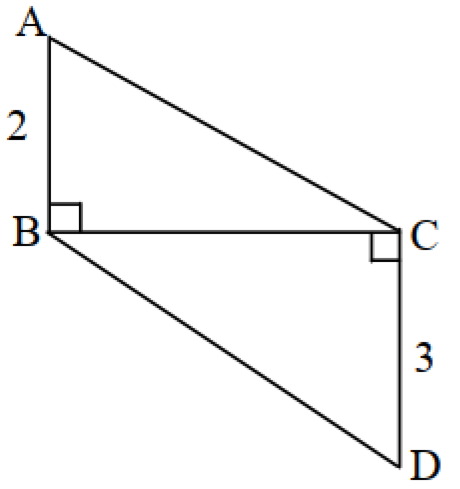
MSBSHSE Class 10 Mathematics Question Paper 2015 Geometry Paper with Solutions

PART - A

1. Solve any five sub-questions: **[5]**

(i) In the following figure, seg AB ⊥ seg BC, seg DC ⊥ seg BC. If AB = 2 and DC = 3, find A(Δ ABC)/ A(Δ DCB).



Solution:

Given,

AB = 2 and DC = 3

We know that the ratio of areas of two triangles lie on the same base is equal to the ratio of their corresponding heights.

A(Δ ABC)/ A(Δ DCB) = AB/DC = 2/3

(ii) Find the slope and y-intercept of the line y = -2x + 3.

Solution:

Given,

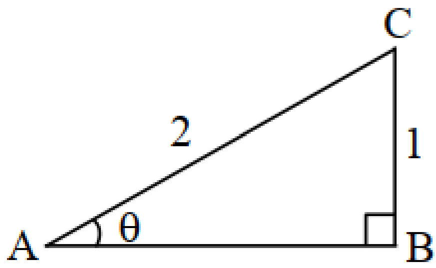
y = -2x + 3

Comparing with the equation of line in slope-intercept form y = mx + c

Slope = m = -2

y-intercept = c = 3

(iii) In the following figure, in ΔABC, BC = 1, AC = 2, ∠B = 90°. Find the value of sin θ.



Solution:
Given,
BC = 1, AC = 2, ∠B = 90°
sin θ = Side opposite to angle θ/ Hypotenuse
= BC/AC
= 1/2

(iv) Find the diagonal of a square whose side is 10 cm.

Solution:
Given,
Side of a square = a = 10 cm
Diagonal of the square = √2 × a
= √2 × 10
= 10√2 cm

(v) The volume of a cube is 1000 cm^3. Find the side of a cube.

Solution:
Given,
Volume of cube = 1000 cm^3
⇒ $(side)^3 = (10)^3$
⇒ Side = 10 cm
Therefore, the side of the cube is 10 cm.

(vi) If two circles with radii 5 cm and 3 cm respectively touch internally, find the distance between their centres.

Solution:
Given,
Two circles with radii 5 cm and 3 cm respectively touch internally.
Distance between their centres = Difference of the radii
= 5 - 3
= 2 cm

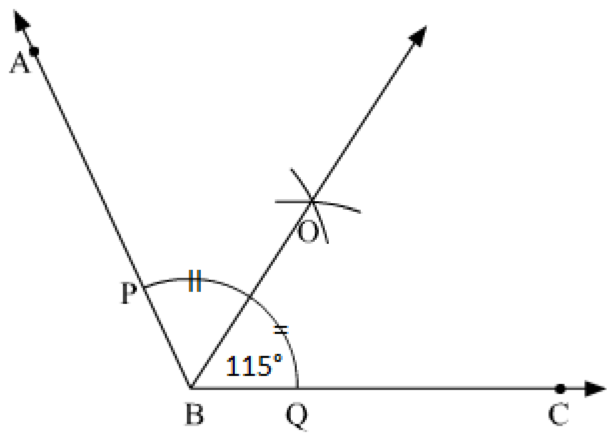
2. Solve any four sub-questions: **[8]**
(i) If sin θ = 5/13, where θ is an acute angle, find the value of cos θ.

Solution:
Given,
sin θ = 5/13

cos θ = $\sqrt{1 - \sin^2\theta}$
= $\sqrt{1 - (5/13)^2}$
= $\sqrt{1 - (25/169)}$
= $\sqrt{(169 - 25)/169}$
= $\sqrt{144/169}$
= 12/13
Therefore, cos θ = 12/13

(ii) Draw ∠ABC of measure 115° and bisect it.

Solution:



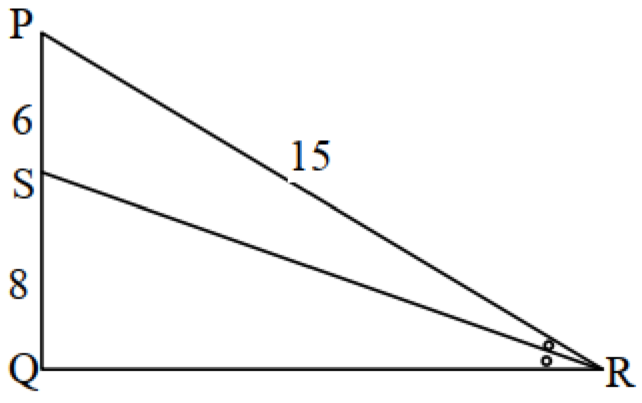
(iii) Find the slope of the line passing through the points C(3, 5) and D(-2, -3).

Solution:
Let the given points be:
C(3, 5) = (x_1, y_1)
D(-2, -3) = (x_2, y_2)
Slope of the line passing through the points (x_1, y_1) and (x_2, y_2)
= $(y_2 - y_1)/ (x_2 - x_1)$
= (-3 - 5)/ (-2 - 3)
= -8/ -5
= 8/5
Therefore, the slope is 8/5.

(iv) Find the area of the sector whose arc length and radius are 10 cm and 5 cm respectively.

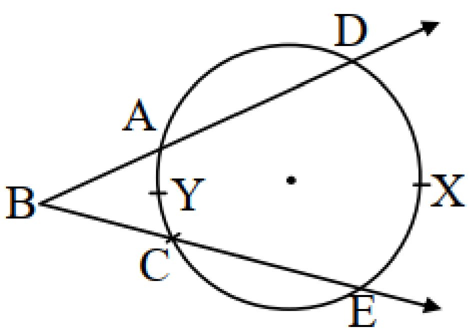
Solution:
Given,
Length of the arc = 10 cm
Radius = r = 5 cm
Area of sector = (r/2) × Length of arc
= (5/2) × 10
= 5 × 5
= 25 cm^2

(v) In the following figure, in ΔPQR, seg RS is the bisector of ∠PRQ, PS = 6, SQ = 8, PR = 15. Find QR.



Solution:
Given,
PS = 6, SQ = 8, PR = 15
seg RS is the bisector of ∠PRQ.
By the angle bisector property,
PR/QR = PS/SQ
15/QR = 6/8
⇒ QR = (15 × 8)/6
⇒ QR = 20

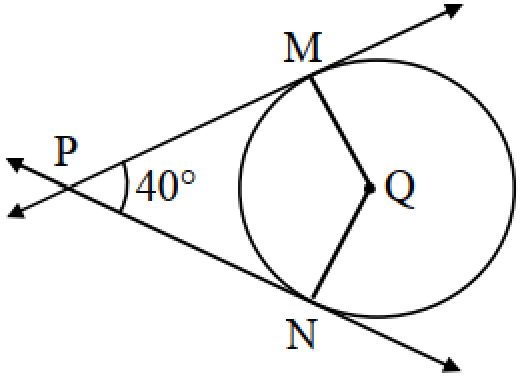
(vi) In the following figure, if m(arc DXE) = 100° and m(arc AYC) = 40°, find ∠DBE.



Solution:
Given,
m(arc DXE) = 100°
m(arc AYC) = 40°
By the inscribed angle theorem,
m∠AEB = (1/2) × m∠AYC
= (1/2) × 40°
= 20°
And
m∠EAD = (1/2) × m∠DXE
= (1/2) × 100°
= 50°

Now, by the exterior angle theorem:
$m\angle DBE + 20° = 50°$
$m\angle DBE = 50° - 20° = 30°$

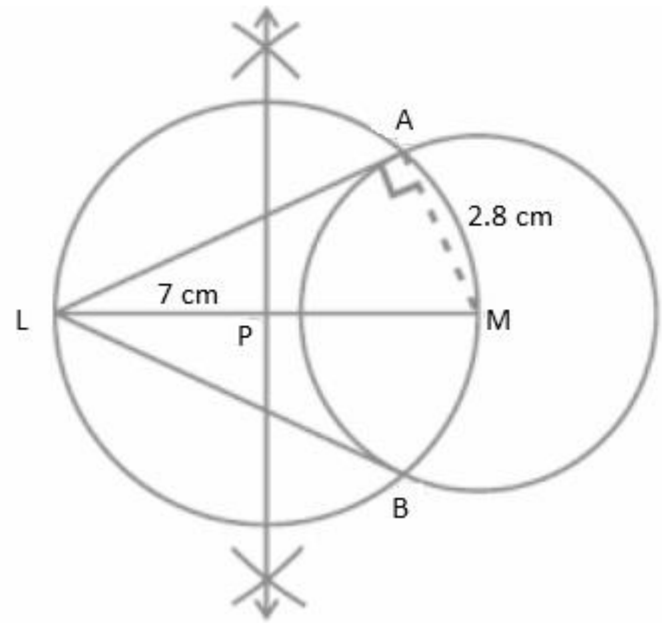
3. Solve any three sub-questions: **[9]**
(i) In the following figure, Q is the centre of a circle and PM, PN are tangent segments to the circle. If $\angle MPN = 40°$, find $\angle MQN$.



Solution:
Given,
$\angle MPN = 40°$
We know that the radius is perpendicular to the tangent through the point of contact.
$\angle PMQ = \angle PNQ = 90°$
In quadrilateral PMQN,
$\angle MPN + \angle PNQ + \angle MQN + \angle PMQ = 360°$
$40° + 90° + \angle MQN + 90° = 360°$
$\angle MQN + 220° = 360°$
$\angle MQN = 360° - 220°$
$\angle MQN = 140°$

(ii) Draw the tangents to the circle from the point L with radius 2.8 cm. The point, 'L' is at a distance 7 cm from the centre 'M'.

Solution:



(iii) The ratio of the areas of two triangles with the common base is 6 : 5. Height of the larger triangle is 9 cm, then find the corresponding height of the smaller triangle.

Solution:

Let H be the height of the larger triangle and h be the height of the smaller triangle.

We know that the ratio of the areas of two triangles with a common base is equal to the ratio of their corresponding heights.

⇒ $6/5 = H/h$

⇒ $6/5 = 9/h$ (given height of the larger triangle is 9 cm)

⇒ $h = (9 \times 5)/6$

⇒ $h = 15/2 = 7.5$ cm

Hence, the corresponding height of the smaller triangle is 7.5 cm.

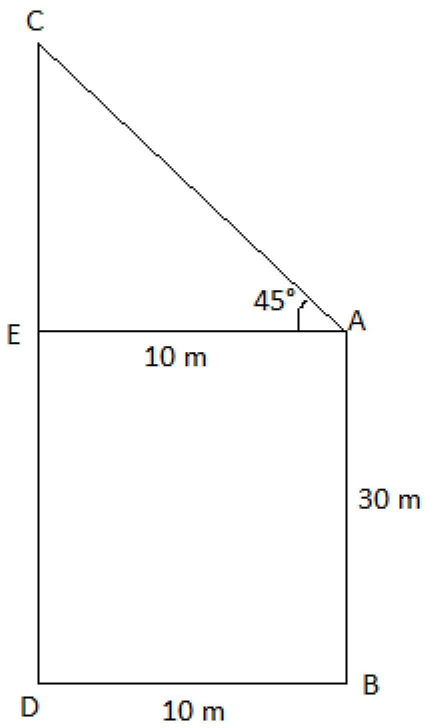
(iv) Two buildings are in front of each other on either side of a road of width 10 metres. From the top of the first building which is 30 metres high, the angle of elevation to the top of the second is 45°. What is the height of the second building?

Solution:

Let AB be the first building and CD be the second building.

BD = Width of the road = 10 m

AB = 30 m



BD = AE = 10 m
AB = ED = 30 m
In right triangle AEC,
tan 45° = CE/AE
1 = CE/10
CE = 10 m
CD = CE + ED
= 10 + 30
= 40 m
Therefore, the height of the second building is 40 m.

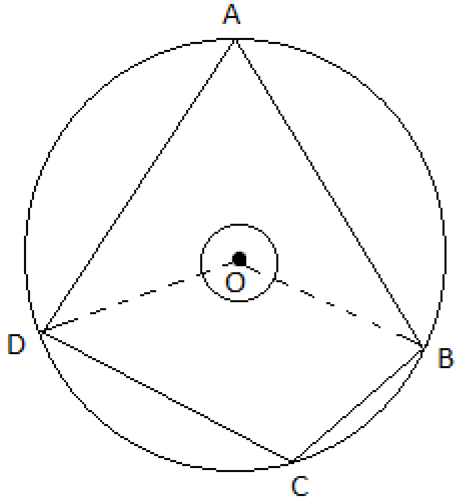
(v) Find the volume and surface area of a sphere of radius 4.2 cm. ($\pi = 22/7$)

Solution:
Given,
Radius of the sphere = r = 4.2 cm
Volume of sphere = $(4/3)\pi r^3$
= $(4/3) \times (22/7) \times 4.2 \times 4.2 \times 4.2$
= 310.464 cm^3
The surface area of the sphere = $4\pi r^2$
= $4 \times (22/7) \times 4.2 \times 4.2$
= 221.76 cm^2
Therefore, the volume of the sphere is 310.464 cm^3 and the surface area of the sphere is 24.64 cm^2.

4. Solve any two sub-questions: **[8]**
(i) Prove that "the opposite angles of a cyclic quadrilateral are supplementary".

Solution:

Given,
ABCD is a cyclic quadrilateral of a circle with centre O.
Construction: Join OB and OD.
To prove: ∠BAD + ∠BCD = 180°

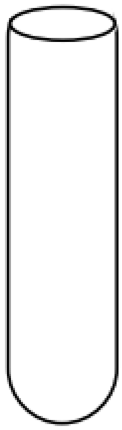


Proof:
We know that the angle subtended by the arc at the centre is twice the angle subtended by it at the remaining part of the circle.
∠BOD = 2∠BAD....(i)
Also,
reflex∠BOD = 2∠BCD....(ii)
Adding (i) and (ii),
2∠BAD + 2∠BCD = ∠BOD + reflex ∠BOD
2(∠BAD + ∠BCD) = 360°
∠BAD + ∠BCD = 360°/2
∠BAD + ∠BCD = 180°
Hence proved.

(ii) Prove that $\sin^6\theta + \cos^6\theta = 1 - 3 \sin^2\theta. \cos^2\theta$.

Solution:
LHS = $\sin^6\theta + \cos^6\theta$
$= (\sin^2\theta)^3 + (\cos^2\theta)^3$
$= (\sin^2\theta + \cos^2\theta) [(\sin^2\theta)]^2 + (\cos^2\theta)^2 - \sin^2\theta \cos^2\theta]$
$= (1) [(\sin^2\theta + \cos^2\theta)^2 - 2 \sin^2\theta \cos^2\theta - \sin^2\theta \cos^2\theta]$
$= (1)^2 - 3 \sin^2\theta \cos^2\theta$
$= 1 - 3 \sin^2\theta. \cos^2\theta$
= RHS
Hence proved.

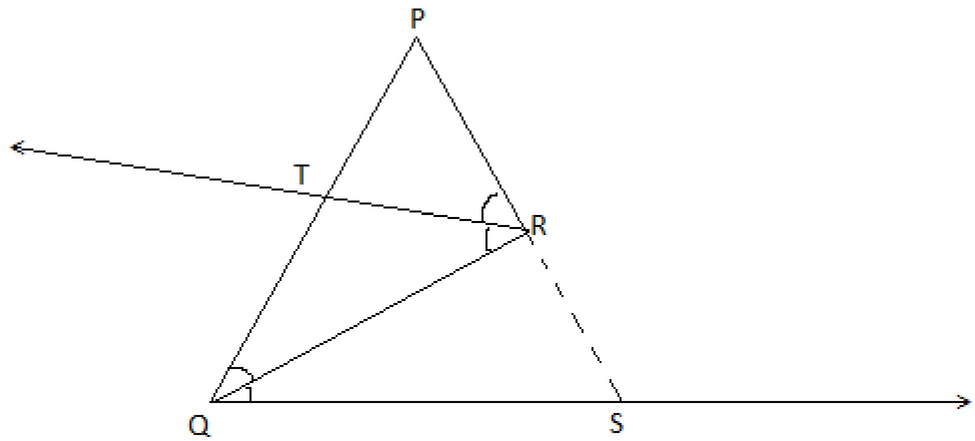
(iii) A test tube has a diameter 20 mm and the height is 15 cm. The lower portion is a hemisphere. Find the capacity of the test tube. ($\pi = 3.14$)



Solution:
Given,
Diameter of the test tube = 20 mm
Radius of cylindrical part = Radius of the hemisphere = r = 20/2 = 10 mm = 1 cm
Height of the test tube = 15 cm
Height of the cylindrical part = h = 15 - 1 = 14 cm
Volume (capacity) of the test tube = Volume of cylinder + Volume of hemisphere
$= \pi r^2 h + (2/3)\pi r^3$
$= 3.14 \times 1 \times 1 \times 14 + (2/3) \times 3.14 \times 1 \times 1 \times 1$
$= 43.96 + 2.09$
$= 46.05$
Hence, the capacity of the test tube is 46.05 cm^3.

5. Solve any two sub-questions: **[10]**
(i) Prove that the angle bisector of a triangle divides the side opposite to the angle in the ratio of the remaining sides.

Solution:
Given,
Triangle PQR in which RT is the angle bisector of ∠QRP.
Construction:
Draw angle bisector RT from R which intersects PQ at T.
Extend PR and QR so that they intersect each other at S.



To prove: PT/TQ = PR/QR
Proof:
RT || QS and PS is the transversal.
∠PRT = ∠RSQ (corresponding angles)
Now, BC is the transversal.
∠TRQ = ∠RQS (alternate angles)
∠PRT = ∠TRQ (given)
From the all above,
∠RQS = ∠RSQ
In triangle RQS,
QR = RS (sides opposite to equal angles are equal)
In triangle PQS,
RT || QS
PT/TQ = PR/RS
PT/TQ = PR/RQ
Hence proved.

(ii) Write down the equation of a line whose slope is 3/2 and which passes through point P, where P divides the line segment AB joining A(-2, 6) and B(3, -4) in the ratio 2 : 3.

Solution:
Given,
P divides the line segment AB joining A(-2, 6) and B(3, -4) in the ratio 2 : 3.
A(-2, 6) = (x_1, y_1)
B(3, -4) = (x_2, y_2)
m : n = 2 : 3
Using the section formula,
P = $[(mx_2 + nx_1)/ (m + n), (my_2 + ny_1)/ (m + n)]$
= [(6 - 6)/ (2 + 3), (-8 + 18)/ (2 + 3)]
= (0/5, 10/5)
= (0, 2)
Therefore, P = (0, 2)
Equation of the line passing through p(0, 2) and having slope 3/2 is:
y - 2 = (3/2) (x - 0)
2(y - 2) = 3x
2y - 4 = 3x

$3x - 2y + 4 = 0$

(iii) ΔRST ~ ΔUAY. In ΔRST, RS = 6 cm, ∠S = 50°, ST = 7.5 cm. The corresponding sides of ΔRST and ΔUAY are in the ratio 5 : 4. Construct ΔUAY.

Solution:
Given,
ΔRST ~ ΔUAY
In ΔRST,
RS = 6 cm, ∠S = 50°, ST = 7.5 cm
ΔRST and ΔUAY are in the ratio 5 : 4. (given)
RS/UA = ST/AY = RT/UY = 5/4
∠S = ∠A = 50°
Now,
RS/UA = 5/4
6/UA = 5/4
UA = (6 × 4)/5
UA = 4.8 cm
Similarly,
ST/AY = 5/4
7.5/AY = 5/4
AY = (7.5 × 4)/5
AY = 6 cm