Exercise 1.5 Page: 24

1. Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$

Solution:

We know that, $\sqrt{5} = 2.2360679\ldots$

Here, 2.2360679…is non-terminating and non-recurring.

Now, substituting the value of $\sqrt{5}$ in $2 - \sqrt{5}$, we get,

$2 - \sqrt{5} = 2 - 2.2360679\ldots = -0.2360679$

Since the number, $-0.2360679\ldots$, is non-terminating non-recurring, $2 - \sqrt{5}$ is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

Solution:

$(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23}$

$= 3$

$= 3/1$

Since the number 3/1 is in p/q form, $(3 + \sqrt{23}) - \sqrt{23}$ is rational.

(iii) $2\sqrt{7}/7\sqrt{7}$

Solution:

$2\sqrt{7}/7\sqrt{7} = (2/7) \times (\sqrt{7}/\sqrt{7})$

We know that $(\sqrt{7}/\sqrt{7}) = 1$

Hence, $(2/7) \times (\sqrt{7}/\sqrt{7}) = (2/7) \times 1 = 2/7$

Since the number, 2/7 is in p/q form, $2\sqrt{7}/7\sqrt{7}$ is rational.

(iv) $1/\sqrt{2}$

Solution:

Multiplying and dividing numerator and denominator by $\sqrt{2}$ we get,

$(1/\sqrt{2}) \times (\sqrt{2}/\sqrt{2}) = \sqrt{2}/2$ (since $\sqrt{2} \times \sqrt{2} = 2$)

We know that, $\sqrt{2} = 1.4142\ldots$

Then, $\sqrt{2}/2 = 1.4142/2 = 0.7071..$

Since the number, 0.7071..is non-terminating non-recurring, $1/\sqrt{2}$ is an irrational number.

(v) 2π

Solution:

We know that, the value of $\pi = 3.1415$

Hence, $2\pi = 2 \times 3.1415.. = 6.2830\ldots$

Since the number, 6.2830…, is non-terminating non-recurring, 2π is an irrational number.

2. Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

Solution:

$(3+\sqrt{3})(2+\sqrt{2})$

Opening the brackets, we get, $(3\times2)+(3\times\sqrt{2})+(\sqrt{3}\times2)+(\sqrt{3}\times\sqrt{2})$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(2+\sqrt{2})$

Solution:

$(3+\sqrt{3})(2+\sqrt{2}) = 3^2-(\sqrt{3})^2 = 9-3$

$= 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Solution:

$(\sqrt{5}+\sqrt{2})^2 = \sqrt{5}^2+(2\times\sqrt{5}\times\sqrt{2})+\sqrt{2}^2$

$= 5+2\times\sqrt{10}+2 = 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Solution:

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5}^2-\sqrt{2}^2) = 5-2 = 3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter, (say d). That is, π =c/d. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Solution:

There is no contradiction. When we measure a value with a scale, we only obtain an approximate value. We never obtain an exact value. Therefore, we may not realize whether c or d is irrational. The value of π is almost equal to 22/7 or 3.142857…

4. Represent ($\sqrt{9.3}$) on the number line.

Solution:

Step 1: Draw a 9.3 units long line segment, AB. Extend AB to C such that BC=1 unit.

Step 2: Now, AC = 10.3 units. Let the centre of AC be O.

Step 3: Draw a semi-circle of radius OC with centre O.

Step 4: Draw a BD perpendicular to AC at point B intersecting the semicircle at D. Join OD.

Step 5: OBD, obtained, is a right angled triangle.

Here, OD 10.3/2 (radius of semi-circle), OC = 10.3/2 , BC = 1

OB = OC – BC

$\Rightarrow (10.3/2)-1 = 8.3/2$

Using Pythagoras theorem,

We get,

$OD^2= BD^2+OB^2$

$\Rightarrow (10.3/2)^2 = BD^2+(8.3/2)^2$

$\Rightarrow BD^2 = (10.3/2)^2-(8.3/2)^2$

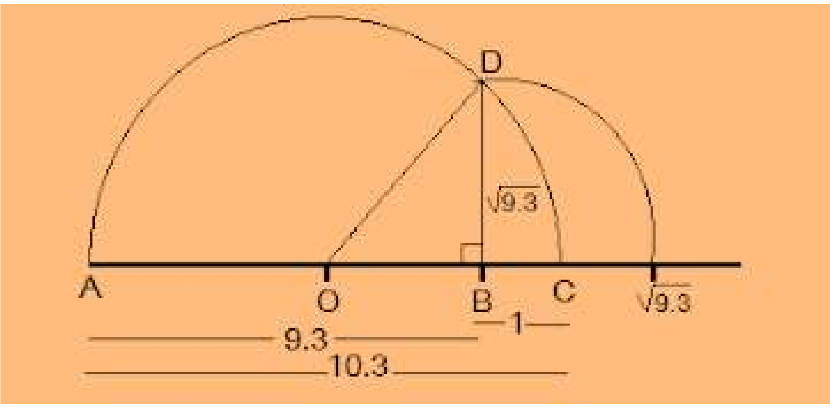
$\Rightarrow (BD)^2 = (10.3/2)-(8.3/2)(10.3/2)+(8.3/2)$

$\Rightarrow BD^2 = 9.3$

$\Rightarrow BD = \sqrt{9.3}$

Thus, the length of BD is $\sqrt{9.3}$ units.

Step 6: Taking BD as radius and B as centre draw an arc which touches the line segment. The point where it touches the line segment is at a distance of $\sqrt{9.3}$ from O as shown in the figure.



5. Rationalize the denominators of the following:

(i) 1/√7

Solution:

Multiply and divide 1/√7 by √7

(1×√7)/(√7×√7) = √7/7

(ii) 1/(√7-√6)

Solution:

Multiply and divide 1/(√7-√6) by (√7+√6)

[1/(√7-√6)]×(√7+√6)/(√7+√6) = (√7+√6)/(√7-√6)(√7+√6)

= (√7+√6)/$\sqrt{7}^2$-$\sqrt{6}^2$ [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√7+√6)/(7-6)

= (√7+√6)/1

= √7+√6

(iii) 1/(√5+√2)

Solution:

Multiply and divide 1/(√5+√2) by (√5-√2)

[1/(√5+√2)]×(√5-√2)/(√5-√2) = (√5-√2)/(√5+√2)(√5-√2)

= (√5-√2)/($\sqrt{5}^2$-$\sqrt{2}^2$) [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√5-√2)/(5-2)

= (√5-√2)/3

(iv) 1/(√7-2)

Solution:

Multiply and divide 1/(√7-2) by (√7+2)

1/(√7-2)×(√7+2)/(√7+2) = (√7+2)/(√7-2)(√7+2)

= (√7+2)/($\sqrt{7}^2$-2^2) [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√7+2)/(7-4)

= (√7+2)/3