MSBSHSE Class 12th Maths Question Paper With Solutions 2018

QUESTION PAPER CODE J - 265

SECTION - A

Question 1[A]: Select and write the most appropriate answer from the given alternatives in each of the following sub-questions. [6]

[i]:

If A = $\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$, then adjoint of matrix A is

(A) $\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

Answer: (a)

$$Adj(A) = \text{Adj}\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} +(1) & -(4) \\ -(-3) & +(2) \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -4 \\ 3 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$$

[ii]: The principal solutions of sec x = 2 / √3 are

(a) **π** / 3, 11**π** / 6 (b) **π** / 6, 11**π** / 6
(c) **π** / 4, 11**π** / 4 (d) **π** / 6, 11**π** / 4

Answer: (b)
sec x = 2 / √3
cos x = √3 / 2 = cos **π** / 6
= cos (2**π** - **π** / 6)
= cos (11**π** / 6)

[iii]: The measure of the acute angle between the Lines whose direction ratios are 3, 2, 6 and –2, 1, 2 is ______.

(a) $\cos^{-1}$ (1 / 7) (b) $\cos^{-1}$ (8 / 15)
(c) $\cos^{-1}$ (1 / 3) (d) $\cos^{-1}$ (8 / 21)

Answer: (d)
If direction ratios of lines are (a_1, b_1, c_1) and (a_2, b_2, c_2)
The angle between then given as,

$\cos \Theta = (a_1 * a_2 + b_1 * b_2 + c_1 * c_2) / (\sqrt{(a_1^2 + b_1^2 + c_1^2) * (a_2^2 + b_2^2 + c_2^2)})$

$\cos \Theta = (3 * -2 + 2 * 1 + 6 * 2) / \sqrt{(9 + 4 + 36)} * \sqrt{(4 + 1 + 4)}$

$\cos \Theta = 8 / 21$

$\Theta = \cos^{-1} (8 / 21)$

[B] Attempt any three of the following. [6]

[i] Write the negations of the following statements:
[a] All students of this college live in the hostel.
[b] 6 is an even number or 36 is a perfect square.

Solution:
[a] All students of this college live in the hostel.
Negation: Some students of this college do not live in the hostel.
[b] p: 6 is an even number.
q: 36 is a perfect square.
Symbolic form : p v q
$\sim (p \vee q) \equiv \sim p \wedge \sim q$
Negation:
6 is not an even number and 36 is not a perfect square.

[ii] If a line makes angles α, β, γ with the coordinate axes, prove that cos 2α + cos 2β + cos 2γ + 1 = 0.

Solution:
LHS = $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1$
$= 2 \cos^2 \alpha - 1 + 2 \cos^2 \beta - 1 + 2 \cos^2 \gamma - 1 + 1$
$= 2 [\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma] - 2$
$= 2 * 1 - 2$
$= 0$

[iii] Find the distance of the point (1, 2, -1) from the plane x - 2y + 4z - 10 = 0.

Solution:

The distance of the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is

$D = |ax_1 + by_1 + cz_1 + d / \sqrt{a^2 + b^2 + c^2 + d^2}|$

$(x_1, y_1, z_1) = (1, 2, -1)$

$a = 1, b = -2, c = 4$

$D = |1 - 2 * 2 + 4 * (-1) - 10 / \sqrt{1 + 4 + 16}|$

$= |-17 / \sqrt{21}|$

$= 17 / \sqrt{21}$ units

[iv] Find the vector equation of the line which passes through the point with position vector 4i - j + 2k and is in the direction of -2i + j + k.

Solution:

Let a = 4i - j + 2k

b = -2i + j + k

The equation of the line which passes through the point is

r = a + λb

r = 4i - j + 2k + λ (-2i + j + k)

[v] If a = 3i - 2j + 7k, b = 5i + j - 2k, c = i + j - k, then find a . (b x c).

Solution:

a . (b x c) = 3 (- 1 + 2) + 2 (- 5 + 2) + 7 (5 - 1)

= 3 - 6 + 28

= 25

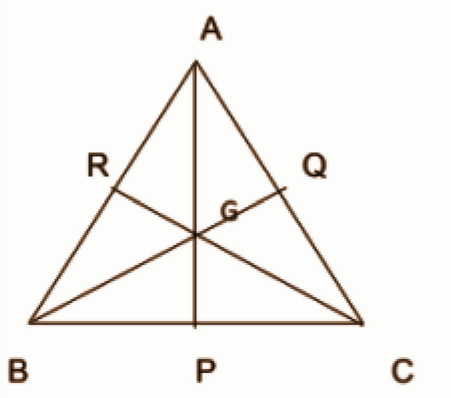
Question 2[A]: Attempt any two of the following. [6]

[i] Using the vector method prove that the medians of a triangle are concurrent.

[ii] Using the truth table, prove the following logical equivalence:

Solution:

[i]



Let us consider a triangle ABC. Let O be the fixed point.

$\overrightarrow{OA}$ = position vector of A = $\vec{a}$

$\overrightarrow{OB}$ = position vector of B = $\vec{b}$

$\overrightarrow{OC}$ = position vector of C= $\vec{c}$

The position vector of the midpoint P is vector OP = ½ vector (OB + OC)
= ½ vector (b + c)
If G divides vector AP in the ratio 2:1, then the position vector of

$$\overrightarrow{OG} = \frac{2\{½(\vec{b} + \vec{c})\} + 1\,(\vec{a})}{2 + 1}$$

$$= \frac{\vec{b} + \vec{c} + \vec{a}}{3}$$

G =

The symmetry of this result shows that the point which divides the other two medians in the ratio 2:1 will also have the same position vector.

$$\frac{\vec{b} + \vec{c} + \vec{a}}{3}$$

Hence, the medians of a triangle are concurrent at G.

[ii] Using the truth table, prove the following logical equivalence:
$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

Solution:

p	q	$p \leftrightarrow q$ [A]	$(p \wedge q)$	~p	~q	$(\sim p \wedge \sim q)$ [B]	A V B
T	T	T	T	F	F	F	T
T	F	F	F	F	T	F	F
F	T	F	F	T	F	F	F
F	F	T	F	T	T	T	T

$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

[iii] If the origin is the centroid of the triangle whose vertices are A (2, p, -3), B (q, -2, 5), R (-5, 1, r), then find the value of p, q and r.

Solution:

Let a, b and c be the position vectors of the triangle ABC whose vertices are A (2, p, -3), B (q, -2, 5), R (-5, 1, r).

a = 2i + pj - 3k

b = qi - 2j + 5k

c = -5i + j + rk

Given that the origin O is the centroid of the triangle ABC,

O = [a + b + c] / 3

a + b + c = 0

2i + pj - 3k + qi - 2j + 5k + [-5i] + j + rk = 0

[2 + q - 5]i + [p - 2 + 1]j + [-3 + 5 + r]k = 0i + 0j + 0k

By the equality of vectors,

2 + q - 5 = 0

q = 3

p - 2 + 1 = 0

p = 1

-3 + 5 + r = 0

r = -2

$p = 1, q = 3, r = -2$

[B] Attempt any two of the following. [8]

[i] Show that a homogeneous equation of degree 2 in x and y that is $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin if $h^2 - ab \geq 0$.

Solution:

Consider a homogeneous equation of degree 2 in x and y

$ax^2 + bhxy + by^2 = 0$ ---- (1)

In this equation at least one of the coefficients a, b or h is non 0. There are 2 cases.

Case 1: If $b = 0$ then the equation,

$ax^2 + 2hxy = 0$

$x (ax + 2hy) = 0$

This is the joint equation of lines $x = 0$ and $(ax + 2hy) = 0$.

These lines pass through the origin.

Case 2: If $b \neq 0$

Multiply both sides of equation (1) by b,

$abx^2 + 2hbxy + b^2y^2 = 0$

$2hbxy + b^2y^2 = -abx^2$

To make LHS a complete square, we add h^2x^2 on both sides,

$b^2y^2 + 2hbxy + h^2y^2 = -abx^2 + h^2x^2$

$(by + hx)^2 = (h^2 - ab)x^2$

$(by + hx)^2 = [(\sqrt{h^2 - ab})x]^2$

$(by + hx)^2 - [(\sqrt{h^2 - ab})x]^2 = 0$

$[(by + hx) + [(\sqrt{h^2 - ab})x]]\ [(by + hx) - [(\sqrt{h^2 - ab})x]] = 0$

It is the joint equation of two lines.

$[(by + hx) + [(\sqrt{h^2 - ab})x]] = 0$ and $[(by + hx) - [(\sqrt{h^2 - ab})x]] = 0$

These lines pass through the origin when $h^2 - ab > 0$.

From the two cases above, it can be concluded that $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin.

[ii] In △ABC prove that tan [c - a] / 2 = [c - a] / [c + a] cot (b / 2).

Solution:

In $\triangle$ABC, by sine rule,

a / sin A = b / sin B = c / sin C = k

a = k sin A, b = k sin B, c = k sin C

[c - a] / [c + a] = [k sin C - k sin A] / [k sin C + k sin A]

= sin C - sin A / sin C + sin A

= 2 cos [(C + A) / 2] sin [(C - A) / 2] / 2 sin [(C + A) / 2] cos [(C - A) / 2]

= cot [(C + A) / 2] tan [(C - A) / 2]

= tan (B / 2) tan [(C - A) / 2]

tan [(C - A) / 2] = [c - a] / [c + a] cot (b / 2)

[iii] Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ **using elementary row transformations.**

Solution:

Write the augmented matrix

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	2	-2	1	0	0
2	-1	3	0	0	1	0
3	0	-2	1	0	0	1

Find the pivot in the 1st column in the 1st row

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	2	-2	1	0	0
2	-1	3	0	0	1	0
3	0	-2	1	0	0	1

Eliminate the 1st column

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	2	-2	1	0	0
2	0	5	-2	1	1	0
3	0	-2	1	0	0	1

Make the pivot in the 2nd column by dividing the 2nd row by 5

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	2	-2	1	0	0
2	0	1	-2/5	1/5	1/5	0
3	0	-2	1	0	0	1

Eliminate the 2nd column

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	0	-6/5	3/5	-2/5	0
2	0	1	-2/5	1/5	1/5	0
3	0	0	1/5	2/5	2/5	1

Make the pivot in the 3rd column by dividing the 3rd row by 1/5

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	0	-6/5	3/5	-2/5	0
2	0	1	-2/5	1/5	1/5	0
3	0	0	1	2	2	5

Eliminate the 3rd column

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	0	0	3	2	6
2	0	1	0	1	1	2
3	0	0	1	2	2	5

There is the inverse matrix on the right

	A_1	A_2	A_3	B_1	B_2	B_3
1	1	0	0	3	2	6
2	0	1	0	1	1	2
3	0	0	1	2	2	5

Question 3[A]: Attempt any two of the following. [6]

[i] Find the joint equation of the pair of lines passing through the origin, which are perpendicular to the lines represented by $5x^2 + 2xy - 3y^2 = 0$.

Solution:

Given the homogeneous equation is $5x^2 + 2xy - 3y^2 = 0$ which is factorisable

$5x^2 + 5xy - 3xy - 3y^2 = 0$

$5x (x + y) - 3y (x + y) = 0$

$(x + y) (5x - 3y) = 0$

$(x + y) = 0$ and $(5x - 3y) = 0$are the two lines represented by the equation

Their slopes are -1 and 5 / 3.

The required two lines respectively are 1 and 3 / 5 and the line passes through the origin and their individual equations are:

$y = 1 * x$ and $y = (-3 / 5) x$

$x - y = 0$ and $3x + 5y = 0$

Their joint equation is $(x - y) (3x + 5y) = 0$

$3x^2 + 2xy - 5y^2 = 0$

[ii] Find the angle between the lines $(x - 1) / 4 = (y - 3) / 1 = z / 8$ and $(x - 2) / 2 = (y + 1) / 2 = (z - 4) / 1$.

Solution:

Let a and b be the vectors in the direction of the lines $(x - 1) / 4 = (y - 3) / 1 = z / 8$ and $(x - 2) / 2 = (y + 1) / 2 = (z - 4) / 1$ respectively.

$a = 4i + j + 8k$ and $b = 2i + 2j + k$

$a . b = 4 * 2 + 1 * 2 + 8 * 1$

$= 8 + 2 + 8$

$= 18$

$a = \sqrt{16 + 1 + 64} = \sqrt{81} = 9$

$b = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$

Let θ be the acute angle between the 2 given lines

$\cos \theta = a . b / |a| . |b|$

$= 18 / 9 * 3$

$= 2 / 3$
$\theta = \cos^{-1} (2 / 3)$

[iii] Write the converse, inverse and contrapositive of the following conditional statement: If an angle is a right angle then its measure is 90º.

Solution:
Inverse - If an angle is a not right angle then its measure is not 90°.
Converse - If the measure of an angle is 90° then it is a right angle.
Contrapositive - If the measure of an angle is not 90° then it is not a right angle.

[B] Attempt any two of the following: [8]

[i] Prove that: $\sin^{-1} (3 / 5) + \cos^{-1} (12 / 13) = \sin^{-1} (56 / 65)$

Solution:
Let $\cos^{-1} (12 / 13) = x$ and $\sin^{-1} (3 / 5) = y$.
$\cos x = 12 / 13$
$\sin x = 5 / 13$
$\sin y = 3 / 5$
$\cos y = 4 / 5$
$\sin (x + y) = \sin x \cos y + \cos x \sin y$
$= (5 / 13) (4 / 5) + (12 / 13) (3 / 5)$
$= [20 + 36] / 65$
$= 56 / 65$
$x + y = \sin^{-1} (56 / 65)$
$\sin^{-1} (3 / 5) + \cos^{-1} (12 / 13) = \sin^{-1} (56 / 65)$

[ii] Find the vector equation of the plane passing through the points A (1, 0, 1), B (1, -1, 1), C (4, -3, 2).

Solution:
Let the points be A (1, 0, 1), B (1, -1, 1), C (4, -3, 2).
$a = i + k$
$b = i - j + k$

c = 4i - 3j + 2k

b - a = -j

c - a = 3i - 3j + k

(b - a) x (c - a) = -i + 3k

Equation of the plane through A, B and C in vector form is

(r - a) . [(b - a) x (c- a)] = 0

(r - a) . (-i + 3k) = 0

r . (-i + 3k) = (i + k) . (-i + 3k)

= -1 + 3

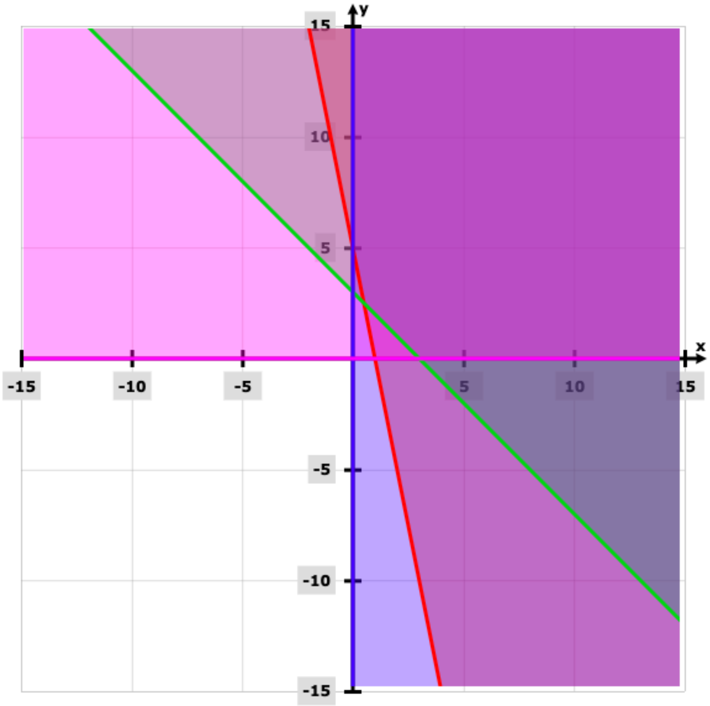
= 2

r . (-i + 3k) = 2

[iii] Minimize Z = 7x + y subject to

5x + y ≥ 5, x + y ≥ 3, x ≥ 0, y ≥ 0.

Solution:



Points	**Minimize (Z = 7x + y)**
(0, 5)	5

(1 / 2, 5 / 2)	6
(3, 0)	21

Z is minimum at x = 0, y = 5 and minimum (Z) = 5.

Question 4[A]: Select and write the appropriate answer from the given alternatives from each of the following sub-questions. [6]

[i] Let the pmf of a random variable X be -
P (x) = (3 - x) / 10 for x = -1, 0, 1, 2
= 0 otherwise
Then E (X) is _____.

(a) 1 (b) 2 (c) 0 (d) -1

Answer: (c)

x	-1	0	1	2
P(x)	4 / 10	3 / 10	2 / 10	1 / 10
x * P(x)	-4 / 10	0	2 / 10	2 / 10

$\sum x * P(x) = 0$

[ii] **If $\int_0^k (1 / 2 + 8x^2)\, dx = \pi / 16$, then the value of k is**

(a) 1 / 2 (b) 1 / 3 (c) (1 / 4) (d) 1 / 5

Answer: (a)

$I = \int_0^k (1 / 2 + 8x^2)\, dx = \pi / 16$

$(1 / 2) * (1 / 2) * [\tan^{-1} (2x)]_0^k = \pi / 16$

$\tan^{-1} (2k) - \tan^{-1} 0 = \pi / 4$

$2k = 1$

$k = 1 / 2$

[iii] Integrating factor of linear differential equation $x * (dy / dx) + 2y = x^2 \log x$ is

(a) $1 / x^2$ (b) $1 / x$ (c) x (d) x^2

Answer: (d)

$x * (dy / dx) + 2y = x^2 \log x$

$dy / dx + (2y / x) = x \log x$

$P = 2 / x$

$IF = e^{\int 2 / x \, dx} = e^{2\log x} = x^2$

[B] Attempt any three of the following. [6]

[i] Evaluate $\int (e^x [\cos x - \sin x] / \sin^2 x) \, dx$.

Solution:

$I = \int (e^x [\cos x - \sin x] / \sin^2 x) \, dx$

$= \int e^x [\cos x / \sin^2 x] - [\sin x / \sin^2 x] \, dx$

$= \int e^x [\cot x * \mathrm{cosec}\, x] - [\mathrm{cosec}\, x]$

$= \int e^x [f(x) + f'(x)] \, dx = e^x f(x) + c$

$I = -e^x \mathrm{cosec}\, x + c$

[ii] If $y = \tan^2 (\log x^3)$, find dy / dx.

Solution:

$y = \tan^2 (\log x^3)$

$y = [\tan (3 \log x)]^2$

On differentiating both sides,

$dy / dx = 2 [\tan (3 \log x)] * \sec^2 (3 \log x) * (3 / x)$

$dy / dx = (6 / x) \tan (\log x^3) \sec^2 (\log x^3)$

[iii] Find the area of the ellipse $x^2 / 1 + y^2 / 4 = 1$.

Solution:

Required area = $\int_0^1 (y\, dx)$

$x^2 / 1 + y^2 / 4 = 1$

$y = -2\sqrt{1 - x^2}$

Required area = $4 \int_0^1 (-2\sqrt{1 - x^2})\, dx$

$= 8 [(x / 2) * \sqrt{1 - x^2} + (1 / 2) \sin^{-1} (x / 1)]_0^1$

$= 8 * (1 / 2) * (\pi / 2)$

$= 2\pi$ square units

[iv]: Obtain the differential equation by eliminating the arbitrary constants from the following equation: $y = c_1e^{2x} + c_2e^{-2x}$.

Solution:

$y = c_1e^{2x} + c_2e^{-2x}$

On differentiating both sides by x,

$dy / dx = 2c_1e^{2x} - 2c_2e^{-2x}$

Again differentiating both sides by x,

$d^2y / dx^2 = 4c_1e^{2x} + 4c_2e^{-2x}$

$= 4 (c_1e^{2x} + c_2e^{-2x})$

$= 4y$

$d^2y / dx^2 - 4y = 0$

[v] Given X ~ B (n, p) if n = 10, p = 0.4, find E (X) and Var (X).

Solution:

n = 10

p = 0.4

q = 1 - p = 1 - 0.4 = 0.6

E (X) = np = 10 * 0.4 = 4

Var (X) = npq = 10 * 0.4 * 0.6 = 2.4

Question 5[A]: Attempt any two of the following. [6]

[i] Evaluate ∫1 / [3 + 2 sinx + cos x] dx.

Solution:

$I = \int dx / (2\sin x + \cos x + 3)$

$= \int dx / [2(\sin x + 1) + (\cos x + 1)]$

$= \int dx / [2\{\cos (x / 2) + \sin (x / 2)\}^2 + 2 \{\cos (x / 2)\}^2]$

$= (1 / 2)*\int dx / [\{\cos (x / 2) + \sin (x / 2)\}^2 + \{\cos (x / 2)\}^2]$

$= (1 / 2)*\int \{\sec (x / 2)^2\}*dx / [1 + [1 + \{\tan (x / 2)\}^2]]$ [Multiplying both numerator and denominator by $\{\sec (x / 2)^2\}$.

Let, $z = [1 + \tan (x / 2)]$

$dz = (1 / 2)*[\{\sec (x / 2)\}^2]*dx$

$I = \int dz / \{1 + (z^2)\}$

$= \tan^{-1} z + K$

$= \tan^{-1}\{1 + \tan (x / 2)\} + K$

[ii] If $x = a \cos^3 t$, $y = a\sin^3 t$, then show that $dy / dx = - (y / x)^{⅓}$.

Solution:

$x = a \cos^3 t$, $y = a \sin^3 t$

Differentiating with respect to t,

$(dx) / (dt) = a (d) / (dt) (\cos t)^3$

$= a . 3 (\cos t)^2 (d) / (dt) (\cos t)$

$= 3a \cos^2 t (- \sin t)$

$= -3a \cos^2 t \sin t$

$(dy) / (dt) = a (d) / (dt) (\sin t)^3$

$= a . 3 (\sin t)^2 (d) / (dt) (\sin t)$

$= 3a \sin^2 t . \cos t$

$(dy) / (dx) = (dy) / (dt) / (dx) / (dt)$

$= (3a \sin^2 t \cos t) / (-3a \cos^2 t \sin t)$

$= -(\sin t) / (\cos t)$ ... (1)

Now $x = a \cos^3 t$

$\cos^3 t = (x) / (a)$

$\cos t = ((x) / (a))^{(1/3}$

$y = a \sin^3 t$

$\sin^3 t = (y) / (a)$
$\sin t = ((y) / (a))^{(1/3)}$
therefore from (1),
$(dy) / (dx) = - (y^{(1/3)} / a^{(1/3)}) / (x^{(1/3)} / a^{(1/3)})$
$= - ((y) / (x))^{(1/3)}$

[iii] Examine the continuity of the function f (x) = log 100 + log (0.01 + x) / 3x for x ≠ 0, 100 / 3 for x = 0, at x = 0.

Solution:

$$f(x) = \begin{cases} \frac{\log 100 + \log(0.01 + x)}{3x}, & x \neq 0 \\ \frac{100}{3}, & x = 0 \end{cases}$$

Checking continuity at $x = 0$

$$\Rightarrow \lim_{x \to 0^+} \frac{\log 100 + \log(0.01 + x)}{3x} = \lim_{h \to 0} \frac{\log 100 + \log(0.01 + 0 + h)}{3(0 + h)} = \lim_{h \to 0} \frac{\frac{1}{(0.01 + h)}}{3(1)} = \frac{100}{3}$$ ... (1) (By L' Hospital Rule)

$$\Rightarrow \lim_{x \to 0^-} f(x) = \lim_{h \to 0} \frac{\log 100 + \log(0.01 + 0 - h)}{3(0 - h)} = \lim_{h \to 0} \frac{0 + \frac{-1}{0.01 - h}}{3(-1)} = \frac{100}{3}$$ .... (2) (L' Hospital Rule)

$$\Rightarrow f(0) = \frac{100}{3} \to 3$$

From (1), (2) and (3)

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(x) = \frac{100}{3}$$

$\Rightarrow f(x)$ is continuous at $x = 0$

[B] Attempt any two of the following. [8]

[i] Find the maximum and minimum value of the function f (x) = 2x^3 - 21x^2 + 36x - 20.

Solution:
$f(x) = 2x^3 - 21x^2 + 36x - 20$
$f'(x) = 6x^2 - 42x + 36$
For finding the critical points, $f'(x) = 0$
$6x^2 - 42x + 36 = 0$

$x^2 - 7x + 6 = 0$
$(x - 6)(x - 1) = 0$
Finding maxima and minima
$f''(x) = 12x - 42$
For $x = 6$,
$f''(6) = 30 > 0$
Minima for $x = 1$,
$f''(1) = 12 - 42 = -30 < 0$
Maximum values of $f(x)$ at $x = 1$, $f(1) = -3$
Minimum values of $f(x)$ at $x = 6$, $f(6) = -128$

[ii] Prove that $\int (1 / a^2 - x^2)\, dx = (1 / 2a) \log [(a + x) / (a - x)] + c$.

Solution:
$\int (1 / a^2 - x^2)\, dx$
$= \int (1 / (a + x) / (a - x)]\, dx$
$= (1 / 2a) \int [(a + x) / (a - x) / (a + x) / (a - x)]\, dx$
$= (1 / 2a) \int (1 / (a + x)) + (1 / (a - x))\, dx$
$= (1 / 2a) [\int (1 / (a + x))\, dx + \int (1 / (a - x))\, dx]$
$= (1 / 2a) [\log |a + x| - \log |a - x|] + c$
$= (1 / 2a) \log [(a + x) / (a - x)] + c$

[iii] Show that $\int_{-a}^{a} f(x)\, dx = 2 \int_0^a f(x)\, dx$, if $f(x)$ is an even function.
$= 0$ if $f(x)$ is an odd function.

Solution:
$\int_{-a}^{a} f(x)\, dx = \int_{-a}^{0} f(x)\, dx + \int_0^a f(x)\, dx$ ----- (1)
$\int_a^a f(x)\, dx = I + \int_0^a f(x)\, dx$
$I = \int_{-a}^{0} f(x)\, dx$
Put $x = -t$
$dx = -dt$
When $x = -a$, $t = a$ and when $x = 0$, $t = 0$
$I = \int_a^0 f(-t)(-dt)$

$= - \int_a^0 f(-t)\,dt$

$= - \int_a^0 f(-x)\,dt$

Equation (1) becomes,

$\int_{-a}^a f(x)\,dx = \int_0^a f(-x)\,dx + \int_0^a f(x)\,dx$

$= \int_0^a [f(-x) + f(x)]\,dx$ ---- (2)

Case (i): If f (x) is an even function, then f (-x) = f (x).

Equation (2) becomes,

$\int_{-a}^a f(x)\,dx = \int_0^a [f(x) + f(x)]\,dx = 2 \int_0^a f(x)\,dx$

Case (ii): If f (x) is an odd function, then f (-x) = - f (x).

Thus the equation becomes,

$\int_{-a}^a f(x)\,dx = \int_0^a [-f(x) + f(x)]\,dx = 0$

Question 6[A]: Attempt any two of the following. [6]

[i] If f (x) = [x^2 - 9 / x - 3] + α, for x > 3

= 5, for x = 3

= $2x^2$ + 3x + β, for x < 3

is continuous at x = 3, find α and β.

Solution:

Since f (x) is continuous at x = 3,

$\lim_{x\to3-} f(x) = \lim_{x\to3+} f(3)$ ---- (1)

$\lim_{x\to3+} f(x) = \lim_{x\to0} [x^2 - 9 / x - 3] + \alpha$

$= \lim_{x\to3} [(x - 3)(x + 3) / (x - 3)] + \alpha$

$= \lim_{x\to0} [(x + 3) + \alpha]$

$= (3 + 3) + \alpha$

$= 6 + \alpha$

$\lim_{x\to3-} f(x) = \lim_{x\to3} [2x^2 + 3x + \beta]$

$= 2(3)^2 + 3 * 3 + \beta$

$= 27 + \beta$

Also, f (3) = 5 ---- given

From (1),

$\alpha + 6 = 27 + \beta = 5$

$\alpha + 6 = 5$ and $27 + \beta = 5$
$\alpha = 1$ and $\beta = -22$

[ii] Find dy / dx if $y = \tan^{-1} (5x + 1) / (3 - x - 6x^2)$.

Solution:
Let $y = \tan^{-1} (5x + 1) / (3 - x - 6x^2)$
$= \tan^{-1} (5x + 1) / (1 + 2 - x - 6x^2)$
$= \tan^{-1} (5x + 1) / (1 - (3x + 2) (2x - 1))$
$= \tan^{-1} ((3x + 2) (2x - 1) / (1 - (3x + 2) (2x - 1))$
$= \tan^{-1} (3x + 2) + \tan^{-1} (2x - 1)$
$dy / dx = [3 / 1 + (3x + 2)^2] + [1 / 1 + (2x - 1)^2]$
$= [3 / 9x^2 + 12x + 5] + [1 / 2x^2 - 2x + 1]$

[iii] A fair coin is tossed 9 times. Find the probability that it shows the head exactly 5 times.

Solution:
Let X = number of heads that shows up
N = 9
p = 1 / 2
q = 0.5
$P (x = 5) = {}^9C_5 (0.5)^5 (0.5)^4$
= 3024 / 24
= 126 / 512
= 0.2460

[B] Attempt any two of the following. [8]

[i] Verify Rolle's theorem for the following function: $f (x) = x^2 - 4x + 10$ on [0, 4].

Solution:
Since f (x) is a polynomial,
(i) It is continuous on [0, 4]

(ii) It is differentiable on (0, 4)
(iii) f (0) = 10, f (4) = 16 - 16 + 10 = 10
f (0) = f (4) = 10
Thus all the conditions on Rolle's theorem are satisfied.
The derivative of f (x) should vanish for at least one point c in (0, 4). To obtain the value of c, the following steps are followed.
f (x) = x^2 - 4x + 10
f' (x) = 2x - 4 = 2(x - 2)
f' (x) = 0
(x - 2) = 0
x = 2
c = 2 in (0,4)
We know that $2 \in (0, 4)$
Thus Rolle's theorem is verified.

[ii] Find the particular solution of the differential equation y (1 + log x) dy / dx - x log x = 0 when y = e^2 and x = e.

Solution:
y (1 + log x) dy / dx - x log x = 0
y (1 + log x) dy / dx = x log x
y (1 + log x) dx = x log x * dy
Separating the variables
(1 / y) dy = (1 + log x) / x log x dx
$\int(1 / y)\, dy = \int(1 + \log x) / x \log x\, dx$
log |y| = log |x log x| + log c
log |y| = log |cx log x|
y = cx log x is the general solution.
Given x = e, y = e^2
e^2 = ce log e
e^2 = ce
c = e
y = xe log x

[iii] Find the variance and standard deviation of the random variable X whose probability distribution is given below

x	**0**	**1**	**2**	**3**
P (X = x)	**1 / 8**	**3 / 8**	**3 / 8**	**1 / 8**

Solution:

x	0	1	2	3
P (X = x)	1 / 8	3 / 8	3 / 8	1 / 8
$p_i x_i$	0	3 / 8	6 / 8	3 / 8
$p_i x_i^2$	0	3 / 8	12 / 8	9 / 8

E (X) = $\boldsymbol{\mu} = \sum p_i x_i = 12 / 8 = 3 / 2$

VAar (X) = $\sum p_i x_i^2 - \boldsymbol{\mu}^2$

$= 3 - (3 / 2)^2$

$= 3 / 4$

Standard deviation = $\sqrt{\text{Var (X)}} = \sqrt{3 / 4} = \sqrt{3} / 2$