MSBSHSE Class 10 Mathematics Question Paper 2016 Geometry Paper with Solutions

PART - A

1. Solve any five sub-questions: **[5]**

(i) $\Delta DEF \sim \Delta MNK$. If DE = 2, MN = 5, then find the value of $A(\Delta DEF)/A(\Delta MNK)$.

Solution:

Given,

$\Delta DEF \sim \Delta MNK$

DE = 2, MN = 5

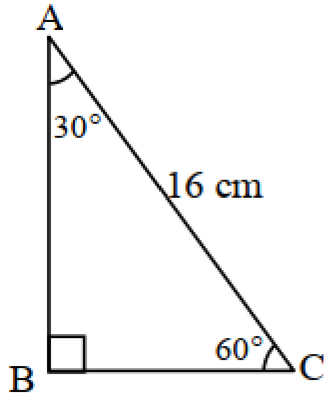
We know that the ratio of areas of similar triangles is equal to squares of the ratio of their corresponding sides.

$A(\Delta DEF)/A(\Delta MNK) = DE^2/MN^2$

$= (2)^2/(5)^2$

$= 4/25$

(ii) In the following figure, in ΔABC, $\angle B = 90°$, $\angle C = 60°$, $\angle A = 30°$, AC = 16 cm. Find BC.



Solution:

Given,

ΔABC is a 30° - 60° - 90° triangle.

Side opposite to 30° = BC

BC = (1/2) × Hypotenuse

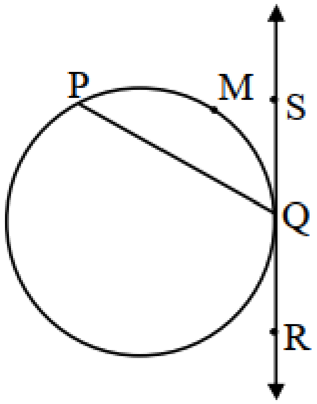
= (1/2) × AC

= (1/2) × 16

= 8 cm

Therefore, BC = 8 cm

(iii) In the following figure, m(arc PMQ) = 110°, find ∠PQS.



Solution:
Given,
m(arc PMQ) = 110°
We know that the measure of the angle formed by the intersection of a chord and tangent of the circle equal to the half the angle made by the arc with the chord.
∠PQS = (½) × m(arc PMQ)
= (1/2) × 110°
= 55°

(iv) If the angle θ = -30°, find the value of cos θ.

Solution:
Given,
θ = -30°
We know that,
cos (-θ) = cos θ
cos θ = cos (-30°)
= cos 30°
= √3/2

(v) Find the slope of the line with inclination 60°.

Solution:
Given,
Inclination of line = θ = 60°
The slope of the line = tan θ
= tan 60°
= √3

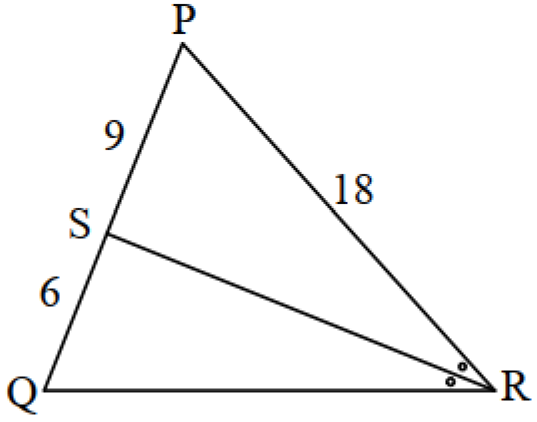
(vi) Using Euler's formula, find V if E = 10, F = 6.

Solution:
Given,
E = 10, F = 6
Using Euler's formula,
F + V = E + 2
6 + V = 10 + 2

V = 12 - 6
V = 6

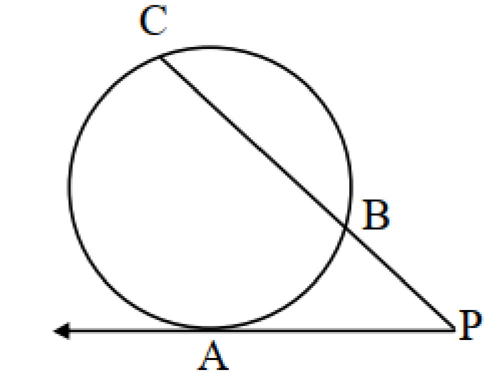
2. Solve any four sub-questions: **[8]**

(i) In the following figure, in ΔPQR, seg RS is the bisector of ∠PRQ. If PS = 9, SQ = 6, PR = 18, find QR.



Solution:
Given that, in triangle PQR, seg RS is the bisector of ∠PRQ.
PS = 9, SQ = 6, PR = 18
By the angle bisector property,
PR/QR = PS/SQ
18/QR = 9/6
⇒ QR = (18 × 6)/9
⇒ QR = 12

(ii) In the following figure, a tangent segment PA touching a circle in A and a secant PBC are shown. If AP = 12, BP = 9, find BC.

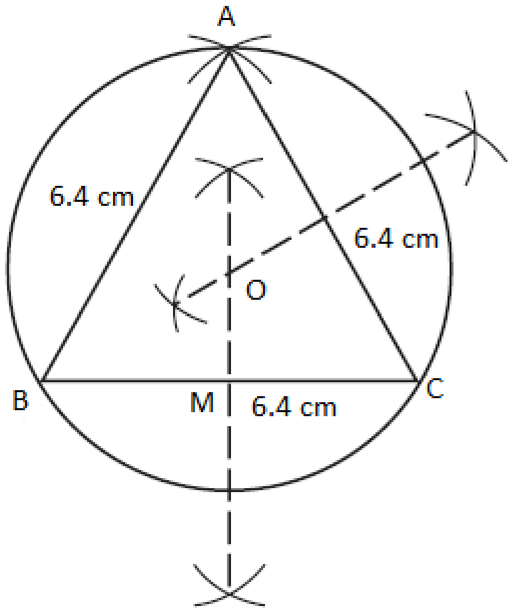


Solution:
Given,
PA = tangent
PBC = secant
By the tangent - secant theorem,
$PB \times PC = PA^2$
$9 \times PC = (12)^2$ [given AP = 12, BP = 9]

PC = 144/9
PC = 16
Now,
PC = PB + BC
16 = 9 + BC
⇒ BC = 16 - 9
⇒ BC = 7

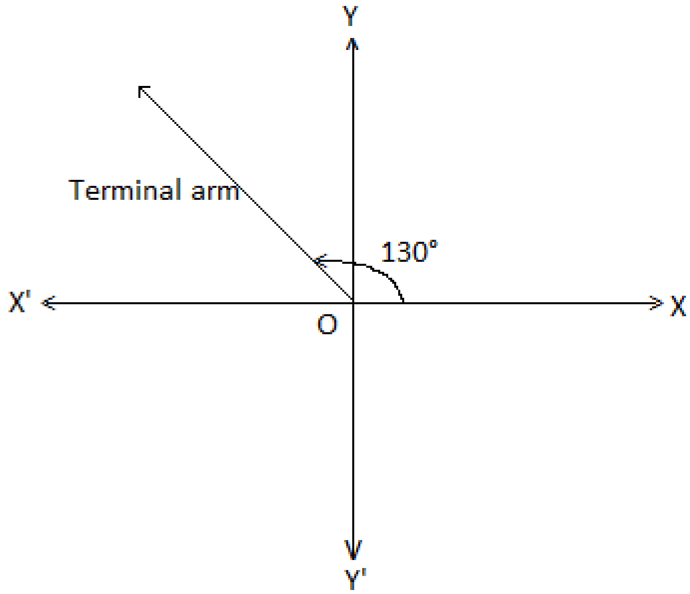
(iii) Draw an equilateral Δ ABC with side 6.4 cm and construct its circumcircle.

Solution:



(iv) For the angle in standard position if the initial arm rotates 130° in an anticlockwise direction, then state the quadrant in which the terminal arm lies. (Draw the Figure and write the answer.)

Solution:
Given,
The initial arm rotates 130° in the anticlockwise direction from the standard position.
The measure of angle 130° lies between 90° and 180°.
Hence, the terminal arm lies in quadrant II.



(v) Find the area of the sector whose arc length and radius are 16 cm and 9 cm respectively.

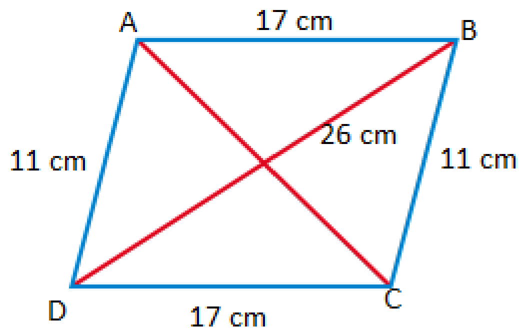
Solution:
Given,
Length of the arc = 16 cm
Radius = r = 9 cm
Area of sector = (r/2) × Length of arc
= (9/2) × 16
= 9 × 8
= 72 cm^2

(vi) Find the surface area of a sphere of radius 1.4 cm. (π = 22/7)

Solution:
Given,
Radius of the sphere = r = 1.4 cm
The surface area of the sphere = $4\pi r^2$
= 4 × (22/7) × 1.4 × 1.4
= 24.64 cm^2
Therefore, the surface area of the sphere is 24.64 cm^2.

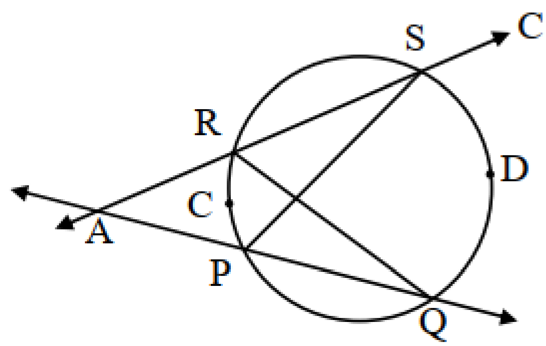
3. Solve any three sub-questions: [9]
(i) Adjacent sides of a parallelogram are 11 cm and 17 cm. If the length of one of its diagonals is 26 cm, find the length of the other.

Solution:
Given,
Adjacent sides of a parallelogram are 11 cm and 17 cm.
The length of one diagonal is 26 cm.



AB = CD = 17 cm
BC = AD = 11 cm
BD = 26 cm
We know that,
Sum of squares of sides of a parallelogram = Sum of squares of its diagonals
$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$
$(17)^2 + (11)^2 + (17)^2 + (11)^2 = AC^2 + (26)^2$
$289 + 121 + 289 + 121 = AC^2 + 676$
$AC^2 = 820 - 676$
$AC^2 = 144$
AC = 12 cm
Therefore, the length of the other diagonal is 12 cm.

(ii) In the following figure, secants containing chords RS and PQ of a circle intersect each other in point A in the exterior of a circle. If m(arc PCR) = 26°, m(arc QDS) = 48°, then find:
a. m ∠PQR
b. m ∠SPQ
c. m ∠RAQ



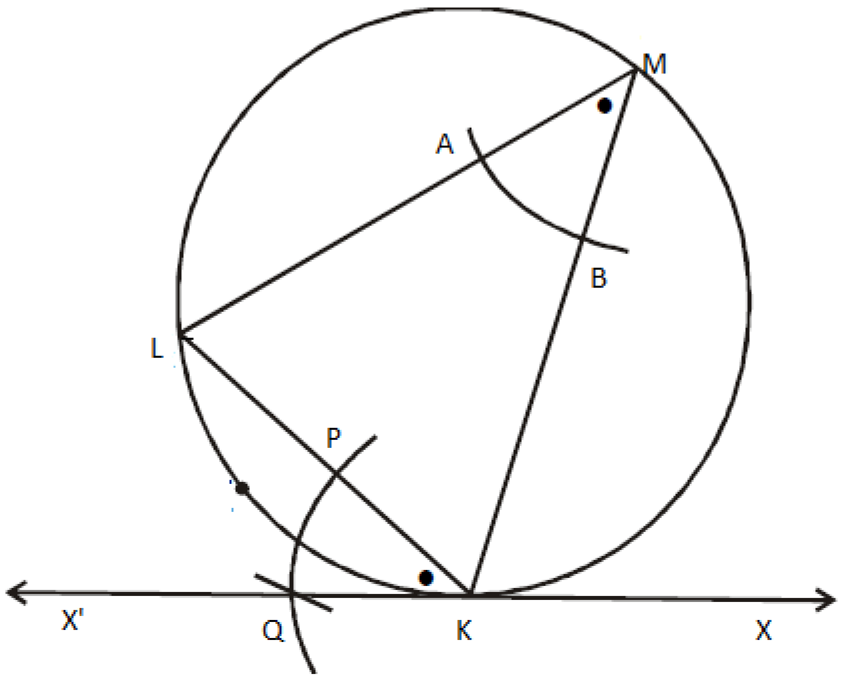
Solution:
Given,
m(arc PCR) = 26°
m(arc QDS) = 48°
By the inscribed angle theorem,
a. ∠PQR = 1/2 × m(arc PCR)
= (1/2) × 26°
= 13°
∠PQR = ∠AQR = 13....(i)

b. ∠SPQ = 1/2 × m(arc QDS)
= (1/2) × 48°
= 24°
∠SPQ = 24°

c. In triangle AQR,
∠RAQ + ∠AQR = ∠SRQ (remote interior angle theorem)
∠SRQ = ∠SPQ (angles subtended by the same arc)
Therefore,
∠RAQ + ∠AQR = ∠SPQ
m∠RAQ = m∠SPQ - m∠AQR
= 24° - 13° [From (i) and (ii)]
= 11°
m∠RAQ = 11°

(iii) Draw a circle of radius 3.5 cm. Take any point K on it. Draw a tangent to the circle at K without using centre of the circle.

Solution:



Therefore, XKX' is the required tangent to the circle with a radius of 3.5 cm.

(iv) If sec α = 2/√3, then find the value of (1 - cosec α)/ (1 + cosec α), where α is in IV quadrant.

Solution:
Given,
sec α = 2/√3
Thus, sec α = r/x = 2/√3
Let r = 2k and x = √3k
We know that,

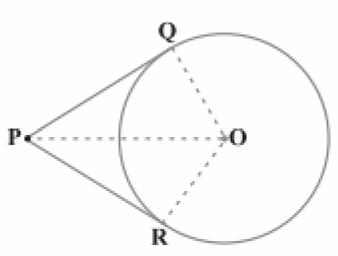
$r^2 = x^2 + y^2$
$(2k)^2 = (\sqrt{3}k)^2 + y^2$
$y^2 = 4k^2 - 3k^2$
$y^2 = k^2$
$y = \pm k$
Given that α lies in quadrant IV.
Therefore, y = -k
cosec α = r/y = 2k/-k = -2
Now,
(1 - cosec α)/ (1 + cosec α)
= [1 - (-2)]/ [1 + (-2)]
= (1 + 2)/ (1 - 2)
= -3

(v) Write the equation of the line passing through the pair of points (2, 3) and (4, 7) in the form of y = mx + c.

Solution:
Let the given points be:
$A(2, 3) = (x_1, y_1)$
$B(4, 7) = (x_2, y_2)$
Equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is:
$(x - x_1)/ (x_2 - x_1) = (y - y_1)/ (y_2 - y_1)$
(x - 2)/ (4 - 2) = (y - 3)/ (7 - 3)
(x - 2)/ 2 = (y - 3)/ 4
4(x - 2) = 2(y - 3)
4x - 8 = 2y - 6
4x - 8 - 2y + 6 = 0
4x - 2y - 2 = 0
2(2x - y - 1) = 0
2x - y - 1 = 0
y = 2x - 1
This is of the form y = mx + c [m = 2, c = -1]
Hence, the required equation of line is y = 2x - 1.

4. Solve any two sub-questions: [8]
(i) Prove that "The length of the two tangent segments to a circle drawn from an external point are equal".

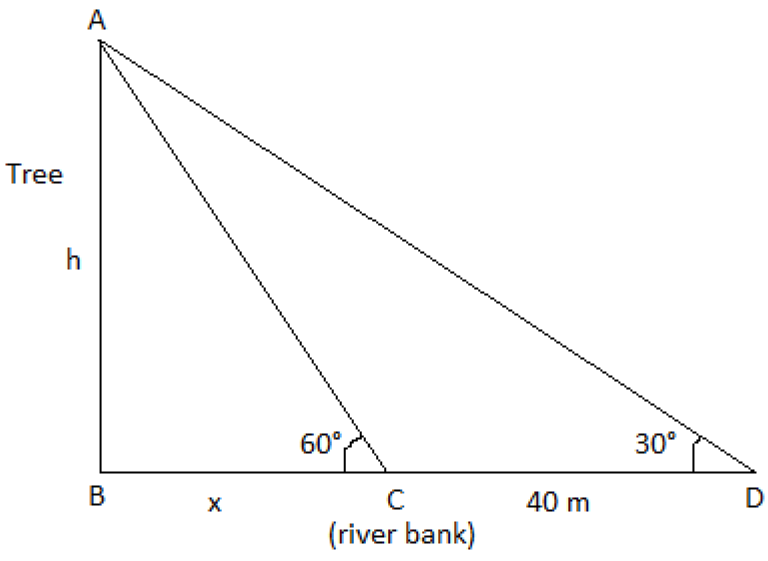
Solution:
Given,
Given,
PQ and PR are the tangents to the circle with centre O from an external point P.
To prove: PQ = PR
Construction:
Join OQ, OR and OP.



Proof:
We know that the radius is perpendicular to the tangent through the point of contact.
∠OQP = ∠ORP = 90°
In right ΔOQP and ORP,
OQ = OR (radii of the same circle)
OP = OP (common)
By RHS congruence criterion,
ΔOQP ≅ ΔORP
By CPCT,
PQ = PR
Hence proved.

(ii) A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and width of the river. (√3 = 1.73)

Solution:
Let AB be the tree and BC be the width of the river.

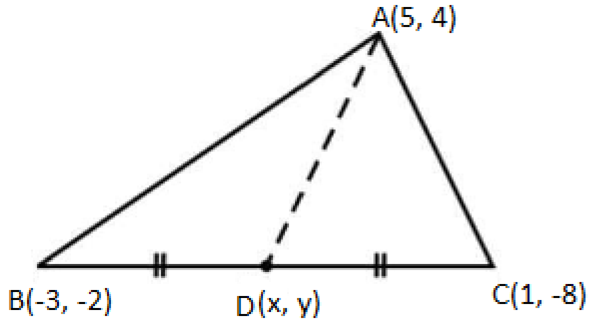


CD = 40 m
BC = x and AB = h
In right triangle ABC,
tan 60° = AB/BC
√3 = h/x
h = √3x....(i)

In right triangle ABD,
tan 30° = AB/BD
$1/\sqrt{3} = h/(x + 40)$
$x + 40 = h\sqrt{3}$
$x + 40 = (\sqrt{3}x)\sqrt{3}$ [From (i)]
$x + 40 = 3x$
$2x = 40$
$x = 20$ m
Substituting $x = 20$ in (i),
$h = 20\sqrt{3}$
$= 20 \times 1.73$
$= 34.6$ m
Therefore, the height of the tree is 34.6 m and the width of the river is 20 m.

(iii) A(5, 4), B(-3, -2) and C(1, -8) are the vertices of a triangle ABC. Find the equations of median AD and the line parallel to AC passing through point B.

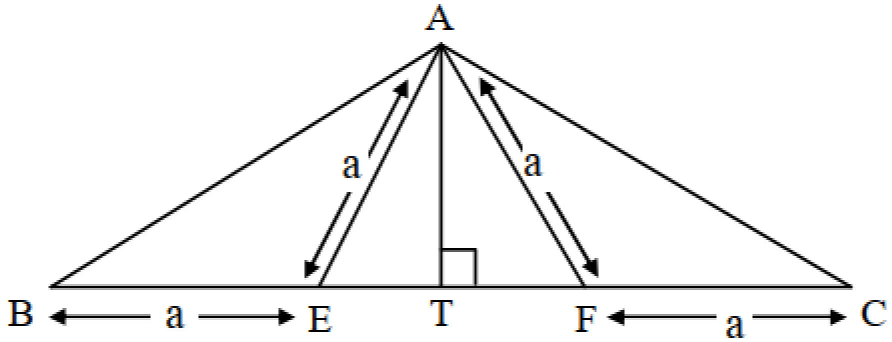
Solution:
Given,
Vertices of a triangle ABC are A(5, 4), B(-3, -2) and C(1,-8).
A(5, 4) = (x_1, y_1)
B(-3, -2) = (x_2, y_2)
C(1, -8) =(x_3, y_3)
Let D(x, y) be the median of triangle ABC.



D is the midpoint of BC.
$D(x, y) = [(x_2 + x_3)/2, (y_2 + y_3)/2]$
$= [(-3 + 1)/2, (-2 - 8)/2]$
$= (-2/2, -10/2)$
$= (-1, -5)$
$D(-1, -5) = (x_4, y_4)$
Equation of median AD is
$(x - x_1)/ (x_4 - x_1) = (y - y_1)/(y_4 - y_1)$
$(x - 5)/ (-1 - 5) = (y - 4)/ (-5 - 4)$
$(x - 5)/(-6) = (y - 4)(-9)$
$-9(x - 5) = -6(y - 4)$
$-9x + 45 = -6y + 24$
$9x - 45 - 6y + 24 = 0$
$9x - 6y - 21 = 0$

3(3x - 2y - 7) = 0
3x - 2y - 7 = 0
Hence, the required equation of median AD is 3x - 2y - 7 = 0.
We know that the line parallel to AC = Slope of AC
Slope of AB = $(y_3 - y_1)/ (x_3 - x_1)$
= (-8 - 4)/ (1 - 5)
= -12/-4
= 3
Thus, m = 3
Equation of the line parallel to AC and passing through the point B(-3, -2) is
$y - y_2 = m(x - x_2)$
y - (-2) = 3[x - (-3)]
y + 2 = 3x + 9
3x + 9 - y - 2 = 0
3x - y + 7 = 0

5. Solve any two sub-questions: **[10]**
(i) In the following figure, AE = EF = AF = BE = CF = a, AT ⊥ BC. Show that AB = AC = √3 × a

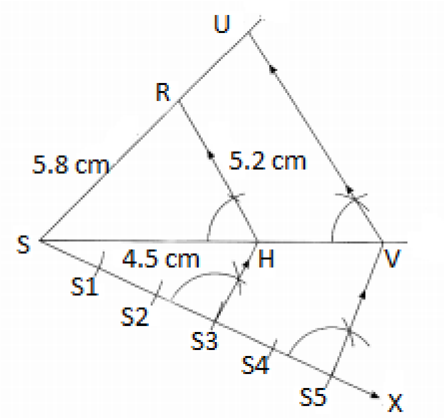


Solution:
Given,
AE = EF = AF = BE = CF
AT ⊥ EF
ΔAEF is an equilateral triangle.
ET = TF = a/2
BT = CT = a + (a/2)....(i)
In right triangle ATB and ATC,
AT = AT (common)
∠ATB = ∠ATC (right angles)
BT = CT [From (i)]
By SAS congruence criterion,
ΔATB ≅ ΔATC
BY CPCT,
AB = AC
In triangle AEF,
AE = AF = EF (given)
ΔAEF is an equilateral triangle.
Altitude of an equilateral triangle = AT = (√3/2)a
In right triangle ATB,

$AB^2 = AT^2 + BT^2$
$= [(\sqrt{3}/2)a]^2 + [a + (a/2)]^2$
$= (3a^2/4) + (3a/2)^2$
$= (3a^2/4) + (9a^2/4)$
$= 12a^2/4$
$= 3a^2$
$AB = \sqrt{3}a$
Therefore, $AB = AC = \sqrt{3} \times a$

(ii) Δ SHR ~ Δ SVU. In Δ SHR, SH = 4.5 cm, HR = 5.2 cm, SR = 5.8 cm and SH/SV = 3/5.
Construct Δ SVU.

Solution:



(iii) Water flows at the rate of 15 m per minute through a cylindrical pipe, having the diameter 20 mm. How much time will it take to fill a conical vessel of base diameter 40 cm and depth 45 cm?

Solution:
Given,
Diameter of cylindrical pipe = 20 mm
Radius of the cylindrical pipe = r = 20/2 = 10 mm = 1 cm
Speed of water = h = 15 m per minute = 1500 cm/min
Volume of cylindrical pipe = Volume of water flowing per minute
$= \pi r^2 h$
$= (22/7) \times 1 \times 1 \times 1500$
$= 33000/7 \text{ cm}^3$
Also,
Diameter of conical vessel = 40 cm
Radius of conical vessel = R = 40/2 = 20 cm
Depth = H = 45
Capacity of the conical vessel = Volume of cone
$= (1/3) \pi R^2 H$
$= (1/3) \times (22/7) \times 20 \times 20 \times 45$
$= 396000/21 \text{ cm}^3$
Time required to fill the vessel = Volume of conical vessel/ Volume of water flowing per minute
$= (396000/21)/ (33000/7)$

= (396000 × 7)/ (21 × 33000)
= 4
Hence, the time required to fill the conical vessel is 4 min.