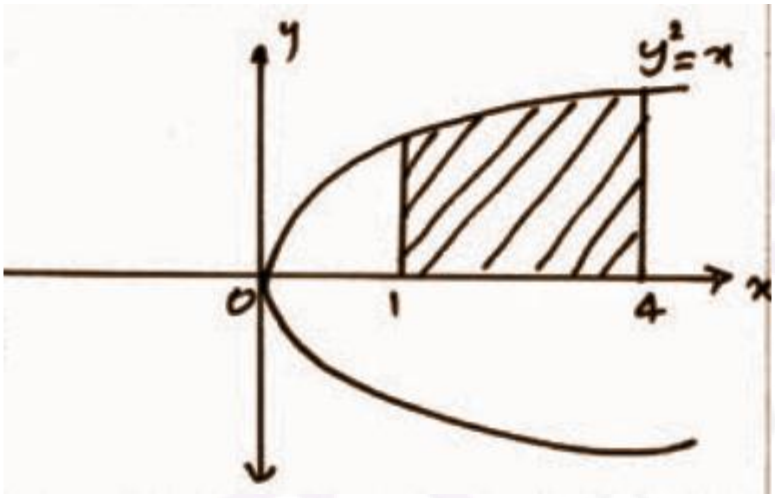
**Solution:**

Area = $\int_a^b f(x)\, dx$

$= \int_1^{\pi/4} \sqrt{x}\, dx$

$= (x^{3/2} / (3/2))_1^{\pi/4}$

= 14 / 3 square units




**Question 6: Find the general solution of the differential equation x dy / dx + 2y = $x^2$ log x.**

**Solution:**

dy / dx + (2 / x)y = x log x

P = (2 / x), Q = x log x

IF = $e^{\int P dx}$

$= e^{\int (2/x)\, dx}$

$= e^{2 \log x}$

$= x^2$

Solutions are

y * (IF) = ∫Q * (IF) dx

$yx^2 = \int x^2 * x \log x\, dx$

$= \int x^3 \log x\, dx$

$= \log x * (x^4 / 4) - \int x^4 / 4x\, dx$

$= x^4 / 4 \log x - (x^4 / 16) + c$

**Question 7: A manufacturer produces nuts and bolts. It takes t hour of work on Machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on Machine A and t hour on Machine B to produce a package of bolts. He earns a profit of Rs. 17.50 per package on nuts and Rs. 7.00 per package on bolts. Formulate the above LPP if the machines operate for at most l2 hours a day.**

**Solution:**
Let x be the packet of nuts and y be the packet of bolts.
Maximise $Z = 17.5x + 7y$
Subject to
$x + 3y \leq 12$
$3x + y \leq 12$
$x, y \geq 0$

**Question 8 to 17 carries 4 scores each. Answer any eight questions. [8 * 4 = 32]**

**Question 8: Let A = N x N and "*" be a binary operation on A defined by (a, b) * (c, d) = (a + c, b + d).**
**[a] Find (1, 2) * (2, 3)**
**[b] Prove that "*" is commutative.**
**[c] Prove that "*" is associative.**

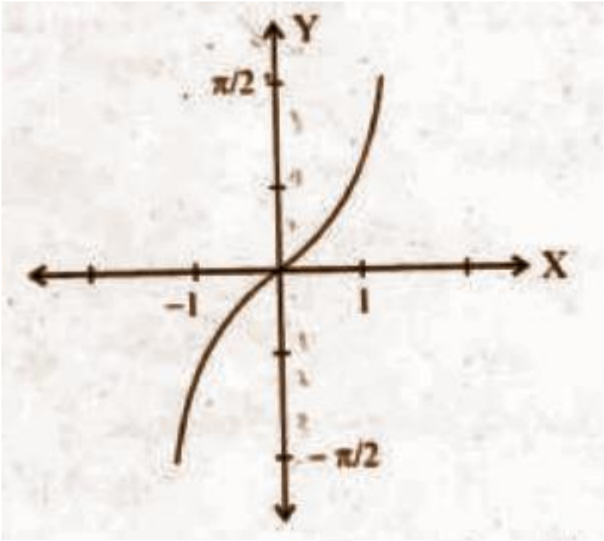
**Solution:**
[a] (1, 2) * (2, 3) = (1 + 2, 2 + 3) = (3, 5)

[b] (c, d) * (a, b) = (c + a, d + b)
= (a + c, b + d)
= (a, b) * (c, d)

[c] (a, b) * [(c, d) * (e, f)] = (a, b) * [(c + e, d + f)]
= (a + c + e, b + d + f)

$[(a, b) * (c, d)] * (e, f) = [(a + c, b + d)] * (e, f)$
$= (a + c + e, b + d + f)$
So, $(a, b) * [(c, d) * (e, f)] = [(a, b) * (c, d)] * (e, f)$

**Question 9[a]:**




**Identify the function from the above graph.**

[i] $\tan^{-1} x$ [ii] $\sin^{-1} x$ [iii] $\cos^{-1} x$ [iv] $\text{cosec}^{-1} x$

**Answer: [ii]**

**[b] Find the domain and range of the function represented by the graph.**

**Solution:**

Domain → [–1, 1]

Range → [- **π** / 2, **π** / 2]

**[c] Prove that $\tan^{-1} (1 / 2) + \tan^{-1} (2 / 11) = \tan^{-1} (3 / 4)$.**

**Solution:**

$\tan^{-1} x + \tan^{-1} y = \tan^{-1} [(x + y) / (1 - xy)]$
$\tan^{-1} (1 / 2) + \tan^{-1} (2 / 11) = \tan^{-1} [(1 / 2 + 2 / 11) / (1 - [(1 / 2 * 2 / 11)]]$
$= \tan^{-1} (15 / 20)$
$= \tan^{-1} (3 / 4)$

**Question 10[a]: d $(a^x)$ dx = ______.**

[i] $a^x$ [ii] log $(a^x)$ [iii] $a^x$ log a [iv] $xa^{x-1}$

**Answer: [iii]**

**[b] Find dy / dx if $x^y = y^x$.**

**Solution:**

log $x^y$ = log $y^x$

y log x = x log y

y * (1 / x) + log x (dy / dx) = x * (1 / y) (dy / dx) + log y

(log x - (x / y)) dy / dx = log y - (y / x)

dy / dx = [log y - (y / x)] / [(log x - (x / y)]

**Question 11[a]: Find the slope of the tangent to the curve $y = (x - 2)^2$ at x = 1.**

**Solution:**

$y = (x - 2)^2$

dy / dx = 2(x - 2)

Slope = -2

**[b] Find a point at which the tangent to the curve $y = (x - 2)^2$ is parallel to the chord joining the points A (2, 0) and B (4, 4).**

**Solution:**

The slope of AB = $[y_2 - y_1] / [x_2 - x_1]$

= [4 - 0] / [4 - 2]

= 2

Here,

2 (x - 2) = 2

x = 3, y = 1

The point is (3, 1).

**[c] Find the equation of the tangent to the above curve and parallel to the line AB.**

**Solution:**

The equation of the tangent is $y - y_0 = m (x - x_0)$

$y - 1 = 2 (x - 3)$

$2x - y - 5 - 0$

**Question 12: $\int_0^2 (x^2 + 1)\, dx$ as the limit of a sum.**

**Solution:**

$a = 0, b = 2, h = [b - a] / h$

$\int_0^2 (x^2 + 1)\, dx = \lim_{h\to 0} h [1 + (h^2 + 1) + [(2h)^2 + 1] + \ldots\ldots + [(n - 1)h]^2 + 1]$

$= \lim_{h\to 0} h [n + h^2 (1^2 + 2^2 + 3^2 + \ldots. + (n - 1)^2]$

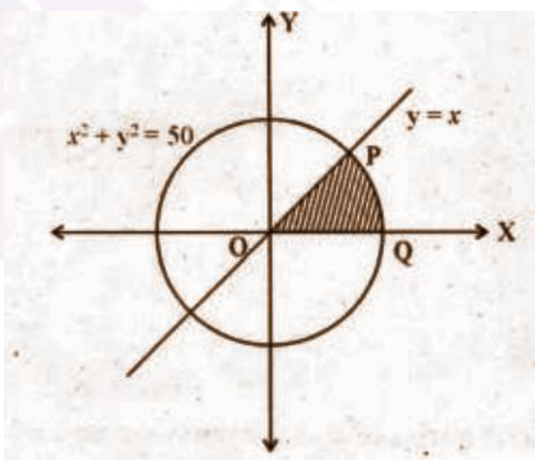
$= \lim_{h\to 0} [nh + h^3 n (n - 1) (2n - 1) / 6]$

$= 2 + [2 (2 - 0) (4 - 0)] / 6$

$= 2 + (8 / 3)$

$= 14 / 3$

**Question 13: Consider the following figure:**




**[a] Find the point of intersection P of the circle $x^2 + y^2 = 50$ and the line $y = x$.**

**Solution:**

$x^2 + y^2 = 50$
$x^2 + x^2 = 50$
$2x^2 = 50$
$x^2 = 50 / 2$
$x = \sqrt{25}$
$x = \pm 5$
$y = \pm 5$
The point of intersection is P (5, 5).

**[b] Find the area of the shaded region.**

**Solution:**
Required area $= \int_0^5 x\, dx + \int_5^{\sqrt{50}} \sqrt{50 - x^2}\, dx$
$= (x^2 / 2)_0^5 + [(x / 2) \sqrt{50 - x^2} + (50 / 2) \sin^{-1} (x / \sqrt{50})]$
$= 25 / 2 + 25\pi / 2 - (25 / 2) - (25\pi / 4)$
$= 25\pi / 4$

**Question 14[a]: The degree of the differential equation $xy (d^2y / dx^2)^2 + x^4 (dy / dx)^3 - y (dy / dx) = 0$ is _____.**

(i) 4 (ii) 3 (iii) 2 (iv) 1

**Answer: [iii]**

**[b] Find the general solution of the differential equation $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$.**

**Solution:**
$(\sec^2 x / \tan x)\, dx = [- \sec^2 y / \tan y]\, dy$
$\int(\sec^2 x / \tan x)\, dx = - \int[\sec^2 y / \tan y]\, dy$
$\log \tan x = - \log \tan y + c$
$\log \tan x + \log \tan y = c$

**Question 15[a]: Prove that for any vectors a, b, c, [a + b, b + c, c + a] = 2 [a . b . c].**

**Solution:**
[a + b, b + c, c + a] = (a + b) . [(b + c) x (c + a)]
= (a + b) . [b x c + b x a + c x c + c x a]
= (a + b) . [b x c + b x a + c x a]
= a . (b x c) + a . (b x a) + a . (c x a) + b . (b x c) + b . (b x a) + b . (c x a)
= 2 [a . b . c]

**[b] Show that if [a + b, b + c, c + a] are coplanar, then a, b, c are also coplanar.**

**Solution:**
[a b c] = 0
Hence a, b, c are also coplanar.

**Question 16[a]: Find the equation of a plane which makes x, y, z intercepts respectively as 1, 2, 3.**

**Solution:**
x / 1 = y / 2 = z / 3 = 1
6x + 3y + 2z = 6

**[b] Find the equation of a plane passing through the point (1, 2, 3) which is parallel to the above plane.**

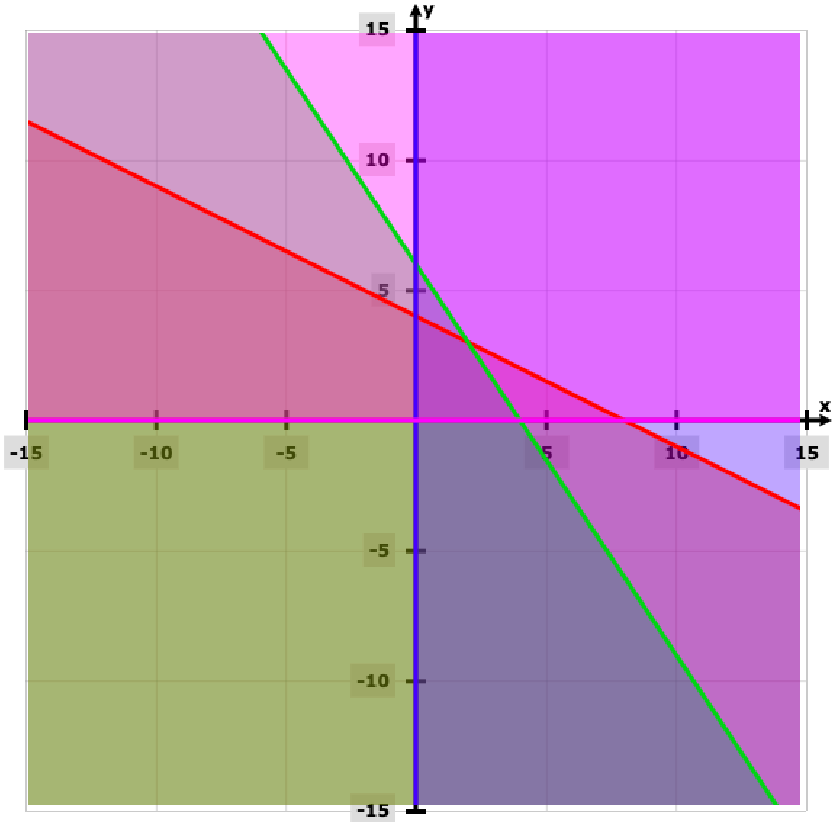
**Solution:**
A plane parallel to the given plane is 6x + 3y + 2z = k.
Since it passes through (1, 2, 3), k = 18.
The equation of the plane is 6x + 3y + 2z = 18.

**Question 17: Solve the LPP given below graphically.**
**Minimise Z = - 3x + 4y**
**Subject to**
$x + 2y \leq 8$

$3x + 2y \leq 12$

$x \geq 0, y \geq 0.$

**Solution:**




Z = - 3x + 4y

| Points | Z value |
|---|---|
| (0, 0) | 0 |
| (4, 0) | -12 |
| (2, 3) | 6 |
| (0, 4) | 16 |

The value of Z is minimum at (4, 0).

**Question 18 to 24 carries 6 scores each. Answer any five questions. [5 * 6 = 30]**

**Question 18[a]: Find x and y if** $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$.

**Solution:**

2x - y = 10 ---- (1)

3x + y = 5 ---- (2)

__________

5x = 15

x = 15 / 5

x = 3

Substitute the value of x in (1),

2 * 3 - y = 10

6 - y = 10

6 - 10 = y

y = - 4

So, x = 3 and y = -4.

**[b] Express the matrix** $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ **as a sum of symmetric and skew-symmetric matrices.**

**Solution:**

$$A^T = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$$

$$P = (\frac{1}{2})\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$$

$$Q = (\frac{1}{2})\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$$

$$A = P + Q$$

**Question 19[a]: Prove that** $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0$.

**Solution:**

$$\begin{bmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{bmatrix}$$

$R_2 - R_1$

$$\begin{bmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{bmatrix}$$

$R_2 - R_3$

$$\begin{bmatrix} a & b & c \\ x & y & z \\ x & y & z \end{bmatrix}$$

$R_2 - R_3$

$$\begin{bmatrix} a & b & c \\ 0 & 0 & 0 \\ x & y & z \end{bmatrix}$$

Hence proved.

**[b] If** $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$,

**[i] Prove that $B = A^{-1}$.**

**[ii] Using $A^{-1}$ solve the system of linear equations given below.**

**$x - y + 2z = 1$**

**$2y - 3z = 1$**

**$3x - 2y + 4z = 2$**

**Solution:**

[i] $AB = I$

$B = A^{-1}$

[ii] $A^{-1} = \text{adj } A / |A| = B$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

$$x = 0, y = 5, z = 3$$

**Question 20[a]: Prove that the function defined by f (x) = cos ($x^2$) is a continuous function.**

**Solution:**

f (x) = cos x

g (x) = $x^2$

Both are continuous functions.

Composition of two continuous functions is continuous.

f (g (x)) = f o g (x) = cos ($x^2$) is continuous.

**[b] [i]** If $y = e^{a\cos^{-1}x}, -1 \le x \le 1$, show that $\dfrac{dy}{dx} = \dfrac{-ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}$.

**[ii] Hence prove that (1 - $x^2$) $d^2y / dx^2$ - x (dy / dx) - $a^2y$ = 0.**

**Solution:**

[i]

$$\frac{dy}{dx} = ae^{acos^{-1}x} * \frac{-1}{\sqrt{1-x^2}}$$

$$= \frac{ae^{acos^{-1}x}}{\sqrt{1-x^2}}$$

[ii] $(\sqrt{1 - x^2})$ dy / dx = -ay

(1 - $x^2$) $(dy / dx)^2$ = $a^2y^2$

(1 - $x^2$) 2 (dy / dx) * ($d^2y / dx^2$) + $(dy / dx)^2$ * (2x) = $2a^2y$ (dy / dx)

(1 - $x^2$) ($d^2y / dx^2$) - x (dy / dx) - $a^2y$ = 0

**Question 21:**
**[a] $\int \sin mx \, dx$**
**[b] $\int 1 \, dx / \sqrt{x^2} + 2x + 2$**
**[c] $\int x \, dx / (x + 1)(x + 2)$**

**Solution:**
[a] $\int \sin mx \, dx$
$= (-\cos mx / m) + c$

[b] $x^2 + 2x + 2 = (x + 1)^2 + 1$
$\int dx / \sqrt{x^2} + 2x + 2 = \int dx / \sqrt{(x + 1)^2} + 1$
$= \log [(x + 1)] + \sqrt{x^2} + 2x + 2 + c$

[c] $\int x \, dx / (x + 1)(x + 2)$
$x / (x + 1)(x + 2) = A / (x + 1) + B / (x + 2)$
$A = -1, B = 2$
$\int x \, dx / (x + 1)(x + 2) = \int (-1 / (x + 1) \, dx + \int 2 / (x + 2) \, dx$
$= -\log (x + 1) + 2 \log (x + 2) + c$

**Question 22[a]: If a = 3i + 2j + 2k, b = i + 2j - 2k**
**[i] Find a + b, a - b.**
**[ii] Find a unit vector perpendicular to both a + b and a - b.**

**Solution:**
[i] a + b = 4i + 4j
a - b = 2i + 4k

[ii] Unit vector perpendicular to both = $16i - 16j - 8k / \sqrt{576}$
$= 2i - 2j - k / 3$

**[b] Consider the points A (1, 2, 7), B (2, 6, 3), C (3, 10, -1).**
**[i] Find AB, BC.**

**[ii] Prove that A, B, C are collinear points.**

**Solution:**

[i] AB = i + 4j - 4k

BC = i + 4j - 4k

[ii] AB = BC

Hence A, B, C are collinear points.

**Question 23[a]: Find the angle between the lines x - 2 / 2 = y - 1 / 5 = z + 3 / -3 and x + 2 / -1 = y - 4 / 8 = z - 5 / 4.**

**[b] Find the shortest distance between the pair of lines.**

**r = (i + 2j + 3k) + λ [i - 3j + 2k]**

**r = (4i + 5j + 6k) + μ [2i + 3j + k]**

**Solution:**

$a_1 = 2$, $b_1 = 5$, $c_1 = -3$ and $a_2 = -1$, $b_2 = 8$, $c_2 = 4$

$\cos\theta = a_1a_2 + b_1b_2 + c_1c_2 / \sqrt{a_1^2 + b_1^2 + c_1^2} * \sqrt{a_2^2 + b_2^2 + c_2^2}$

$= -2 + 40 - 12 / \sqrt{4 + 25 + 9}\ \sqrt{1 + 64 + 16}$

$= 26 / 9\sqrt{38}$

$\theta = \cos^{-1} [26 / 9\sqrt{38}]$

[b] $a_1 = i + 2j + 3k$, $b_1 = i - 3j + 2k$

$a_2 = 4i + 5j + 6k$, $b_2 = 2i + 3j + k$

Shortest distance = $|(a_2 - a_1) . (b_1 \times b_2) / |b_1 \times b_2||$

$(a_2 - a_1) = 3i + 3j + 3k$

$(b_1 \times b_2) = - 9i + 3j + 9k$

Shortest distance = $[-27 + 9 + 27] / \sqrt{171}$

$= 3 / \sqrt{19}$

**Question 24[a]: The probability distribution of a random variable is given by P (x). What is ∑P (x)?**

**Solution:**

$\sum P(x) = 1$

**[b] The following is the probability distribution function of a random variable.**

| x | - 5 | - 4 | - 3 | - 2 | - 1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P (x) | k | 2k | 3k | 4k | 5k | 7k | 8k | 9k | 10k | 11k | 12k |

**[i] Find k**
**[ii] Find P (x > 3)**
**[iii] Find P (-3 < x < 4)**
**[iv] Find P (x < - 3)**

**Solution:**
[i] k + 2k + 3k + 4k + 5k + 7k + 8k + 9k + 10k + 11k + 12k = 1
72k = 1
k = 1 / 72

[ii] P (x > 3) = P (x = 4) + P (x = 5)
= 11 / 72 + 12 / 72
= 23 / 72

[iii] P (-3 < x < 4)
= P (x = 2) + P (x = - 1) + P (x = 0) + P (x = 1) + P (x = 2) + P (x = 3)
= 43k
= 43 / 72

[iv] P (x < -3)
= P (x = - 5) + P (x = - 4)
= 3k
= 3 / 72