MSBSHSE Class 10 Mathematics Question Paper 2017 Algebra Paper with Solutions

PART - A

1. Attempt any five of the following subquestions: **[5]**

(i) State whether the following sequence is an Arithmetic Progression or not:
3, 6, 12, 24,.......

Solution:
Given,
3, 6, 12, 24,....
First term = 3
Second term - First term = 6 - 3 = 3
Third term - Second term = 12 - 6 = 6
Common difference is not the same throughout the sequence.
Hence, the given sequence is not an Arithmetic progression.

(ii) If one root of the quadratic equation is $3 - 2\sqrt{5}$, then write another root of the equation.

Solution:
Given,
One root of the quadratic equation is $3 - 2\sqrt{5}$
The other root will be the conjugate of the first one.
Hence, the other root is $3 + 2\sqrt{5}$.

(iii) There are 15 tickets bearing the numbers from 1 to 15 in a bag and one ticket is drawn from this bag at random. Write the sample space (S) and n(S).

Solution:
Given that a bag contains 15 tickets bearing the numbers from 1 to 15.
Sample space = S = {1, 2, 3, 4, 5,...., 15}
n(S) = 15

(iv) Find the class mark of class 35 - 39.

Solution:
Given class:
35 - 39
Class mark = [Upper class limit + Lower class limit]/2
= (39 + 35)
= 74/2
= 37

(v) Write the next two terms of A.P. whose first term is 3 and the common difference is 4.

Solution:
Given,
First term = a = 3
Common difference = d = 4
Second term = a + d = 3 + 4 = 7
Third term = a + 2d = 3 + 2(4) = 3 + 8 = 11
Hence, the next two terms of the AP are 7 and 11.

(vi) Find the values of a, b, c for the quadratic equation $2x^2 = x + 3$ by comparing with standard form $ax^2 + bx + c = 0$.

Solution:
Given,
$2x^2 = x + 3$
$2x^2 - x - 3 = 0$
Comparing with the standard form $ax^2 + bx + c = 0$,
a = 2, b = -1, c = -3

2. Attempt any four of the following subquestions: [8]
(i) Find the first two terms of the sequence for which S_n is given below:
$S_n = n^2(n + 1)$.

Solution:
Given,
$S_n = n^2(n + 1)$
When n = 1,
$S_1 = 1^2(1 + 1) = 1(2) = 2$
When n = 2
$S_2 = 2^2(2 + 1) = 4(3) = 12$
$S_1 = a_1 = 2$
$a_1 + a_2 = S2$
$2 + a_2 = 12$
$a_2 = 12 - 2 = 10$
Therefore, the first term is 2 and the second term is 10.

(ii) Find the value of discriminant (Δ) for the quadratic equation:
$x^2 + 7x + 6 = 0$.

Solution:
Given,
$x^2 + 7x + 6 = 0$
Comparing with the standard form $ax^2 + bx + c = 0$,
a = 1, b = 7, c = 6
Discriminant $(\Delta) = b^2 - 4ac$
$= (7)^2 - 4(1)(6)$
$= 49 - 24$
$= 25$

(iii) Write the equation of the X-axis. Hence, find the point of intersection of the graph of the equation x + y = 5 with the X-axis.

Solution:
The equation of X-axis is $y = 0$
Given,
$x + y = 5$
Substituting $y = 0$,
$x + 0 = 5$
$x = 5$
The point of intersection of the graph represents the equation $x + y = 5$ with the X-axis is (5, 0).

(iv) For a certain frequency distribution, the values of Assumed mean (A) = 1300, $\sum f_i d_i = 900$ and $\sum f_i = 100$. Find the value of mean (x bar).

Solution:
Given,
Assumed mean (A) = 1300
$\sum f_i d_i = 900$ and $\sum f_i = 100$
Mean (x bar) = $A + (\sum f_i d_i / \sum f_i)$
$= 1300 + (900/100)$
$= 1300 + 9$
$= 1309$

(v) Two coins are tossed simultaneously. Write the sample space (S), n(S), the following event A using set notation, and n(A), where 'A is the event of getting at least one head.'

Solution:
Given,
Two coins are tossed simultaneously.
Sample space = {HH, HT, TH, TT}
n(S) = 4
A = The event of getting at least one head
A = {HT, TH, HH}
n(A) = 3

(vi) Find the value of k for which the given simultaneous equations have infinitely many solutions:
$kx + 4y = 10$;
$3x + 2y = 5$.

Solution:
$kx + 4y = 10$
$3x + 2y = 5$
Comparing with the standard form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,
$a_1 = k$, $b_1 = 4$, $c_1 = -10$
$a_2 = 3$, $b_2 = 2$, $c_2 = -5$
Condition for infinitely many solutions:
$a_1/a_2 = b_1/b_2 = c_1/c_2$
$k/3 = 4/2 = -10/-5$
$k/3 = 2/1 = 2/1$
$k/3 = 2$
$k = 6$

3. Attempt any three of the following subquestions: **[9]**

(i) How many three-digit natural numbers are divisible by 5?

Solution:

Three-digit natural numbers divisible by 5 are:

100, 105, 110, 115,...., 995

This is an AP with a = 100, d = 5 and a_n = 995.

nth term of an AP

$a_n = a + (n - 1)d$

995 = 100 + (n - 1)5

(n - 1)5 = 995 - 100

n - 1 = 895/5

n = 179 + 1

n = 180

Hence, the number of three-digit natural numbers which are divisible by 5 are 180.

(ii) Solve the following quadratic equation by factorization method:

$3x^2 - 29x + 40 = 0$.

Solution:

Given,

$3x^2 - 29x + 40 = 0$

$3x^2 - 24x - 5x + 40 = 0$

3x(x - 8) - 5(x - 8) = 0

(3x - 5)(x - 8) = 0

3x - 5 = 0, x - 8 = 0

x = 5/3, x = 8

(iii) Solve the following simultaneous equations by using Cramer's rule:

3x - y = 7;

x + 4y = 11.

Solution:

Given,

3x - y = 7

x + 4y = 11

$$D = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = (3 \times 4) - (-1 \times 1) = 12 + 1 = 13 \neq 0$$

$$D_x = \begin{vmatrix} 7 & -1 \\ 11 & 4 \end{vmatrix} = (7 \times 4) - (-1 \times 11) = 28 + 11 = 39$$

$$D_y = \begin{vmatrix} 3 & 7 \\ 1 & 11 \end{vmatrix} = (3 \times 11) - (7 \times 1) = 33 - 7 = 26$$

Using Cramer's rule,
$x = D_x/D = 39/13 = 3$
$y = D_y/D = 26/13 = 2$
Therefore, the solution of the given pair of equations is (x, y) =(3, 2).

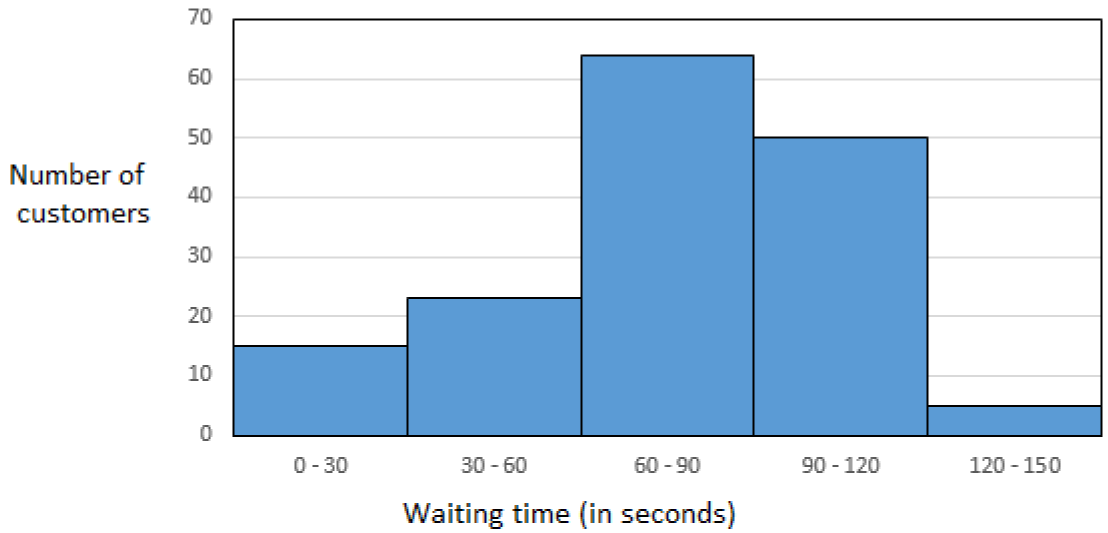
(iv) Two dice are thrown. Find the probability of the event that the product of numbers on their upper faces is 12.

Solution:
Give,
Two dice are thrown.
Sample pace (S) = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
n(S) = 36
Let A be the event of getting the product of numbers on their upper faces is 12.
A = {(2, 6), (6, 2), (3, 4), (4, 3)}
n(A) = 4
P(A) = n(A)/n(S)
= 4/36
= 1/9
Hence, the required probability is 1/9.

(v) The following is the frequency distribution of waiting time at the ATM centre; draw histogram to represent the data:

Waiting time (in seconds)	Number of customers
0 - 30	15
30 - 60	23
60 - 90	64
90 - 120	50
120 - 150	5

Solution:
Histogram
Scale:
x - axis: 1 cm = 30 seconds
y - axis: 1 cm = 10 customers



4. Attempt any two of the following subquestions: [8]

(i) Three horses A, B, and C are in a race, A is twice as likely to win as B and B is twice as likely to win as C. What are their probabilities of winning?

Solution:

Let P(A), P(B), and P(C) be the probability of winning in a race by three horses A, B, and C respectively.

According to the given,

$P(A) = 2P(B)$

$P(B) = 2P(C)$

Now, $P(A) = 2P(B) = 2[2P(C)] = 4P(C)$

We know that,

$P(A) + P(B) + P(C) = 1$

$4P(C) + 2P(C) + P(C) = 1$

$7P(C) = 1$

$P(C) = 1/7$

Therefore,

$P(B) = 2/7$

$P(A) = 4/7$

Hence, the required probabilities are 4/7, 2/7 and 1/7.

(ii) The following is the distribution of the size of certain farms from a taluka (tehsil):

Size of Farms (in acres)	Number of Farms
5 - 15	7
15 - 25	12
25 - 35	17
35 - 45	25

45 - 55	31
55 - 65	5
65 - 75	3

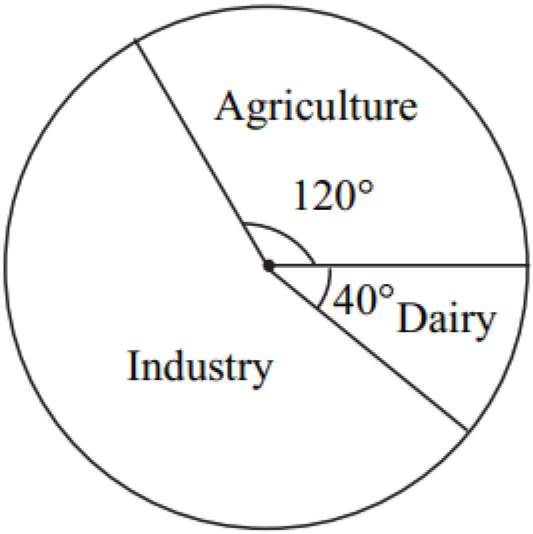
Find the median size of farms.

Solution:

Size of Farms (in acres)	Number of Farms (frequency)	Cumulative frequency
5 - 15	7	7
15 - 25	12	19
25 - 35	17	36
35 - 45	25	61
45 - 55	31	92
55 - 65	5	97
65 - 75	3	100

N/2 = 100/2 = 50
Cumulative frequency greater than and nearest to 50 is 61, which lies in the class interval 35 - 45.
Median class = 35 - 45
The lower limit of the median class = l = 35
Frequency of the median class = f = 25
Cumulative frequency of the class preceding the median class = cf = 36
Class height = h = 10
Median = l + {[(N/2) - cf]/ f} × h
= 35 + [(50 - 36)/ 25] × 10
= 35 + (140/25)
= 35 + 5.6
= 40.6

(iii) The following pie diagram represents the sector-wise loan amount in crores of rupees distributed by a bank. From the information answer the following questions:



a. If the dairy sector receives Rs. 20 crores, then find the total loan disbursed.
b. Find the loan amount for the agriculture sector and also for the industrial sector.
c. How much additional amount did the industrial sector receive than the agriculture sector?

Solution:

Sector	Measure of a central angle
Agriculture	120°
Dairy	40°
Industry	360 - (120° + 40°) = 200°
Total	360°

a. Dairy sector = Rs. 20 crores
40° = Rs. 20 crores
Total amount = (360°/ 40) × 20 = Rs. 180 crores
b. The loan amount for the agriculture sector
= (120°)/ 360°) × 180
= Rs. 60 crores
The loan amount for the industrial sector
= (200°)/ 360°) × 180
= Rs. 100 crores
c. The additional amount received by the industrial sector than the agricultural sector
= Rs. (100 - 60) crores
= Rs. 40 crores

5. Attempt any two of the following subquestions: [10]
(i) If the cost of bananas is increased by Rs. 10 per dozen, one can get 3 dozen less for Rs. 600. Find the original cost of one dozen bananas.

Solution:

Let x be the cost (in Rs.) of one dozen bananas.
Let y be the number of bananas can get for Rs. 600.
$xy = 600$
$y = 600/x$....(i)
According to the given,
$(x + 10)(y - 3) = 600$
$(x + 10)[(600/x) - 3] = 600$ [from (i)]
$(x + 10)[(600 - 3x)/ x] = 600$
$(x + 10)(600 - 3x) = 600x$
$600x - 3x^2 + 6000 - 30x = 600x$
$3x^2 - 6000 + 30x = 0$
$3(x^2 + 10x - 2000) = 0$
$x^2 + 10x - 2000 = 0$
$x^2 + 50x - 40x - 2000 = 0$
$(x + 50)(x - 40) = 0$
$x = -50, x = 40$
Cost cannot be negative.
Therefore, $x = 40$
Hence, the original cost of one dozen bananas is Rs. 40.

(ii) If the sum of first p terms of an A.P. is equal to the sum of first q terms, then show that the sum of its first (p + q) terms is zero where $p \neq q$.

Solution:
We know that the sum of the first n terms of an AP is $S_n = n/2 [2a + (n - 1)d]$
Given,
$S_p = S_q$
$p/2 [2a + (p - 1)d] = q/2 [2a + (q - 1)d]$
$p[2a + (p - 1)d] = q[2a + (q - 1)d]$
$2ap + (p - 1)dp = 2aq + (q - 1)dq$
$2ap - 2aq + (p - 1)dp - (q - 1)dq = 0$
$2a(p - q) + d [p^2 - p - q^2 + q] = 0$
$2a(p - q) + d [p^2 - q^2 - (p - q)] = 0$
$2a(p - q) + d [(p + q)(p - q) - 1(p - q)] = 0$
$2a(p - q) + d(p - q)[p + q - 1] = 0$
$(p - q) [2a + d(p + q - 1)] = 0$
$2a + (p + q - 1)d = 0$....(i)
Sum of the first (p + q) terms
$S_{(p + q)} = (p + q)/2 [2a + (p + q - 1)d]$
$= (p + q)/2 [0]$ (from (i))
$= 0$
Hence proved.

(iii) Solve the following simultaneous equations:
$(1/3x) - (1/4y) + 1 = 0$;
$(1/5x) + (1/2y) = 4/15$

Solution:
Given,
$(1/3x) - (1/4y) + 1 = 0$

$(1/5x) + (1/2y) = 4/15$
Substituting $1/x = a$ and $1/y = b$,
$(1/3)a - (1/4)b = -1$
$(4a - 3b)/12 = -1$
$4a - 3b = -12$….(i)
And
$(1/5)a + (1/2)b = 4/15$
$(2a + 5b)/ 10 = 4/15$
$2a + 5b = 8/3$….(ii)
(i) - (ii) × 2,
$4a - 3b - (4a + 10b) = -12 - (16/3)$
$-3b - 10b = (-36 - 16)/3$
$-13b = - 52/3$
$b = 4/3$
Substituting $b = 4/3$ in (ii),
$2a + 5(4/3) = 8/3$
$2a = (8/3) - (20/3)$
$2a = -12/3$
$a = -4/2$
$a = -2$
Now,
$1/x = a$
$1/x = -2$
$x = -1/2$
$1/y = b$
$1/y = 4/3$
$y = 3/4$
Therefore, the solution of the given system of equations is $x = -1/2$ and $y = 3/4$.