MSBSHSE Class 10 Mathematics Question Paper 2016 Algebra Paper with Solutions

PART - A

1. Attempt any five of the following sub-questions: **[5]**

(i) Write the first two terms of the sequence whose nth term is $t_n = 3n - 4$.

Solution:

Given,

nth term of a sequence is $t_n = 3n - 4$

When n = 1,

$t_1 = 3(1) - 4 = 3 - 4 = -1$

When n = 2,

$t_2 = 3(2) - 4 = 6 - 4 = 2$

Therefore, the first two terms of the given sequence are -1 and 2.

(ii) Find the value of a, b, c in the following quadratic equation:

$2x^2 - x - 3 = 0$

Solution:

Given,

$2x^2 - x - 3 = 0$

Comparing with the standard form $ax^2 + bx + c = 0$,

a = 2, b = -1, c = -3

(iii) Write the quadratic equation whose roots are -2 and -3.

Solution:

Let α and β be the zeroes of the quadratic equation.

Given,

$\alpha = -2, \beta = -3$

$\alpha + \beta = -2 - 3 = -5$

$\alpha\beta = -2(-3) = 6$

Hence, the quadratic equation is $x^2 - (\alpha + \beta) + \alpha\beta = 0$

$x^2 - (-5)x + 6= 0$

$x^2 + 5x + 6 = 0$

(iv) Find the value of the determinant:

$$\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix}$$

Solution:

$$\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix}$$

= 4(1) - (-2)(3)
= 4 + 6
= 10

(v) Write the sample space for selecting a day randomly of the week.

Solution:
A week contains 7 days.
The sample space for selecting a day randomly of the week is
S = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
Also, n(S) = 7

(vi) Find the class mark of the classes 20 − 30 and 30 − 40.

Solution:
Given class:
20 − 30 and 30 − 40
Class mark = [Upper class limit + Lower class limit]/2
Class mark of class 20 - 30 = (30 + 20)/2 = 50/2 = 25
Class mark of class 30 - 40 = (40 + 30)/2 = 70/2 = 35

2. Attempt any four of the following sub-questions: [8]
(i) If for an A.P. the first term is 11 and the common difference is (-2), then find the first three terms of A.P.

Solution:
Given,
First term = a = 11
Common difference = d = -2
Second term = a + d = 11 - 2 = 9
Third term = a + 2d = 11 + 2(-2) = 11 - 4 = 7
Hence, the first three terms of the AP are 11, 9, and 7.

(ii) Solve the following quadratic equation using factorization method:
$x^2 + 11x + 24 = 0$.

Solution:
Given,
$x^2 + 11x + 24 = 0$
Using factorisation method,
$x^2 + 3x + 8x + 24 = 0$
x(x + 3) + 8(x + 3) = 0
(x + 8)(x + 3) = 0
x + 8 = 0, x + 3 = 0
x = -8, x = -3

(iii)

If the value of determinanats $\begin{vmatrix} x & -5 \\ 3 & 4 \end{vmatrix}$ is 31, then find the value of x.

Solution:
Given,

$$\begin{vmatrix} x & -5 \\ 3 & 4 \end{vmatrix} = 31$$

x(4) - (-5)(3) = 31
4x + 15 = 31
4x = 31 - 15
4x = 16
x = 16/4
x = 4

(iv) A die is thrown, then find the probability of the following events:
A is an Event: getting a number divisible by 3.
B is an Event: getting a number less than 5.

Solution:
Given,
A die is thrown.
Sample space = S = {1, 2, 3, 4, 5, 6}
n(S) = 6
A = The event of getting a number divisible by 3
= {3, 6}
n(A) = 2
P(A) = n(A)/n(S)
= 2/6
= 1/3
B = The event of getting a number less than 5
= {1, 2, 3, 4}
n(B) = 4
P(B) = n(B)/n(S)
= 4/6
= 2/3

(v) Below is the given frequency distribution of words in an essay:

Number of Words	600 - 800	800 - 1000	1000 - 1200	1200 - 1400	1400 - 1600
Number of Candidates	14	22	30	18	16

Find the mean number of words written.

Solution:

Number of words	Number of Candidates (f_i)	Class mark (x_i)	$d_i = x_i - A$	$u_i = (x_i - A)/200$	f_iu_i
600 - 800	14	700	-400	-2	-28
800 - 1000	22	900	-200	-1	-22
1000 - 1200	30	1100 = A	0	0	0
1200 - 1400	18	1300	200	1	18
1400 - 1600	16	1500	400	2	32
	$\sum f_i = 100$				$\sum f_iu_i = 0$

Assumed mean = A = 1100

Mean = A + ($\sum f_iu_i / \sum f_i$)

= 1100 + (0/100)

= 1100 + 0

= 1100

(vi) The marks obtained by a student in an examination are given below:

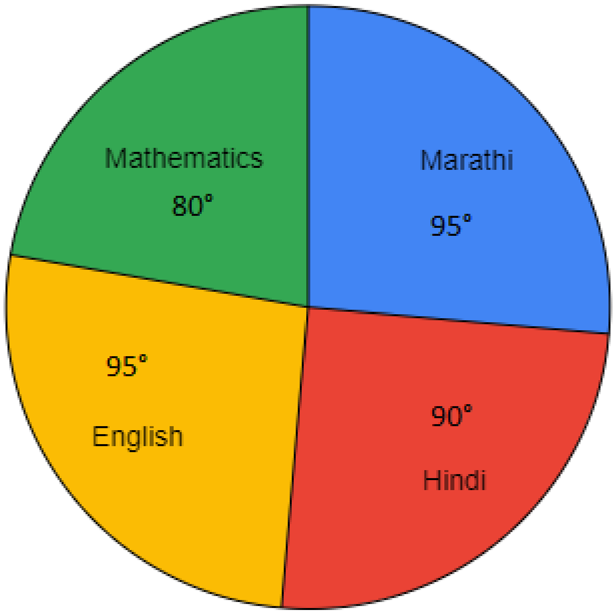
Subject	Marathi	Hindi	English	Mathematics	Total
Marks	95	90	95	80	360

Represent the data using a pie diagram.

Solution:

Subject	Marks	Measure of central angle
Marathi	95	(95/360) × 360° = 95°
Hindi	90	(90/360) × 360° = 90°
English	95	(95/360) × 360° = 95°
Mathematics	80	(80/360) × 360° = 80°
Total	360	360°

Pie chart:



3. Attempt any three of the following sub-questions: **[9]**

(i) Solve the following quadratic equation using the formula method:
$6x^2 - 7x - 1 = 0$.

Solution:
Given,
$6x^2 - 7x - 1 = 0$
Comparing with the standard form $ax^2 + bx + c = 0$,
a = 6, b = -7, c = -1
$x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$
$= [-(-7) \pm \sqrt{\{(-7)^2 - 4(6)(-1)\}}]/ 2(6)$
$= [7 \pm \sqrt{(49 + 24)}]/ 12$
$= (7 \pm \sqrt{73})/ 12$
$x = (7 + \sqrt{73})/ 12$, $x = (7 - \sqrt{73})/ 12$

(ii) There are three boys and two girls. A committee of two is to be formed. Find the probability of the following events:
Event A: The committee contains at least one boy.
Event B: The committee contains one boy and one girl.

Solution:
Given,
Three boys: B1, B2, B3
Two girls: G1, G2
A committee of two is to be formed.
Sample space = S = $\{B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$
n(S) = 10
A = The event that the committee contains at least one boy
$= \{B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2\}$
n(A) = 9

P(A) = n(A)/n(S)
= 9/10
B = The event that the committee contains one boy and one girl
= {B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2}
n(B) = 6
P(B) = n(B)/ n(S)
= 6/10
= 3/5

(iii) The measurements (in mm) of the diameters of the head of the screws are given below:

Diameter (in mm)	33 - 35	36 - 38	39 - 41	42 - 44	45 - 47
No. of screws	9	21	30	20	18

Calculate the mean diameter of the head of a screw by the 'Assumed Mean Method'.

Solution:
Given classes are discontinuous.
Let us convert them into continuous classes as shown below.

Diameter (in mm)	No. of screws (f_i)	Class mark (x_i)	$d_i = x_i - A$	f_id_i
32.5 - 35.5	9	34	-6	-54
35.5 - 38.5	21	37	-3	-63
38.5 - 41.5	30	40 = A	0	0
41.5 - 44.5	20	43	3	60
44.5 - 47.5	18	46	6	108
	$\sum f_i = 98$			$\sum f_id_i = 51$

Assumed mean = A = 40
Mean = A + ($\sum f_id_i$/ $\sum f_i$)
= 40 + (51/98)
= 40 + 0.52
= 40.52

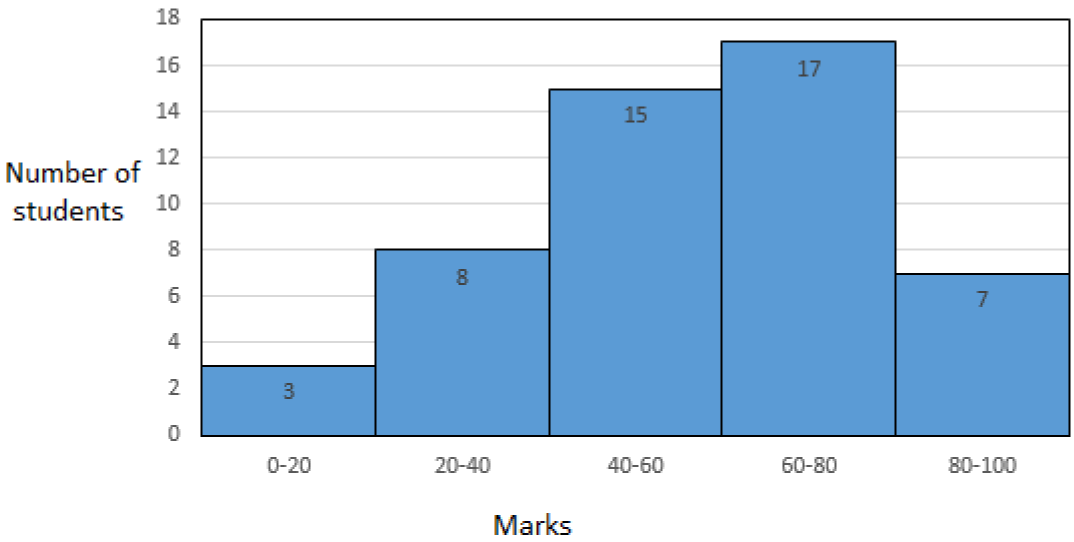
(iv) The marks scored by students in Mathematics in a certain examination are given below:

Marks scored	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Number of students	3	8	15	17	7

Draw histogram for the above data.

Solution:

Histogram
Scale:
X-axis: 2 cm = 20 marks
Y-axis: 1 cm = 2 students



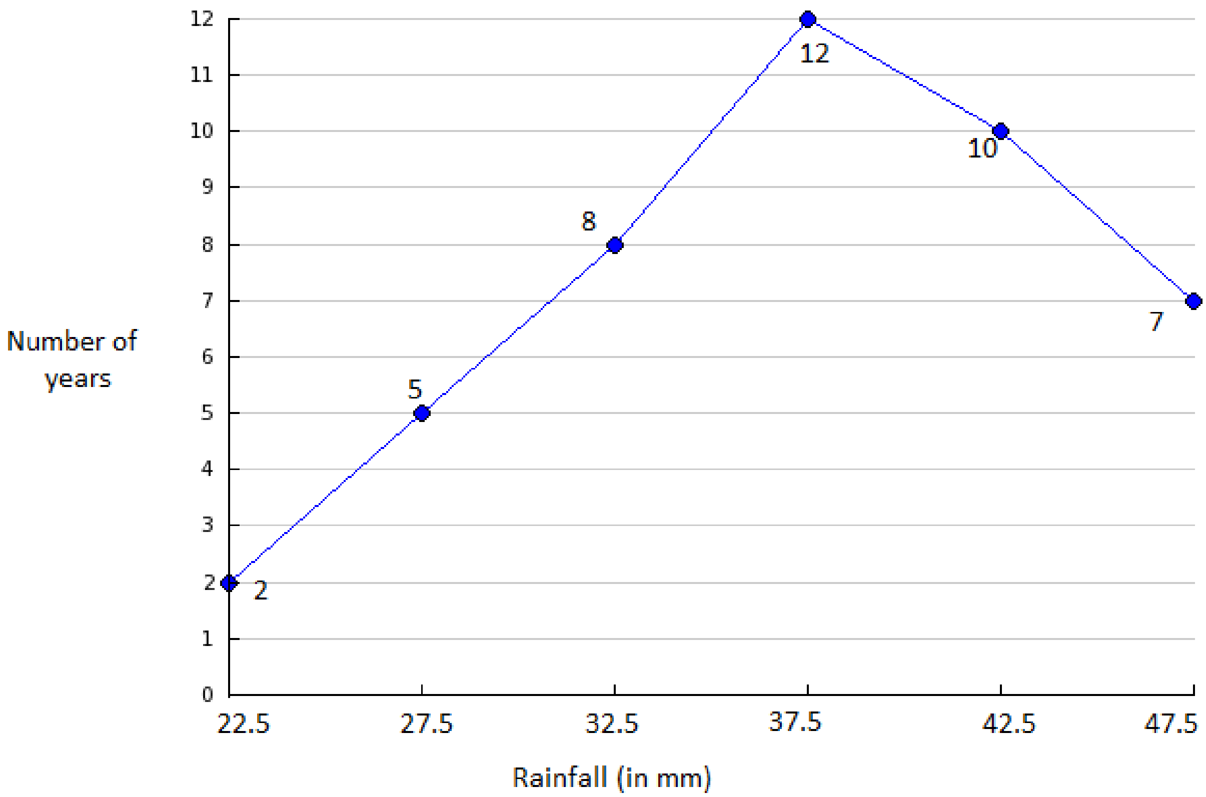
(v) Draw a frequency polygon for the following frequency distribution:

Rainfall (in mm)	20 - 25	25 - 30	30 - 35	35 - 40	40 - 45	45 - 50
No. of Years	2	5	8	12	10	7

Solution:

Rainfall (in mm)	No. of Years	Class mark
20 - 25	2	22.5
25 - 30	5	27.5
30 - 35	8	32.5
35 - 40	12	37.5
40 - 45	10	42.5
45 - 50	7	47.5

Frequency polygon



4. Attempt any two of the following sub-questions: **[8]**

(i) The 11th term and the 21st term of an A.P. are 16 and 29 respectively then find:

a. The first term and common difference.

b. The 34th term.

c. 'n' such that $t_n = 55$.

Solution:

a. Let a be the first term and d be the common difference of an AP.

Given,

$t_{11} = 16$

a + 10d = 16….(i)

And

$t_{21} = 29$

a + 20d = 29….(ii)

Subtracting (i) from (ii),

20d - 10d = 29 - 16

10d = 13

d = 13/10 = 1.3

Substituting d = 1.3 in (i),

a + 10(1.3) = 16

a = 16 - 13

a = 3

Therefore, the first term is 3 and the common difference is 1.3.

b. $t_{34} = a + 33d$
= 3 + 33(1.3)
= 3 + 42.9
= 45.9
Therefore, the 34th term is 45.9.

c. $t_n = 55$
a + (n - 1)d = 55
3 + (n - 1)(1.3) = 55
(n - 1) 1.3 = 55 - 3
n - 1 = 52/1.3
n = 40 + 1
n = 41

(ii) Solve the following simultaneous equations:
[7/(2x + 1)] + [13/ (y + 2)] = 27;
[13/ (2x + 1)] + [7/ (y + 2)] = 33

Solution:
[7/(2x + 1)] + [13/ (y + 2)] = 27
[13/ (2x + 1)] + [7/ (y + 2)] = 33
Substituting 1/(2x + 1) = a and 1/(y + 2) = b in the above equations.
7a + 13b = 27….(i)
13a + 7b = 33….(ii)
From (i),
7a = 27 - 13b
a = (27 - 13b)/7….(iii)
Substituting (iiii) in (ii),
13[(27 - 13b)/7] + 7b = 33
(351 - 169b + 49b)/ 7 = 33
351 - 120b = 231
120b = 351 - 231
120b = 120
b = 1
Substituting b = 1 in (iii),
a = [27 - 13(1)]/ 7
= (27 - 13)/7
= 14/7
a = 2
Now,
1/(2x + 1) = a
1/(2x + 1) = 2
2x + 1 = ½
2x = -1 + ½
2x = -½
x = -¼
Also, 1/(y + 2) = b
1/(y + 2) = 1
y + 2 = 1

y = -1
Therefore, x = -¼ and y = -1.

(iii) In a certain race, there are three boys A, B, C. The winning probability of A is twice than B and the winning probability of B is twice than C. If P(A) + P(B) + P(C) = 1, then find the probability of each boy.

Solution:
Let P(A), P(B), and P(C) be the probability of winning in a race by three boys A, B, and C respectively.
According to the given,
P(A) = 2P(B)
P(B) = 2P(C)
Now, P(A) = 2P(B) = 2[2P(C)] = 4P(C)
We know that,
P(A) + P(B) + P(C) = 1
4P(C) + 2P(C) + P(C) = 1
7P(C) = 1
P(C) = 1/7
Therefore,
P(B) = 2/7
P(A) = 4/7
Hence, the required probabilities are 4/7, 2/7, and 1/7.

5. Attempt any two of the following sub-questions: [10]
(i) The divisor and quotient of the number 6123 are the same and the remainder is half the divisor. Find the divisor.

Solution:
Given,
Dividend = 6123
Let x be the divisor.
According to the given,
Divisor = Quotient = x
Remainder = x/2
We know that,
Dividend = (Divisor × Quotient) + Remainder
$6123 = x^2 + (x/2)$
$6123 \times 2 = 2x^2 + x$
$2x^2 + x - 12246$
Comparing with the standard form $ax^2 + bx + c = 0$,
a = 2, b = 1, c = -12246
$x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$
$= [-1 \pm \sqrt{\{(1)^2 - 4(2)(-12246)\}}]/ 2(2)$
$= [-1 \pm \sqrt{(1 + 97968)}]/ 4$
$= (-1 \pm \sqrt{97969})/ 4$
x = (-1 ± 313)/ 4,
x = (-1 + 313)/4, x = (-1 - 313)/4
x = 312/4, x = -314/4
x = 78, x = 78.5
x cannot be negative.
Therefore, the divisor is 78.

(ii) Find the sum of all numbers from 50 to 350 which are divisible by 6. Hence find the 15th term of that A.P.

Solution:
The numbers which are divisible by 6 from 50 to 350 are:
54, 60, 66, ..., 348
This is an AP with a = 54, d = 6 and t_n = 348
$t_n = a + (n - 1)d$
348 = 54 + (n - 1)6
(n - 1)6 = 348 - 54
n - 1 = 294/6
n = 49 + 1
n = 50
$S_n = n/2 (a + tn)$
$S_{50} = (50/2) (54 + 348)$
= 25 × 402
= 10050
Therefore, the sum of the AP is 10050.
15 term = t_{15}
= a + 14d
= 54 + 14 × 6
= 54 + 84
= 138
Hence, the 15th term of the AP is 138.

(iii) A three-digit number is equal to 17 times the sum of its digits. If 198 is added to the number, the digits are interchanged. The addition of the first and third digit is 1 less than the middle digit. Find the number.

Solution:
Let x, y and z be the digits of a three-digit number.
Numerical value = 100x + 10y + z
According to the given,
100x + 10y + z = 17(x + y + z)
100x + 10y + z = 17x + 17y + 17z
⇒ 83x - 7y - 16z = 0....(i)
Also,
(100x + 10y + z) + 198 = 100z + 10y + x
100x + z + 198 = 100z + x
⇒ 99z - 99x = 198
⇒ z - x = 2
⇒ z = x + 2....(ii)
Again, given that,
x + z = y - 1
x + x + 2 = y - 1 [From (ii)]
2x + 2 + 1 = y
y = 2x + 3....(iii)
From (i), (ii) and (iii),
83x - 7(2x + 3) - 16(x + 2) = 0
83x - 14x - 21 - 16x - 32 = 0
53x = 53

x = 1
Substituting x = 1 in (ii) and (iii),
y = 2(1) + 3 = 2 + 3 = 5
z = 1 + 2 = 3
Hence, the three-digit number is 153.