KBPE Class 12th Maths Question Paper With Solution 2019

QUESTION PAPER CODE SY 27

Question 1 to 7 carries 3 scores each. Answer any six questions. [6 * 3 = 18]

Question 1[a]: If f (x) = sin x, g (x) = x^2 ; $x \in R$; then find (fog) (x).
[b] Let u and v be two functions defined on R as u (x) = 2x – 3 and v (x) = (3 + x) / 2 . Prove that u and v are inverse to each other.

Solution:
[a] f (x) = sin x
g (x) = x^2
(fog) (x) = f (g (x))
= f (x^2)
= sin (x^2)

[b] (u o v) x = u (v (x)) = u ((3 + x) / 2))
= [2 (3 + x) / 2] - 3 = x
(u o v) = I
(v o u) x = v (u (x))
= v (2x - 3)
= (3 + 2x - 3) / 2 = x
(v o u) = I

Question 2[a]: For the symmetric matrix A = $\begin{bmatrix} 2 & x & 4 \\ 5 & 3 & 8 \\ 4 & y & 9 \end{bmatrix}$. **Find the values of x and y.**
(b) From Part(a), verify AA’ and A + A’ are symmetric matrices.

Solution:

[a] x = 5 and y = 8

[b]

$$A = \begin{pmatrix} 2 & 5 & 4 \\ 5 & 3 & 8 \\ 4 & 8 & 9 \end{pmatrix} A' = \begin{pmatrix} 2 & 5 & 4 \\ 5 & 3 & 8 \\ 4 & 8 & 9 \end{pmatrix}$$

$$AA' = \begin{pmatrix} 2 & 5 & 4 \\ 5 & 3 & 8 \\ 4 & 8 & 9 \end{pmatrix} \begin{pmatrix} 2 & 5 & 4 \\ 5 & 3 & 8 \\ 4 & 8 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 45 & 57 & 84 \\ 57 & 98 & 116 \\ 84 & 116 & 161 \end{pmatrix}$$

$$A + A' = \begin{pmatrix} 2 & 5 & 4 \\ 5 & 3 & 8 \\ 4 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 2 & 5 & 4 \\ 5 & 3 & 8 \\ 4 & 8 & 9 \end{pmatrix} = \begin{pmatrix} 4 & 10 & 8 \\ 10 & 6 & 16 \\ 8 & 16 & 18 \end{pmatrix}$$

AA' and A + A' are symmetric matrices.

Question 3[a]: Find the slope of the tangent line to the curve $y = x^2 - 2x + 1$.
(b) Find the equation of the tangent to the above curve which is parallel to the line 2x – y + 9 = 0.

Solution:

[a] $y = x^2 - 2x + 1$

dy / dx = 2x - 2

Slope of tangent line = 2x - 2

[b] Since the tangent is parallel to 2x – y + 9 = 0, the slopes are the same.

2x – y + 9 = 0

y = 2x + 9

Slope = 2 [y = mx + c]

2x - 2 = 2

x = 2 and y = 1

The point is (2, 1).

The equation of the tangent is $(y - y_1) = (dy / dx)(x - x_1)$

$(y - 1) = 2(x - 2)$

$y - 2x + 3 = 0$

Question 4[a]: If $\int f(x)\,dx = \log|\tan x| + C$. Find $f(x)$.

[b] Evaluate $\int 1 / [\sqrt{1 - 4x^2}]\,dx$

Solution:

[a] $\int f(x)\,dx = \log|\tan x| + C$

$f(x) = \sec^2 x / \tan x$ or $2 \operatorname{cosec} 2x$

[b] $\int 1 / [\sqrt{1 - 4x^2}]\,dx$

$= \int 1 / [\sqrt{4(1/4 - x^2)}]\,dx$

$= (1/2) \int 1 / [\sqrt{(1/2)^2 - x^2}]\,dx$

$= (1/2) \sin^{-1}(x / [1/2]) + C$

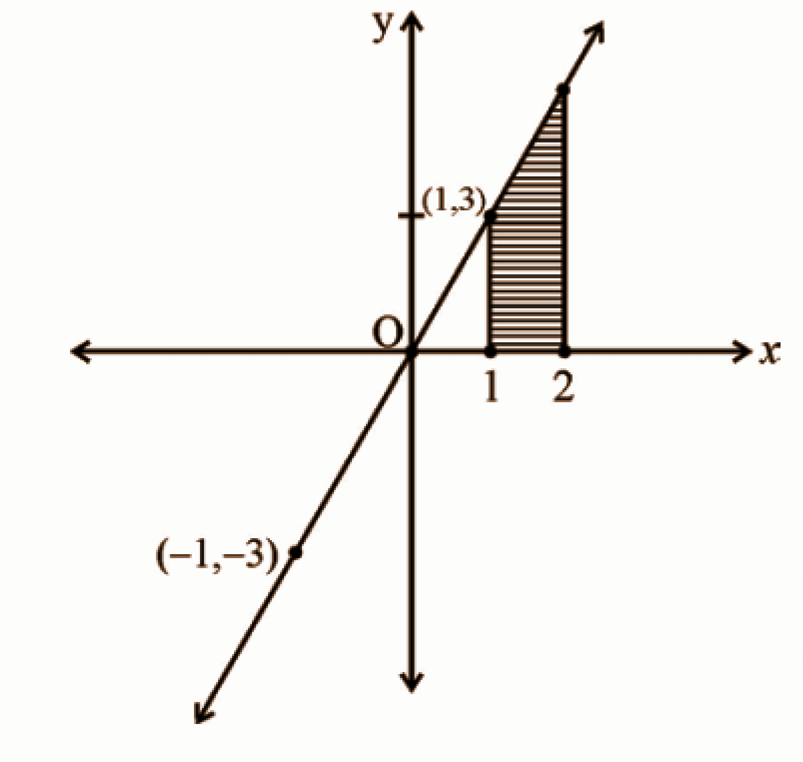
$= (1/2) \sin^{-1} 2x + C$

Question 5[a]: Area bounded by the curve y = f (x) and the lines x = a, x = b and the x axis = ______

(i) $\int_a^b x\,dy$ (ii) $\int_a^b x^2\,dy$ (iii) $\int_a^b y\,dx$ (iv) $\int_a^b y^2\,dx$

Answer: (iii)

[b] Find the area of the shaded region using integration.



Solution:

The curve is $y = 3x$.

Area = $\int_1^2 y\,dx$

$= \int_1^2 3x\,dx$

$= 3\,(x^2 / 2)_1^2$

$= 9 / 2$

Question 6[a]: The order of the differential equation formed by y = A sin x + B cos x + c, where A and B are arbitrary constants is

(i) 1 (ii) 2 (iii) 0 (iv) 3

(b) Solve the differential equation $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$.

Solution:

[a] (ii) or (iv)

[b] $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$[\sec^2 x / \tan x]\, dx + [\sec^2 y / \tan y]\, dy = 0$

$\int [\sec^2 x / \tan x]\, dx + \int [\sec^2 y / \tan y]\, dy = 0$

log |tan x| + log |tan y| = log C
tan x . tan y = C

Question 7: A factory produces three items P, Q and R at two plants A and B. The number of items produced and operating costs per hour is as follows :

Plant	Item produced per hour			Operating cost
	P	Q	R	
A	20	15	25	Rs. 1000
B	30	12	23	Rs. 800

It is desired to produce at least 500 items of type P, at least 400 items of type Q and at least 300 items of type R per day.
(a) Is it a maximization case or a minimization case. Why?
(b) Write the objective function and constraints.

Solution:
Minimum Z = 1000x + 800y
Subject to
$20x + 30y \geq 500$
$15x + 12y \geq 400$
$25x + 23y \geq 300$
$x, y \geq 0$

Questions 8 to 17 carry 4 scores each. Answer any eight. (8 * 4 = 32)

Question 8[a]: The function P is defined as "To each person on the earth is assigned a date of birth". Is this function one-one ? Give reason.
(b) Consider the function f : [0, π / 2] → R
given by f (x) = sin x and g : [0, π / 2] → R

given by g (x) = cos x.
(i) Show that f and g are one-one functions.
(ii) Is f + g one-one? Why?
(c) The number of one-one functions from a set containing 2 elements to a set containing 3 elements is ______

(i) 2 (ii) 3 (iii) 6 (iv) 8

Solution:

[a] The function is not one-one because different persons can have the same birthday.

[b] [i] f (x) = sin x
$f(x_1) = f(x_2)$
$\sin x_1 = \sin x_2$
$x_1 = x_2$
Thus f is one to one.
g (x) = cos x
$g(x_1) = g(x_2)$
$\cos x_1 = \cos x_2$
$x_1 = x_2$
Thus g is one to one.

[ii] (f + g) (x) = sin x + cos x
$(f + g)(x_1) = (f + g)(x_2)$
$\sin x_1 + \cos x_1 = \sin x_2 + \cos x_2$
$\sin x_1 - \sin x_2 = \cos x_2 - \cos x_1$
$\cos (x_1 + x_2) / 2 = \sin (x_1 + x_2) / 2$
$x_1 = (\pi / 2) - x_2$
f + g is not a one-one function.

[c] (iii)

Question 9: If $A = \sin^{-1} (2x / [1 + x^2])$, $B = \cos^{-1} [1 - x^2] / [1 + x^2]$, $C = \tan^{-1} [2x / 1 - x^2]$ satisfies the condition $3A - 4B + 2C = \pi / 3$. Find the value of x.

Solution:

A = $\sin^{-1}$ (2x / [1 + x^2]) = $\sin^{-1}$ (sin 2θ) = 2θ

B = $\cos^{-1}$ [1 - x^2] / [1 + x^2] = $\cos^{-1}$ (cos 2θ) = 2θ

C = $\tan^{-1}$ [2x / 1 - x^2] = $\tan^{-1}$ (tan θ) = 2θ

3A - 4B + 2C = **π** / 3

3 (2θ) - 4 (2θ) + 2 (2θ) = **π** / 3

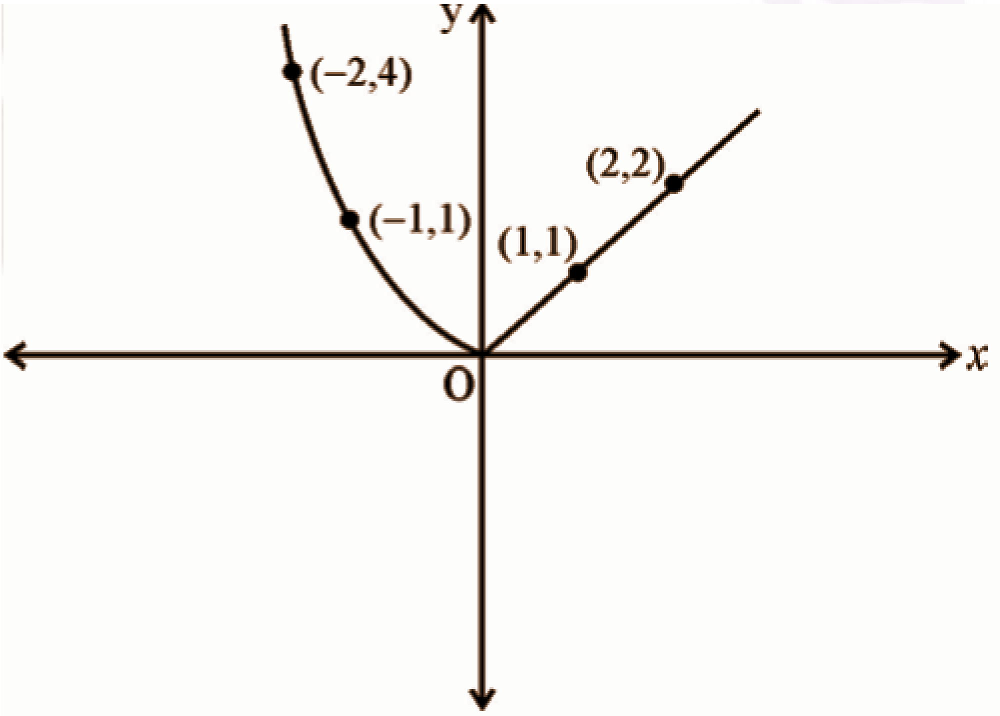
6θ - 8θ + 4θ = **π** / 3

2θ = **π** / 3

$\tan^{-1}$ x = **π** / 6

x = 1 / √3

Question 10[a]: Write the function whose graph is shown below.



(b) Discuss the continuity of the function obtained in part (a).

(c) Discuss the differentiability of the function obtained in part (a).

Solution:

[a] f (x) = $\begin{cases} x^2 & x < 0 \\ 0 & x = 0 \\ x & x > 0 \end{cases}$

[b] $\lim_{x \to 0+} f(x) = \lim_{x \to 0+} (x) = 0$

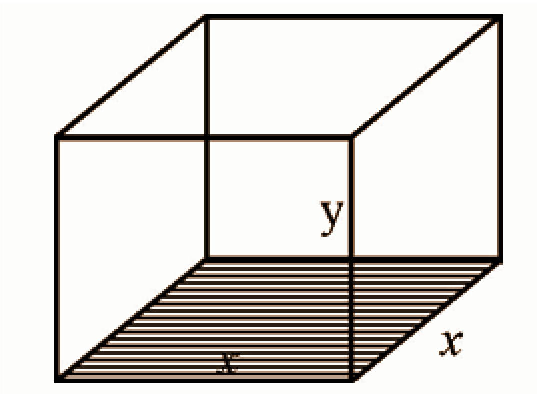
$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} (x^2) = 0$

$f(0) = 0$

f (x) is continuous at x = 0.

[c] f (x) is not differentiable at x = 0.

Question 11: A cuboid with a square base and given volume 'V' is shown in the figure.



(a) Express the surface area 's' as a function of x.
(b) Show that the surface area is minimum when it is a cube.

Solution:

[a] Surface area (S) = $2x^2 + 4xy$

$v = x^2y$

$= 2x^2 + 4x * (v / x^2)$

$= 2x^2 + (4v / x)$

[b] $ds / dx = 4x - (4v / x^2)$

$ds / dx = 0$

$v = x^3$

$x = (v)^{1/3}$

$d^2s / dx^2 = 4 + (8v / x^3) = 4 + 8 = 12 > 0$

Hence, S is minimum.

$y = v / x^2 = x^3 / x^2 = x$

Therefore, cuboid becomes a cube.

Question 12[a]: If $2x + 4 = A(2x + 3) + B$, find A and B.
[b] Using part (a) evaluate $\int\{(2x + 4) / [x^2 + 3x + 1]\}dx$.

Solution:
[a] A = 1 and B = 1

[b] $\int\{(2x + 4) / [x^2 + 3x + 1]\}dx$
$= \int\{(2x + 3) / [x^2 + 3x + 1]\}dx + \int\{1 / [x^2 + 3x + 1]\}dx$
$= \log|x^2 + 3x + 1| + \int\{1 / [x^2 + 3x + (9 / 4) - (9 / 4) + 1]\}dx$
$= \log|x^2 + 3x + 1| + \int 1 / [(x + (3 / 2)^2 - (\sqrt{5} / 2)^2]$
$= \log|x^2 + 3x + 1| + (1 / 2 * (\sqrt{5} / 2)) \log|(x + (3 / 2) - (\sqrt{5} / 2)) / (x + (3 / 2) + (\sqrt{5} / 2))$
$= \log|x^2 + 3x + 1| + [1 / \sqrt{5}] \log|(x + [3 - \sqrt{5}] / 2) / (x + [3 + \sqrt{5}] / 2)|$

Question 13: Consider the differential equation $\cos^2x\,(dy / dx) + y = \tan x$.
Find
(a) its degree
(b) the integrating factor
(c) the general solution.

Solution:
[a] one

[b] IF $= e^{\int P\,dx}$
$= e^{\int \sec^2 x\,dx}$
$= e^{\tan x}$

[c] $y * e^{\tan x} = \int(\tan x / \cos^2 x) * e^{\tan x}\,dx$
Put $u = \tan x$
$y * e^{\tan x} = \int ue^u\,du$
$= \tan x\, e^{\tan x} - e^{\tan x} + C$

Question 14: The position vectors of three points A, B, C are given to be i + 3 j + 3k, 4i + 4k and –2 i + 4 j + 2k respectively.
(a) Find AB and AC.
(b) Find the angle between AB and AC.
(c) Find a vector which is perpendicular to both AB and AC having a magnitude 9 units.

Solution:
[a] AB = 3i - 3j + k
BC = - 3i + j - k

[b] |AB| = √19
|AC| = √11
cos θ = |[AB . AC] / [|AB| . |AC|]
= |(− 13) / [√19 . √11]|
= 13 / [√19 . √11]
$\theta = \cos^{-1}$ [13 / [√19 . √11]]

[c] n = (AB x AC) / |(AB x AC)|
= 2i - 6k
|AB x AC| = √40
The required vector = 9 (2i − 6k) / √40

Question 15[a]: If a, b, c are coplanar vectors, write the vector perpendicular to a.
(b) If a, b, c are coplanar, prove that a + b, b + c, c + a are coplanar.

Solution:
[a] (a x b) or (b x c) or (a x c) or (b x a) or (c x b) or (c x a)

[b] [a b c] = 0
[a + b, b + c, c + a] = (a + b) . [(b + c) x (c + a)]
= (a + b) . [(b x c) + (b x a) + (c x c) + (c x a)]

= a . (b x c) + b . (c x a)
= 2 [a b c]
= 0
So, a + b, b + c, c + a are coplanar.

Question 16[a]: Write all the direction cosines of the x-axis.
(b) If a line makes angles α, β, γ with x, y, z axes respectively, then prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$.
(c) If a line makes equal angles with the three coordinate axes, find the direction cosines of the lines.

Solution:
[a] 1, 0, 0 or cos 0, cos 90, cos 90

[b] $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
$(1 - \sin^2 \alpha) + (1 - \sin^2 \beta) + (1 - \sin^2 \gamma) = 1$
$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

[c] $\alpha = \beta = \gamma$
$\cos \alpha = \cos \beta = \cos \gamma$
$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
$3 \cos^2 \alpha = 1$
$\cos \alpha = 1 / \sqrt{3}$
Similarly, $\beta = 1 / \sqrt{3}$ and $\gamma = 1 / \sqrt{3}$

Question 17: The activities of a factory are given in the following table:

Items	**Departments**			**Profit per unit**
	Cutting	**Mixing**	**Packing**	
A	**1**	**3**	**1**	**Rs. 5**
B	**4**	**1**	**1**	**Rs. 8**

Maximum time available	**24**	**21**	**9**	

Solve the linear programming problem graphically and find the maximum profit subject to the above constraints.

Solution:

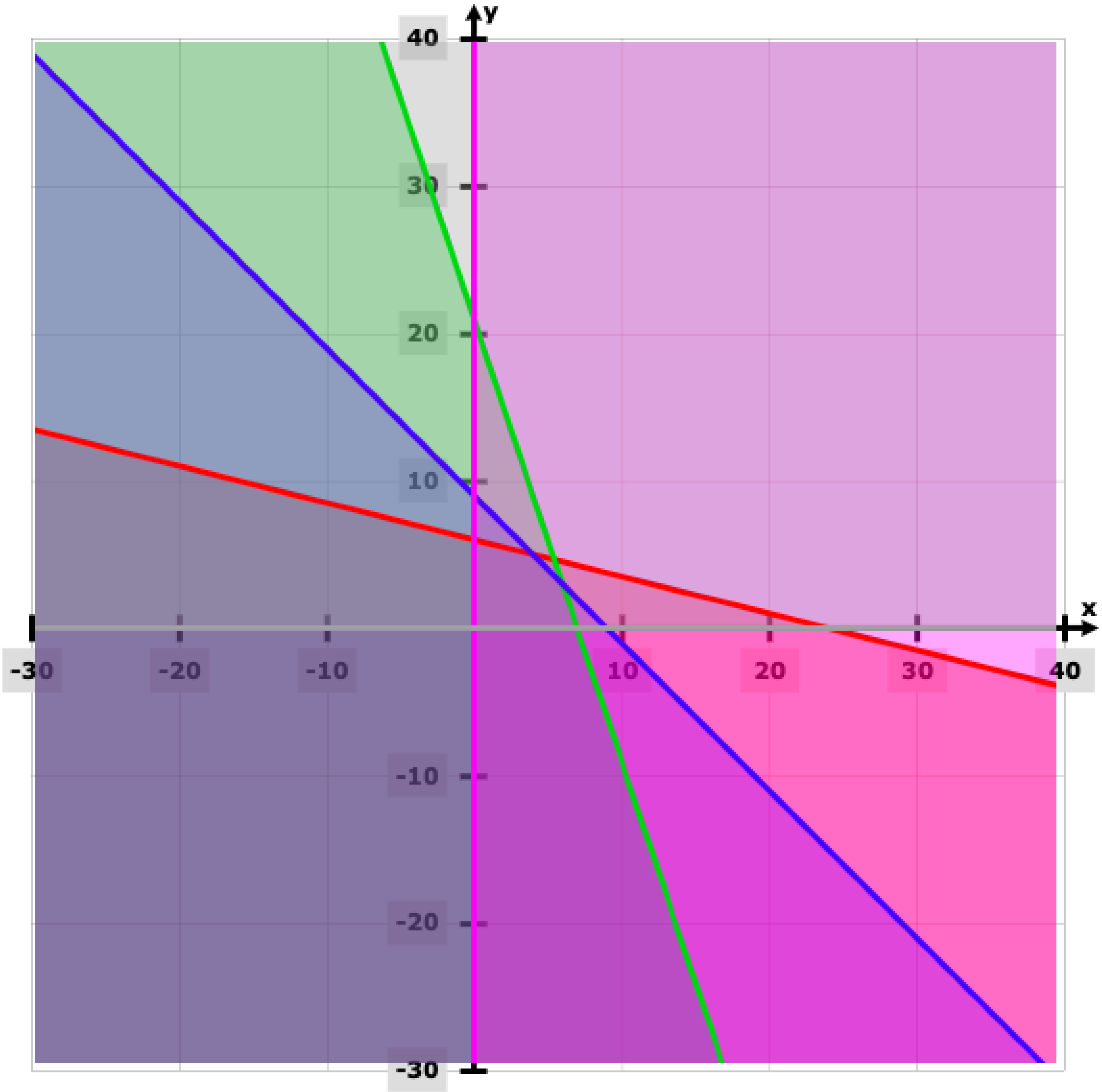
$x + 4y \le 24$

$3x + y \le 21$

$x + y \le 9$

$x, y \ge 0$

Maximise $Z = 5x + 8y$



(0, 0) = 0
(7, 0) = 35
(6, 3) = 54
(4, 5) = 60
(0, 6) = 48
Z is maximum at (4, 5) = 60.

Questions from 18 to 24 carry 6 scores each. Answer any five. (5 * 6 = 30)

Question 18: If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. Show that $A^2 - 5A + 7I = 0$. Hence find A^4 and A^{-1}.

Solution:

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}; 7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 5A + 7I = 0 \Rightarrow A^2 = 5A - 7I$$

$$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$A^4 = A^2.A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 39 & 55 \\ -55 & 16 \end{bmatrix}$$

$$A^2 - 5A + 7I = 0$$

Multiply by A^{-1}

$A^{-1}(A^2 - 5A + 7I) = 0$

$A - 5I + 7A^{-1} = 0$

$7A^{-1} = 5I - A$

$A^{-1} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = (\frac{1}{7}) \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

Question 19: If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then

[a] Find A^{-1}

[b] Use A^{-1} from part (a) solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

Solution:

Write the augmented matrix

	A_1	A_2	A_3	B_1	B_2	B_3
1	2	-3	5	1	0	0
2	3	2	-4	0	1	0
3	1	1	-2	0	0	1

Find the pivot in the 1st column and swap the 3rd and the 1st rows

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	1	-2	0	0	1
2	3	2	-4	0	1	0
3	2	-3	5	1	0	0

Eliminate the 1st column

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	1	-2	0	0	1
2	0	-1	2	0	1	-3
3	0	-5	9	1	0	-2

Find the pivot in the 2nd column in the 2nd row (inversing the sign in the whole row)

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	1	-2	0	0	1
2	0	1	-2	0	-1	3
3	0	-5	9	1	0	-2

Eliminate the 2nd column

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	0	0	0	1	-2
2	0	1	-2	0	-1	3
3	0	0	-1	1	-5	13

Find the pivot in the 3rd column in the 3rd row (inversing the sign in the whole row)

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	0	0	0	1	-2
2	0	1	-2	0	-1	3
3	0	0	1	-1	5	-13

Eliminate the 3rd column

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	0	0	0	1	-2
2	0	1	0	-2	9	-23
3	0	0	1	-1	5	-13

The inverse matrix is on the right.

[b]

$$AX = B$$

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$X = A^{-1}B \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Question 20: Find dy / dx for the following.
[a] $\sin^2 x + \cos^2 y = 1$
(b) $y = x^x$
(c) x = a (t – sin t); y = a (1 + cos t)

Solution:
[a] $\sin^2 x + \cos^2 y = 1$
2 sin x cos x + 2 cos y (- sin y) (dy / dx) = 0
cos y (- sin y) (dy / dx) = - sin x cos x
dy / dx = sin x cos x / sin y cosy

[b] $y = x^x$
log y = x log x
(1 / y) (dy / dx) = x * (1 / x) + log x
dy / dx = y [1 + log x]
= x^x [1 + log x]

[c] x = a (t – sin t); y = a (1 + cos t)
dx / dt = a (1 - cos t)
dy / dt = - a sint
dy / dx = (dy / dt) / (dx / dt)
= - a sint / a (1 - cos t)
= - sint / 1 - cost
= - cot (t / 2)

Question 21: Evaluate the following integrals.

[a] $\int_0^{\pi/2}$ [(sin x) / [sin x + cos x]] dx

[b] $\int_{-\pi/2}^{\pi/2} \sin^7 x$ dx

[c] ∫x sin 3x dx

Solution:

[a] $\int_0^{\pi/2}$ [(sin x) / [sin x + cos x]] dx

I = $\int_0^{\pi/2}$ [(sin x) / [sin x + cos x]] dx

= $\int_0^{\pi/2}$ [sin (π / 2 - x)] / [sin (π / 2 - x + cos (π / 2 - x)]

= $\int_0^{\pi/2}$ [cos x / cos x + sin x] dx

2I = $\int_0^{\pi/2}$ 1 dx

= $[x]_0^{\pi/2}$

= π / 2

I = π / 4

[b] $\int_{-\pi/2}^{\pi/2} \sin^7 x$ dx = 0

Since it is an odd function.

[c] ∫x sin 3x dx

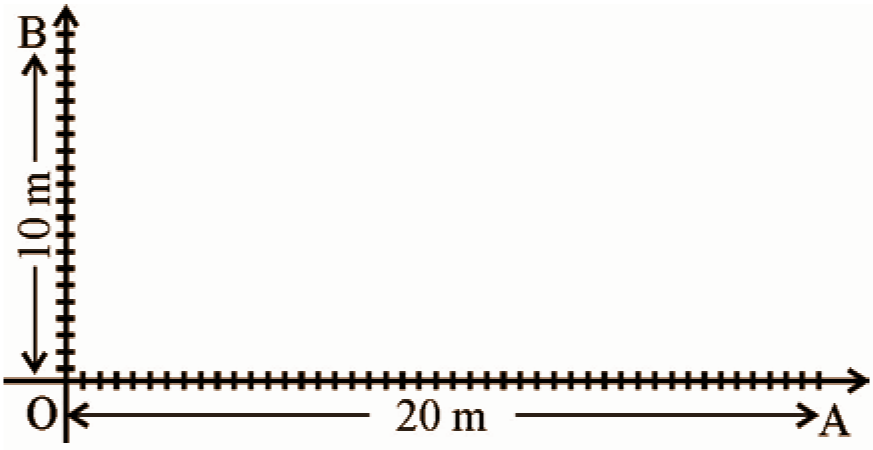
= x ∫sin 3x dx – ∫1 [∫sin 3x dx] dx

= x . [– cos 3x / 3] – ∫[– cos 3x / 3]dx

= [- x cos 3x / 3] + sin 3x / 9 + C

Question 22: (a) Find the area bounded by the curve y = sin x and the lines x = 0, x = 2π, and x-axis.

(b) Two fences are made in a grass field as shown in the figure. A cow is tied at the point O with a rope of length 3 m.



(i) Using integration, find the maximum area of grass that cows can graze within the fences. Choose O as the origin.
(ii) If there are no fences, find the maximum area of grass that cow can graze?

Solution:

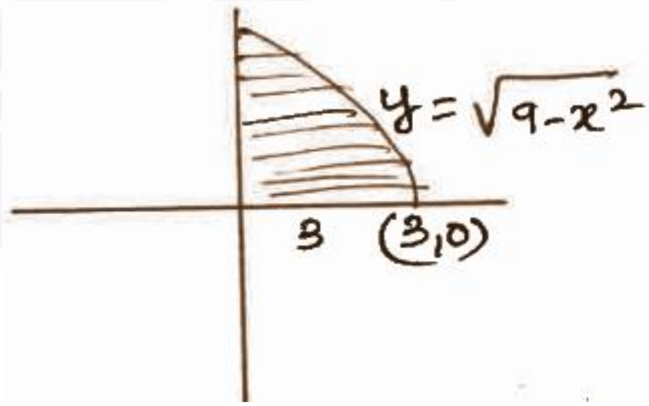
[a] Area = $\int_a^b y\, dx$

$= 4 \int_0^{\pi/2} \sin x\, dx$

$= 4 * 1$

$= 4$

[b] [i]



The equation of the curve is $x^2 + y^2 = 9$.

$y = \sqrt{9 - x^2}$

Area = $\int_a^b y\, dx$

$= \int_0^3 \sqrt{9 - x^2}\, dx$

$= [(x / 2) \sqrt{9 - x^2} + (9 / 2) \sin^{-1} (x / 3)]_0^3$

$= (9 / 2) \sin^{-1} (1)$

$= 9\pi / 4$ square units

[ii] Required area $= 4 * (9\pi / 4)$

$= 9\pi$ square units

Question 23: [a] Find the equation of the plane through the intersection of the planes 3x – y + 2z – 4 = 0 and x + y + z – 2 = 0 and the point (2, 2, 1).
(b) The Cartesian equation of two lines are given by x + 1 / 7 = y + 1 / –6 = z + 1 / 1 and x – 3 / 1 = y – 5 / –2 = z – 7 / 1 . Write the vector equation of these two lines.
(c) Find the shortest distance between the lines mentioned in part (b).

Solution:

[a] $(3x – y + 2z – 4) + k (x + y + z – 2) = 0$

It passes through (2, 2, 1).

$[3 * 2 - 2 + 2 * 1 - 4] + k [2 + 2 + 1 - 2] = 0$

$k = - 2 / 3$

$(3x – y + 2z – 4) - (2 / 3) (x + y + z – 2) = 0$

$7x - 5y + 4z - 8 = 0$

[b] $r = (- i - j - k) + \lambda (7i - 6j + k)$

$r = (3i + 5j + 7k) + \mu (i - 2j + k)$

[c] Shortest distance $= |(a_2 - a_1) . (b_1 \times b_2) / |(b_1 \times b_2)||$

$(a_2 - a_1) = 4i + 6j + 8k$

$(b_1 \times b_2) = - 4i - 6j - 8k$

$SD = |- 116 / \sqrt{116}| = \sqrt{116}$

Question 24: [a] A bag contains 4 red and 4 black balls. Another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag and which is found to be red. Find the probability that the ball is drawn from the first bag.
(b) A random variable X has the following distribution function :

X	0	1	2	3	4
P (x)	**k**	**3k**	**5k**	**7k**	**4k**

(i) Find k.
(ii) Find the mean and the variance of the random variable x.

Solution:
[a] Let E_1 be the event of choosing bag I and E_2 be the event of choosing bag II, also A be the event of choosing a red ball.
$P(E_1) = P(E_2) = 0.5$
$P(A / E_1) = 4 / 8 = 1 / 2$
$P(A / E_2) = 2 / 8 = 1 / 4$
$P(E_1 / A) = [P(E_1) * P(A / E_1)] / [P(E_1) * P(A / E_1) + P(E_2) * P(A / E_2)]$
$= (0.5 * 0.5) / [(0.5 * 0.5) + (0.5 * 0.25)$
$= 2 / 3$

[b] [i] $\sum P_i = 1$
$k + 3k + 5k + 7k + 4k = 1$
$20k = 1$
$k = 1 / 20$

[ii]

X	0	1	2	3	4
P (x)	1 / 20	3 / 20	5 / 20	7 / 20	4 / 20
X * P(x)	0	3 / 20	10 / 20	21 / 20	16 / 20
X^2 * P(x)	0	3 / 20	20 / 20	63 / 20	64 / 20

Mean $= \sum x * P(x) = 50 / 20 = 5 / 2$
Variance $= \sum x^2 * P(x) - [\sum x * P(x)]^2 = 5 / 4$