MSBSHSE Class 12th Maths Question Paper With Solutions 2017

QUESTION PAPER CODE J - 541

SECTION - A

Question 1[A]: Select and write the most appropriate answer from the givenalternatives in each of the following sub-questions.[6]

[i]: If the points A (2, 1, 1), B (0, 1, 4) and C (k, 3, 2) are collinear, then k =____.

(A) 0 (B) 1 (C) 4 (D) - 4

Answer: (c)

Direction ratio of AB = (-2, -2, 3)Direction ratio of BC = (k, 4, -6)If the points are collinear, then, Direction ratio of AB / Direction ratio of BC = constant -2 / k = -2 / 4k = 4

[ii]: The inverse of matrix $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}_{is}$

(A)
$$\frac{1}{13}\begin{bmatrix} 2 & -5\\ 3 & -1 \end{bmatrix}$$

(C) $\frac{1}{13}\begin{bmatrix} -1 & -3\\ 5 & 2 \end{bmatrix}$







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	Α ₁	Α2	B ₁	В2
1	-1	5	1	0
2	-3	2	0	1

Find the pivot in the 1st column in the 1st row (inversing the sign in the whole row)

	Α ₁	A ₂	B ₁	B ₂
1	1	-5	-1	0
2	-3	2	0	1

Eliminate the 1st column

	A ₁	A ₂	В1	B ₂
1	1	-5	-1	0
2	0	-13	-3	1

) Q^Q

Make the pivot in the 2nd column by dividing the 2nd row by -13

	A ₁	A ₂	B ₁	B_2
1	1	-5	-1	0
2	0	1	3/13	-1/13

Eliminate the 2nd column

	Α ₁	A ₂	B ₁	B ₂
1	1	0	2/13	-5/13
2	0	1	3/13	-1/13

There is the inverse matrix on the right

	Α ₁	A ₂	B ₁	B ₂
1	1	0	2/13	-5/13
2	0	1	3/13	-1/13

[iii]: In \triangle ABC, if a = 13, b = 14 and c = 15, then sin [A / 2] is

(A) 1 / 5 (B) $\sqrt{(1/5)}$ (C) 4 / 5 (D) 2 / 5

Answer: (b)

a = 13, b = 14 and c = 15 s = [a + b + c] / 2= [13 + 14 + 15] / 2= 21 sin (A / 2) = $\sqrt{(s - b)}$ (s - c) / b * c = $\sqrt{(21 - 14)}$ (21 - 15) / 14 * 15 = $\sqrt{7} * 6 / 14 * 15$ = $\sqrt{1} / 5$

[B] Attempt any THREE of the following:

[i]: Find the volume of the parallele π ped whose coterminous edges are given by vectors 2i + 3j - 4k, 5i + 7j + 5k and 4i + 5j - 2k.

Solution:

The volume of the parallele π ped = [a b c] a = 2i + 3j - 4k b = 5i + 7j + 5k c = 4i + 5j - 2k V = [a b c] = 2 (-14 - 25) - 3(-10 - 20) - 4 (25 - 28) = 2 (-39) - 3 (-30) - 4 (-3) = -78 + 90 + 12 = 24 cubic units

[ii]: In $\triangle ABC$, prove that, a (b cos C - c cos B) = b² - c².

Solution:

(6)

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Law of cosines: c^2 = a^2 + b^2 - 2ab \cos y

b^2 = a^2 + c^2 - 2ac \cos B

c^2 = a^2 + b^2 - 2ab \cos C

The second equation is subtracted from the first one,

b^2 - c^2 = c^2 - b^2 - 2ac \cos B + 2ab \cos C

2b^2 - 2c^2 = -2ac \cos B + 2ab \cos C

2 (b^2 - c^2) = 2a (-c \cos B + b \cos C)

b^2 - c^2 = a (b \cos C - c \cos B)
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[iii]: If from a point Q (a, b, c) perpendiculars QA and QB are drawn to the YZ and ZX planes respectively, then find the vector equation of the plane QAB.

Solution:

QA and QB are the perpendiculars from the point Q(a, b, c) to YZ and ZX planes. A = (0, b, c) and B = (a, 0, c)

The required plane is passing through O(0, 0, 0), A(0, b, c) and B(a, 0, c)

The vector equation of the plane passing through the O, A, B is

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r. (OA x OB) = 0. (OA x OB)

r. (a x b) = 0

OA = a = 0i + bj + ck

OB = b = ai + 0j + ck

\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & b & c \\ a & 0 & c \end{vmatrix}

OA x OB = \begin{vmatrix} a & 0 & c \end{vmatrix}

= (bc - 0)i - (0 - ac)j + (0 - ab)k

= bci + acj - abk

The vector equation of the required plane is r. (bci + acj - abk) = 0.
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[iv]: Find the cartesian equation of the line passing through the points A(3, 4, -7) and B(6, -1, 1).

Solution:

The cartesian equation of the line passing through the points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) is $(x - x_1) / (x_2 - x_1) = (y - y_1) / (y_2 - y_1) = (z - z_1) / (z_2 - z_1)$ The cartesian equation of the line passing through the points A (3, 4, -7) and B (6, -1, 1) is (x - 3) / (6 - 3) = (y - 4) / (-1 - 4) = (z - (-7)) / (1 - (-7))(x - 3) / (3) = (y - 4) / (-5) = (z + 7) / (8)

[v]: Write the following statement in symbolic form and find its truth value: \forall n ϵ N, n² + n is an even number and n² - n is an odd number.

Solution:

Let $\mathbf{p} \equiv \forall \mathbf{n} \in \mathbf{N}$, $\mathbf{n}^2 + \mathbf{n}$ is an even number Let $\mathbf{q} \equiv \forall \mathbf{n} \in \mathbf{N}$, $\mathbf{n}^2 - \mathbf{n}$ is an odd number The symbolic form of the given statement is $(\mathbf{p} \land \mathbf{q})$ The truth value of a given statement is $\mathbf{p} \equiv \forall \mathbf{n} \in \mathbf{N}$, $\mathbf{n}^2 + \mathbf{n}$ is an even number (T) $\mathbf{q} \equiv \forall \mathbf{n} \in \mathbf{N}$, $\mathbf{n}^2 - \mathbf{n}$ is an odd number (F) (\because from $\mathbf{n} = 1$, $\mathbf{n}^2 - \mathbf{n} = 0$, which is not an odd number) $(\mathbf{p} \land \mathbf{q}) \equiv \mathbf{T} \land \mathbf{F} \equiv \mathbf{F}$ So, the given statement is false.

Question 2[A]: Attempt any TWO of the following: (6)

[i] Using truth tables, examine whether the statement pattern $(p \land q) \lor (p \land r)$ is a tautology, contradiction or contingency.

Solution:

Number of rows $= 2^n = 2^3 = 8$ Number of columns = m + n = 3 + 3 = 6

p q	r	$p\wedgeq$	$p \wedge \mathtt{r}$	(p \land q) \lor
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					(p \wedge r)
Т	Т	Т	Т	Т	Т
Т	Т	F	Т	F	Т
Т	F	Т	F	Т	Т
Т	F	F	F	F	F
F	Т	Т	F	F	F
F	Т	F	F	F	F
F	F	Т	F	F	F
F	F	F	F	F	F

Therefore, it is a contingency.

[ii] Find the shortest distance between the lines: x - 1/2 = y - 2/3 = z - 3/4and x - 2/3 = y - 4/4 = z - 5/5.

Solution:

The lines are x - 1/2 = y - 2/3 = z - 3/4 - ... (1) x - 2/3 = y - 4/4 = z - 5/5 - ... (2)Here x1 = 1, y1 = 2, z1 = 3 and a1 = 3, b1 = 3, c1 = 4 x2 = 2, y2 = 4, z2 = 5 and a2 = 3, b2 = 4, c2 = 5Shortest distance between two lines is

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}$$

= 1 (15 - 16) - 2 (10 - 12) + 2 (8 - 9)
= -1 + 4 - 2
= 1

Denominator = $(15 - 16)^2 + (12 - 10)^2 + (8 - 9)^2 = 1 + 4 + 1 = 6$ d = 1 / $\sqrt{6}$ units

[iii] Find the general solution of the equation $\sin 2x + \sin 4x + \sin 6x = 0$.

Solution:

 $(\sin 2x + \sin 6x) + \sin 4x = 0$ $2\sin 4x \cdot \cos 2x + \sin 4x = 0$ $\sin 4x (2\cos 2x + 1) = 0$ $\sin 4x = 0$ or $2\cos 2x + 1 = 0$ $\sin 4x = 0$ or $\cos 2x = -1/2$ $= -\cos \pi / 3$ $= \cos (\pi - \pi / 3)$ Using sinx = $0 \Rightarrow x = n\pi$ $\sin 4x = 0$ $4\mathbf{x} = \mathbf{n}\boldsymbol{\pi}$ The general solution is $x = (n\pi) / 4$ Using $\cos x = \cos a \Rightarrow x = 2mx \pm a$ $\cos 2x = \cos ((2\pi) / 3)$ $2x = 2m\pi \pm (2\pi) / 3$ The general solution is $x = m\pi \pm \pi / 3$ where m, n is in z.

[B] Attempt any TWO of the following:

(8)

[i] Solve the following equations by method of reduction: x - y + z = 4, 2x + y - 3z = 0, x + y + z = 2.

Solution:

x - y + z = 42x + y - 3z = 0x + y + z = 2

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

 $R_2 - 2R_1 \text{ and } R_3 - R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & -5 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -8 \\ -2 \end{bmatrix}$$

 $x - y + z = 4 \dots (1)$
 $3y - 5z = -8 \dots (2)$
 $2y = -2 \dots (3)$
 $y = -1$
By equation (2)
 $-3 - 5z = -8 - 5z = -5z = 1$
By equation (1)
 $x + 1 + 1 = 4$
 $x = 2$
 $x = 2, y = -1, z = 1$

[ii] If θ is the measure of the acute angle between the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan \theta = |2\sqrt{h^2} - ab / a + b|$ where $a + b \neq 0$ and $b \neq 0$. Find the condition for coincident lines.

Solution:

Let m1 and m2 be the slopes of the lines represented by the equation, $ax^2 + 2hxy + by^2 = 0 ---- (1)$ $y = m_1x$ and $y = m_2x$ The combined equation is $(y - m_1x) (y - m_2x) = 0$ $m_1m_2x^2 - (m_1 + m_2)xy + y^2 = 0 --- (2)$ Since equations (1) and (2) represents the same two lines, on comparing the coefficients,

 $m_{1}m_{2} / a = 1 / b = m_{1} + m_{2} / 2h$ $m_{1} + m_{2} = -2h / b \text{ and } m_{1}m_{2} = a / b$ $(m_{1} - m_{2})^{2} = (m_{1} + m_{2})^{2} - 4m_{1}m_{2}$ $(m_{1} - m_{2})^{2} = (-2h / b)^{2} - 4 (a / b)$ $= 4 (h^{2} - ab) / b^{2}$ The angle between them is given by $tan \theta = |m_{1} - m_{2} / 1 + m_{1} - m_{2}|$ $= |\sqrt{(m_{1} - m_{2})^{2} / 1 + m_{1} - m_{2}|}$ $= |4 \sqrt{(h^{2} - ab) / b^{2} / 1 + (a / b)|}$ $= |2\sqrt{h^{2} - ab / a + b|}$ $tan \theta = |2\sqrt{h^{2} - ab / a + b|}$

[iii] Using the vector method, find incentre of the triangle whose vertices are P (0, 4, 0), Q (0, 0, 3) and R(0, 4, 3).

Solution:

Let p, q, r be the position vectors of vertices P, Q, R of triangle PQR respectively. p = 4j, q = 3k, r = 4j + 3k PQ = q - p = 3k - 4j = - 4j + 3k QR = r - q = 4j + 3k - 3k = 4j RP = p - r = 4j - 4j - 3k = - 3k Let x, y, z be the lengths of opposites of vertices P, Q, R respectively. x = |QR| = 4y = |RP| = 3z = |PQ| = 5The incentre of the triangle = h = (xp + yq + zr) / x + y + z= 4 (4j) + 3 (3k) + 5 (4j + 3k) / 4 + 3 + 5 = 16j + 9k + 20j + 15k / 12 = 3j + 2k

Question 3[A]: Attempt any TWO of the following:

(6)

[i] Construct the switching circuit for the statement $(p \land q) \lor (\sim p) \lor (p \land \sim q)$.

Solution:

Let p = S1 is closed and q = S2 is closed. $\sim p =$ switch S_1 ' and $\sim q = S_2$ '



[ii] Find the joint equation of the pair of lines passing through the origin which are perpendicular respectively to the lines represented by $5x^2 + 2xy - 3y^2 = 0$.

Solution:

Given homogeneous equation is $5x^2 + 2xy - 3y^2 = 0$ $5x^2 + 5xy - 3xy - 3y^2 = 0$ 5x (x + y) - 3y (x + y) = 0 (x + y) (5x - 3y) = 0 x + y = 0 and 5x - 3y = 0 are the two lines represented by the given equation. The slopes are -1 and 5 / 3.

The required two lines are perpendicular to these lines.

The slopes of required lines are 1 and 3/5 and the lines pass through the origin. The individual equations are

y = 1 . x and y = -3/5 x x - y = 0 and 3x + 5y = 0 Their joint equation is (x - y) (3x + 5y) = 0 3x² - 3xy + 5xy - 5y² = 0 3x² + 2xy - 5y² = 0

[iii] Show that $\cos^{-1}(4 / 5) + \cos^{-1}(12 / 13) = \cos^{-1}(33 / 65)$.

Solution:

Let $a = \cos^{-1}(4 / 5)$ and $b = \cos^{-1}(12 / 13)$ $\cos a = 4 / 5$ $\sin^2 a = 1 - \cos^2 a$ $\sin a = \sqrt{1 - \cos^2 a}$ $=\sqrt{1} - (4/5)^2$ $=\sqrt{1} - (16/25)$ $= \sqrt{9} / 25$ = 3 / 5 $\cos b = 12 / 13$ $\sin^2 b = 1 - \cos^2 b$ $\sin b = \sqrt{1 - \cos^2 b}$ $=\sqrt{1} - (12/13)^2$ $=\sqrt{1} - (144 / 169)$ $= \sqrt{25} / 169$ = 5 / 13 $\cos(a+b) = \cos a \cos b - \sin a \sin b$ = (4 / 5) * (12 / 13) - (3 / 5) * (5 / 13)= (48 - 15) / 65= 33 / 65 $\cos(a + b) = 33 / 65$ $a + b = \cos^{-1} (33 / 65)$ $\cos^{-1}(4 / 5) + \cos^{-1}(12 / 13) = \cos^{-1}(33 / 65)$

[B] Attempt any TWO of the following:

(8)

[i] If l, m, n are the direction cosines of a line, then prove that $l^2 + m^2 + n^2 = 1$. Hence find the direction angle of the line with the c-axis which makes direction angles of 135° and 45° with Y and Z axes respectively.

Solution:

Let α , β , γ be the angles made by the line with X, Y, Z axes respectively. $1 = \cos \alpha$, $m = \cos \beta$ and $n = \cos \gamma$

Let $a = a_1i + a_2j + a_3k$ be any non-zero vector along the line. Since i is the unit vector along X-axis, a. i = |a|. $|i| \cos a = a \cos a$ Also, a . $i = (a_1i + a_2j + a_3k)$. i $= a_1 x 1 + a_2 x 0 + a_3 x 0 = a_1$ $a \cos a = a_1$ (1) Since j is the unit vector along Y-axis, a. $\mathbf{j} = |\mathbf{a}|$. $|\mathbf{j}| \cos\beta = \mathbf{a} \cos\beta$ $a \cdot j = (a_1i + a_2j + a_3k) \cdot j$ $= a_1 x 0 + a_2 x 1 + a_3 x 0 = a_2$ $a \cos \beta = a_2$ (2) $a \cos y = a_3 \dots (3)$ From equations (1), (2) and (3), $a^{2}\cos^{2}\alpha + a^{2}\cos^{2}\beta + a^{2}\cos^{2}\nu = a_{1}^{2} + a_{2}^{2} + a_{3}^{2}$ $a^{2}(\cos^{2} a + \cos^{2} \beta + \cos^{2} \nu) = a^{2}$ [a = |a| = $\sqrt{(a_{1}^{2} + a_{2}^{2} + a_{3}^{2})}$] $(\cos^2 a + \cos^2 \beta + \cos^2 \gamma) = 1$ $l^2 + m^2 + n^2 = 1$ $a = ?, \beta = 135, \gamma = 45$ $(\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma) = 45$ $\cos^2 a + \cos^2 135 + \cos^2 45 = 1$ $\cos^2 a + 1 / 2 + 1 / 2 = 1$ $\cos \alpha = 0$ $a = \pi / 2$ or $(3\pi) / 2$

[ii] Find the vector and cartesian equations of the plane passing through the points A (1, 1, 2), B (1, 2, 1) and C (2, 1, 1).

Solution:

The vector equation of the plane passing through the points A (a), B (b) and C (c). r. ((AB) x (AC)) = a ((AB) x (AC))....(1) Let a = i + j - 2k, b = i + 2j + k, c = 2i - j + k(AB) = b - a = (i + 2j + k) - (i + j - 2k) = j + 3k and (AC) = c - a = (2i - j + k) - (i - j - 2k) = i - 2j + 3k (AB) x (AC) = |[i, j, k], [0, 1, 3], [1, -2, 3]| = (3 + 6)i - (0 - 3)j + (0 - 1)k= 9i + 3j - k a. ((AB) x (AC)) = (i + j - 2k) . (9i + 3j - k) = 1 (9) + 1 (3) + (-2) (-1) = 9 + 3 + 2 = 14 From (1), the vector equation of the required plane is r . (9i + 3j - k) = 14 (xi + yj + zk) (9i + 3j - k) = 14 The cartesian equation of the plane is 9x + 3y - z = 14.

[iii] Solve the following L.P.P. by the graphical method: Maximise : Z = 6x + 4y subject to $x \le 2$, $x + y \le 3$, $-2x + y \le 1$, $x \ge 0$, $y \ge 0$.

Solution:



Z = 6x + 4y Z = 6x + 4y Z at (0, 0) = 6 (0) + 4 (0) = 0 Z at (2, 0) = 6 (2) + 4 (0) = 12Z at (2, 1) = 6 (2) + 4 (1) = 16 Z at (2/3, 7/3) = 6(2/3) + (7/3) 4 = 40/3Z at (0,1) = 6(0) + 4(1) = 4Thus, Z is maximized at (2, 1) and its maximum value is 16.

SECTION - II

Question 4[A]: Select and write the appropriate answer from the givenalternatives in each of the following sub-questions:(6)

[i] Derivatives of $\tan^3 \theta$ with respect to $\sec^3 \theta$ at $\theta = \pi / 3$ is _____.

(A) 3/2 (B) $\sqrt{3}/2$ (C) 1/2 (D) $-\sqrt{3}/2$

Answer: (b) $y = \tan^3 \theta$ and $x = \sec^3 \theta$ $dy / d\theta = 3 \tan^2 \theta \sec^2 \theta$ $dx / d\theta = 3 \sec^2 \theta \sec \theta \tan \theta$ $dy / dx = \sin \theta = \sin (\pi / 3) = \sqrt{3} / 2$

[ii] The equation of tangent to the curve $y = 3x^2 - x + 1$ at P(1, 3) is _____.

(A) 5x - y = 2 (B) x + 5y = 16 (C) 5x - y + 2 = 0 (D) 5x = y

Answer: (a)

dy / dx = 6x - 1 at (1, 3) Slope of the tangent at (1, 3) = (6, 1) = 5 Equation of tangent is y - y₁ = m (x - x₁) y - 3 = 5 (x - 1) 5x - y - 2 = 0 y = 5x - 2

[iii] The expected value of the number of heads obtained when three fair coins are tossed simultaneously is _____.

(A) 1	(B) 1.5	(C) 0	(D) 1
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Answer: (b)

p = 1 / 2 q = 1 / 2 n = 3E(X) = np = 3 * (0.5) = 1.5

[B] Attempt any THREE of the following:

(6)

[i] Find dy / dx if x sin y + y sin x = 0.

Solution:

 $x \sin y + y \sin x = 0$ $[x \cos y \, dy / dx + \sin y] + [y \cos x + \sin x \, dy / dx] = 0$ sin y + cos x = dy / dx [- sin x - x cos y]dy / dx = [- sin x - x cos y] / [sin y + cos x]

[ii] Test whether the function, f (x) = x - (1 / x), x \in R, x \neq 0, is increasing or decreasing.

Solution:

f (x) = x - (1 / x) f ' (x) = 1 - (-1 / x²) = 1 + (1 / x²) x \neq 0, for all values of x, x² > 0 1 / x² > 0, [1 + (1 / x²)] is always positive Thus f ' (x) > 0, for all x \in R. It is an increasing function.

[iii] Evaluate: $\int [\sin \sqrt{x} / \sqrt{x}] dx$.

Solution:

Let I = $\int [\sin \sqrt{x} / \sqrt{x}] dx$

Let $\sqrt{x} = t$

 $[1 / \sqrt{x}] dx = 2dt$ I = 2 $\int \sin t dt$ = -2 cos t + c = -2 cos \sqrt{x} + c

[iv] Form the differential equation by eliminating arbitrary constants from the relation $y = Ae^{5x} + Be^{-5x}$.

Solution:

y = Ae^{5x} + Be^{-5x} On differentiating with respect to x, dy / dx = Ae^{5x} (5) + Be^{-5x} (-5) dy / dx = 5Ae^{5x} - 5Be^{-5x} Again differentiating with respect to x, $d^2y / dx^2 = 5Ae^{5x} (5) - 5Be^{-5x} (-5)$ $d^2y / dx^2 = 25Ae^{5x} + 25 Be^{-5x}$ = 25 [Ae^{5x} + Be^{-5x}] = 25y $d^2y / dx^2 - 25y = 0$

[v] The probability that a bomb will hit a target is 0.8. Find the probability that out of 10 bombs dropped, exactly 4 will hit the target.

Solution:

Let r be the number of bombs that hit the target. p = 0.8 q = 1 - p = 1 = 0.8 = 0.2 n = 10 r = 4 $P (r = 4) = {}^{n}C_{r} p^{r} q^{n-r}$ $= {}^{10}C_{4} (0.8)^{4} (0.2)^{10-4}$ $= 210 * (2)^{18} * (1 / 10)^{10}$ $= 55050240 / 10^{10}$ = anti (log 210 + 18 log 2 - 10)

= 0.0055

Question 5[A]: Attempt any TWO of the following:

[i] Solve: $dy / dx = \cos (x + y)$.

Solution:

dy / dx = cos (x + y) ---- (1) Put x + y = v ---- (2) y = v - x dy / dx = dv / dx - 1 ---- (3) Subtracting (2) and (3) in (1), dv / dx - 1 = cos v dv / dx = 1 + cos v dv / dx = 2 cos² (v / 2) [1 / cos² (v / 2)] dv = 2dx sec² (v / 2) dv = 2 dx On integrating both sides, $\int \sec^2 (v / 2) dv = 2 \int dx$ 2 tan (v / 2) = 2x + c tan (v / 2) = x + (c / 2) tan [(x + y) / 2] = x + c

[ii] If u and v are two functions of x, then prove that: $\int uv \, dx = u \int v \, dx - \int [du / dx \int v \, dx] \, dx.$

Solution:

Let $\int v \, dx = w - - - - (1)$ dw / dx = v - - (2) $d(u, w) / dx = u \cdot (d[w] / dx] + w \cdot (d[u] / dx]$ $= u \cdot v + w (du / dx) - - (from 2)$ By the definition of integration, $u \cdot w = \int [u \cdot v + w (du / dx)] \, dx$ $= \int u \cdot v \, dx + \int w \cdot (du / dx) \, dx$

$$\int \mathbf{u} \cdot \mathbf{v} \, d\mathbf{x} = \mathbf{u} \cdot \mathbf{w} - \int \mathbf{w} \cdot (d\mathbf{u} / d\mathbf{x}) \, d\mathbf{x}$$
$$= \mathbf{u} \int \mathbf{v} \, d\mathbf{x} - \int [d\mathbf{u} / d\mathbf{x} \int \mathbf{v} \, d\mathbf{x}] \, d\mathbf{x}$$

[iii] If
$$f(x) = \frac{e^{x^2} - \cos x}{x^2}$$
, for $x \neq 0$, is continuous at $x = 0$, find f (0).

Solution:

f (x) is continuous at x = 0. lim $_{x\to 0}$ f (x) = f (0)

$$\begin{split} f(0) &= \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = \lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2} \\ &= \lim_{x \to 0} \left(\frac{e^{x^2} - 1}{x^2} + \frac{1 - \cos x}{x^2} \right) \\ &= \lim_{x \to 0} \left(\frac{e^{x^2} - 1}{x^2} + \frac{2\sin^2\left(\frac{x}{2}\right)}{x^2} \right) \\ &= \lim_{x \to 0} \left(\frac{e^{x^2} - 1}{x^2} + 2\left(\frac{\sin\left(\frac{x}{2}\right)}{x}\right)^2 \right) \\ &= \lim_{x \to 0} \left(\frac{e^{x^2} - 1}{x^2} + 2\left(\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}} \times \frac{1}{2}\right)^2 \right) \\ &= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \frac{1}{2} \left(\lim_{x \to 0} \frac{\sin\left(\frac{x}{2}\right)}{x} \right)^2 \\ &= 1 + \frac{1}{2} (1)^2 \\ &= \frac{3}{2} \end{split}$$

[B] Attempt any TWO of the following:

[i] If y = f(x) is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then prove that x is a differentiable function of y and dx / dy = 1 / (dy / dx) where dy / dx \neq 0. Hence find d $(\tan^{-1} x) / dx$.

Solution:

Let δy be the increment in y corresponding to an increment δx in x as $\delta x \rightarrow 0$, $\delta v \rightarrow 0.$ Now y is a differentiable function of x. $\lim_{x\to 0} (\delta y / \delta x) = dy / dx$ $(\delta y / \delta x) (\delta x / \delta y) = 1$ $(\delta x / \delta y) = 1 / (\delta y / \delta x)$ Taking limits on both sides as $\delta x \rightarrow 0$, $\lim_{\delta x \to 0} (\delta x / \delta y) = \lim_{\delta x \to 0} [1 / (\delta y / \delta x)] = 1 / \lim_{\delta x \to 0} (\delta y / \delta x)$ $\lim_{\delta x \to 0} (\delta x / \delta y) = 1 / \lim_{\delta x \to 0} (\delta y / \delta x)$ Since the limit in RHS exists, limits on LHS also exist. $\lim_{\delta y \to 0} (\delta x / \delta y) = dx / dy$ dx / dy = 1 / (dy / dx), where $dy / dx \neq 0$ Let $y = \tan^{-1} x$ $\mathbf{x} = \tan \mathbf{y}$ $\cos y = 1 / \sqrt{1 + \tan^2 y} = 1 / \sqrt{1 + x^2}$ $\sec^2 y (dy / dx) = 1$ $dx / dy = \sec^2 y$ $dy / dx = 1 / (dx / dy) = 1 / \sec^2 y = \cos^2 y$ $dy / dx = \cos^2 y$ d (tan⁻¹ x) / dx = cos² y = (cos y)² = (1 / $\sqrt{1 + x^2}$)² d $(\tan^{-1} x) / dx = 1 / (1 + x^2)$

[ii] A telephone company in a town has 5000 subscribers on its list and collects fixed rent charges of 3,000 per year from each subscriber. The company proposes to increase annual rent and it is believed that for every increase of

one rupee in the rent, one subscriber will be discontinued. Find what increased annual rent will bring the maximum annual income to the company.

Solution:

Here, the number of subscribers = 5000 and annual rental charges per subscriber = Rs.3000.

For every increase of 1 rupee in the rent, one subscriber will be discontinued. Let the rent be increased by Rs. x.

New rental charges per year = 3000 + x and the number of subscribers after the increase in rental charges = 5000 - x.

Let R be the annual income of the company.

Then, R = (3000 + x) (5000 - x)

 $= 15000000 - 3000x + 5000x - x^2$

= $15000000 + 2000x - x^2$ and $(d^2R)/dx^2 = -2$

R is maximum if dR / dx = 0

2000 - 2x = 0

If x = 1000,

 $(d^2R) / dx^2)_{(x = 1000)} = -2 < 0$

By the second derivative test, R is maximum when x = 1000.

Thus, the annual income of the company is maximum when the annual rental charges are increased by Rs.1000.

[iii] Evaluate: $\int a^a \sqrt{a - x} / (a + x) dx$.

Solution:

Let
$$I = \int_{-a}^{a} \sqrt{(a - x)} / (a + x) dx$$

 $= \int_{-a}^{a} \sqrt{(a - x)} (a - x) / (a + x) (a - x) dx$
 $= \int_{-a}^{a} (a - x) / \sqrt{a^2} - x^2 dx$
 $= \int_{-a}^{a} a / \sqrt{a^2} - x^2 dx - \int_{-a}^{a} x / \sqrt{a^2} - x^2 dx$
 $= 2a [\sin^{-1} (x / a)]_{0}^{a}$
 $= 2a [\pi / 2 - 0]$
 $= a\pi$

Question 6[A]: Attempt any TWO of the following:

[i] Discuss the continuity of the following function, at x = 0.
f (x) = x / |x|, for x ≠ 0
= 1, for x = 0

Solution:

f(0) = 1 - ... (1)For x > 0, |x| = x $\lim_{x\to 0^{+}} f(x) = \lim_{x\to 0^{+}} x / |x|$ = $\lim_{x\to 0^{+}} f(x) = \lim_{x\to 0^{+}} x / |x|$ = 1 For x < 0, |x| = -x $\lim_{x\to 0^{-}} f(x) = \lim_{x\to 0^{-}} x / |x|$ = $\lim_{x\to 0^{-}} (-x / x)$ = $\lim_{x\to 0^{+}} f(x) \neq \lim_{x\to 0^{-}} f(x)$ f is discontinuous at x = 0. $\lim_{x\to 0} f(x) \text{ does not exist.}$ It is discontinuous at x = 0.

[ii] If the population of a country doubles in 60 years, in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants? [Given : $\log 2 = 0.6912$ and $\log 3 = 1.0986$.]

Solution:

Let P be the population at time t years. Then dP / dt the rate of increase of population is proportional to P.

 $dp / dt \propto p$ dP / dt = kP when k is a constant dP / P = kdtOn integrating we get $\int dp / P = k \int dt + c$ $\log P = kt + c$ Initially when t = 0 let $P = P_0$ $\log P_0 = k \times 0 + c$ $\log P = kt + \log P_0$ $\log P - \log P_0 = kt$ $\log (P / P_0) = kt$ Since the population doubles in 60 years, when $t = 60 P = 2P_0$ $\log (2P_0 / P_0) = 60k$, $\therefore k = [1 / 60] \log 2$ (1) becomes $\log (P / P0) = [t / 60] \log 2$ When population becomes triple when $P = 3P_0$ we get $\log (3P_0 / P_0) = (t / 60) \log 2$ $\log 3 = [t / 60] \log 2$ $\log 3 = [t / 60] \log 2$ $t = 60 (\log 3 / \log 2)$ = 60 (1.0986 / 0.6912) $= 60 \times 1.5894$ = 95.364= 95.4 years The population becomes tripled in 95.4 years.

[iii] A fair coin is tossed 8 times. Find the probability that it shows headsa. exactly 5 timesb. at least once

Solution:

a. Let X = number of heads P = Probability of getting in the first toss p = 0.5 q = 1 - p = 1 - 0.5 = 0.5 n = 8 P (exactly 5 heads) = P [X = 5] P (5) = ${}^{8}C_{5} (0.5)^{5} (0.5)^{8-5}$ = 7 / 32 P (X = 5) = 0.21875

b. P [getting head at least once] = P $[X \ge 1] = 1 - P [X = 0]$ = 1 - P (0) = 1 - ${}^{8}C_{0} (0.5)^{0} (0.5)^{8-0}$ = 255 / 256 = 0.996

[B] Attempt any TWO of the following:

(8)

[i] Evaluate: $\int d\theta / \sin \theta + \sin 2\theta$.

Solution: $I = \int dx / (sinx + sin2x)$ $= \int 1 / (\sin x + 2 \sin x \cos x) dx$ $= \int 1 / (\sin x (1 + 2 \cos x)) dx$ $= \int \sin x / (\sin^2 x (1 + 2 \cos x)) dx$ Let u = cosx $du = -\sin x \, dx$ Also, $\sin^2 x = 1 - \cos^2 x = 1 - u^2$ $I = \int -1 / ((1 - u^2) (1 + 2u)) du$ $= \int 1 / ((1 + u) (1 - u) (1 + 2u)) du$ Using partial fractions, we get 1 / ((1 + u) (1 - u) (1 + 2u)) = A / (1 + u) + B / (1 - u) + C / (1 + 2u)-1 = A (1 - u) (1 + 2u) + B (1 + u) (1 + 2u) + C (1 + u) (1 - u) $-1 = A (1 + u - 2u^2) + B (1 + 3u + 2u^2) + C (1 - u^2)$ $-1 = (-2A + 2B - C)u^2 + (A + 3B)u + (A + B + C)$ Equating the respective coefficients on the LHS and the RHS, -2A + 2B - C = 0 - (1)A + 3B = 0 - (2)A + B + C = -1 ---- (3)Adding the above equations,

A = -3B A = 1/2 C = -1 - A - B C = -4/3 1/((1 + u) (1 - u) (1 + 2u)) = 1/(2 (1 + u)) - 1/(6 (1 - u)) - 4/(3 (1 + 2u)))= I = $\int [1/(2 (1 + u)) - 1/(6 (1 - u)) - 4/(3 (1 + 2u))] du$ = $1/2 \log (1 + u) + 1/6 \log (1 - u) - 4/(2) \log (1 + 2u) + C$ = $1/2 \log (1 + \cos x) + 1/6 \log (1 - \cos x) - 2/3 \log (1 + 2 \cos x) + C$ [ii] Find the area of the region lying between the parabolas y² = 4ax and x² = 4ay.

Solution:

The equations of the parabolas are $v^2 = 4ax....(1)$ $x^2 = 4ay....(2)$ $[x^2/(4a)]^2 = 4ax$ by (2) $x^4 = 64a^3x$ $x [x^3 - (4a)^3] = 0$ x = 0 and x = 4av = 0 and v = 4aThe points of intersection of curves are O(0, 0), P(4a, 4a). The required areas is, A = (Area under parabola $y^2 = 4ax$) - (Area under parabola $x^2 = 4ay$) = $\int_0^{4a} \left[\sqrt{(4ax)dx} - \int_0^{4a} \frac{x^2}{(4a)} \right] dx$ = $\sqrt{(4a)}$. 2 / 3 $[\mathbf{x}^{(3/2)}]_0^{4a}$ - $(1 / (4a)) (1 / 3) [\mathbf{x}^3]_0^{4a}$ = $(4\sqrt{a}) / 3 * 4a\sqrt{(4a)} - (1 / (12a)) * 64a^3$ $= [32/3] a^2 - [16/3] a^2$ $= [16/3]a^2$ sq.units

[iii] Given the probability density function (p.d.f.) of a continuous random variable X as, f (x) = $x^2/3$, -1 < x < 2

= 0, otherwise.

Determine the cumulative distribution function (c.d.f.) of X and hence find P (X < 1), P (X > 0), P (1 < X < 2).

Solution:

CDF of the continuous random variable is given by $F(x) = \int_{-1}^{x} [y^2/3] dx$ $= [y^3/9]_{-1}^{x}$ $= (x^3 + 1)/9, x \text{ in } R$ Consider P (X < 1) = F (1) = (13 + 1)/9 = 2/9 P (x ≤ -2) = 0 P (x ≤ -2) = 0 P (X > 0) = 1 - P (X ≤ 0) = 1 - F(0) = 1 - (0/9 + 1/9) = 8/9P (1 < x < 2) = F (2) - F (1) = 1 - (1/9 + 1/9)= 7/9