Exercise 1.1 Page: 5

1. Is zero a rational number? Can you write it in the form p/q where p and q are integers and q ≠ 0?

Solution:

We know that, a number is said to be rational if it can be written in the form p/q , where p and q are integers and q ≠ 0.

Taking the case of '0',

Zero can be written in the form 0/1, 0/2, 0/3 … as well as , 0/1, 0/2, 0/3 ..

Since it satisfies the necessary condition, we can conclude that 0 can be written in the p/q form, where q can either be positive or negative number.

Hence, 0 is a rational number.

2. Find six rational numbers between 3 and 4.

Solution:

There are infinite rational numbers between 3 and 4.

As we have to find 6 rational numbers between 3 and 4, we will multiply both the numbers, 3 and 4, with 6+1 = 7 (or any number greater than 6)

i.e., $3\times(7/7) = 21/7$

and, $4\times(7/7) = 27/7$. ∴The numbers in between 21/7 and 28/7 will be rational and will fall between 3 and 4.

Hence, 22/7, 23/7, 24/7, 25/7, 26/7, 27/7 are the 6 rational numbers between 3 and 4.

3 Find five rational numbers between 3/5 and 4/5.

Solution:

There are infinite rational numbers between 3/5 and 4/5.

To find out 5 rational numbers between 3/5 and 4/5, we will multiply both the numbers 3/5 and 4/5 with 5+1=6 (or any number greater than 5)

i.e., $(3/5)\times(6/6) = 18/30$

and, $(4/5)\times(6/6) = 24/30$

∴The numbers in between18/30 and 24/30 will be rational and will fall between 3/5 and 4/5.

Hence,19/30, 20/30, 21/30, 22/30, 23/30 are the 5 rational numbers between 3/5 and 4/5.

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

Solution:

True

Natural numbers- Numbers starting from 1 to infinity (without fractions or decimals)

i.e., Natural numbers= 1,2,3,4…

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3…

Or, we can say that whole numbers have all the elements of natural numbers and zero.

∴ Every natural number is a whole number, however, every whole number is not a natural number.

(ii) Every integer is a whole number.

Solution:

False

Integers- Integers are set of numbers that contain positive, negative and 0; excluding fractional and decimal numbers.

i.e., integers= {…-4,-3,-2,-1,0,1,2,3,4…}

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers include whole numbers as well as negative numbers.

∴ Every whole number is an integer, however, every integer is not a whole number.

(iii) Every rational number is a whole number.

Solution:

False

Rational numbers- All numbers in the form p/q, where p and q are integers and q≠0.

i.e., Rational numbers = 0, 19/30, 2, 9/-3, -12/7…

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers include whole numbers as well as negative numbers.

∴ Every whole numbers are rational, however, every rational numbers are not whole numbers.

Exercise 1.2 Page: 8

1.State whether the following statements are true or false. Justify your answers.
(i) Every irrational number is a real number.

Solution:
True
Irrational Numbers - A number is said to be irrational, if it **cannot** be written in the p/q, where p and q are integers and q ≠ 0.
i.e., Irrational numbers = 0, 19/30, 2, 9/-3, -12/7, √2, √5, π, 0.102….

Real numbers - The collection of both rational and irrational numbers are known as real numbers.
i.e., Real numbers = √2, √5, π, 0.102…
∴ Every irrational number is a real number, however, every real numbers are not irrational numbers.

(ii) Every point on the number line is of the form √m where m is a natural number.

Solution:
False
The statement is false since as per the rule, a negative number cannot be expressed as square roots.
E.g., √9 =3 is a natural number.
But √2 = 1.414 is not a natural number.
Similarly, we know that there are negative numbers on the number line but when we take the root of a negative number it becomes a complex number and not a natural number.
E.g., √-7 = 7i, where i = √-1
∴ The statement that every point on the number line is of the form √m, where m is a natural number is false.

(iii) Every real number is an irrational number.

Solution:
False
The statement is false, the real numbers include both irrational and rational numbers. Therefore, every real number cannot be an irrational number.
Real numbers - The collection of both rational and irrational numbers are known as real numbers.
i.e., Real numbers = √2, √5, π, 0.102…
Irrational Numbers - A number is said to be irrational, if it **cannot** be written in the p/q, where p and q are integers and q ≠ 0.
i.e., Irrational numbers = 0, 19/30, 2, 9/-3, -12/7, √2, √5, π, 0.102….
∴ Every irrational number is a real number, however, every real number is not irrational.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution:
No, the square roots of all positive integers are not irrational.
For example,
√4 = 2 is rational.
√9 = 3 is rational.
Hence, the square roots of positive integers 4 and 9 are not irrational. (2 and 3, respectively).

3. Show how √5 can be represented on the number line.

Solution:

Step 1: Let line AB be of 2 unit on a number line.

Step 2: At B, draw a perpendicular line BC of length 1 unit.

Step 3: Join CA

Step 4: Now, ABC is a right-angled triangle. Applying Pythagoras theorem,

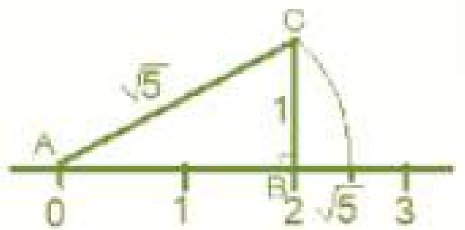
$AB^2+BC^2 = CA^2$

$2^2+1^2 = CA^2 \Rightarrow CA^2 = 5$

$\Rightarrow CA = \sqrt{5}$. Thus, CA is a line of length √5 unit.

Step 5: Taking CA as a radius and A as a center draw an arc touching the number line. The point at which number line get intersected by arc is at √5 distance from 0 because it is a radius of the circle whose center was A.

Thus, √5 is represented on the number line as shown in the figure.



4. Classroom activity (Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP1 of unit length. Draw a line segment P1P2 perpendicular to OP_1 of unit length (see Fig. 1.9). Now draw a line segment P_2P_3 perpendicular to OP_2. Then draw a line segment P_3P_4 perpendicular to OP_3. Continuing in Fig. 1.9 :

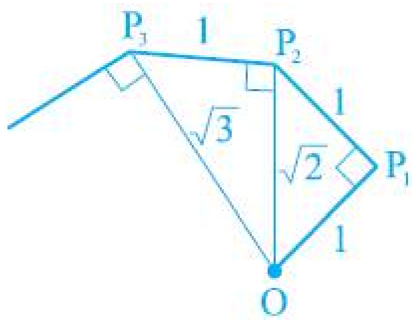
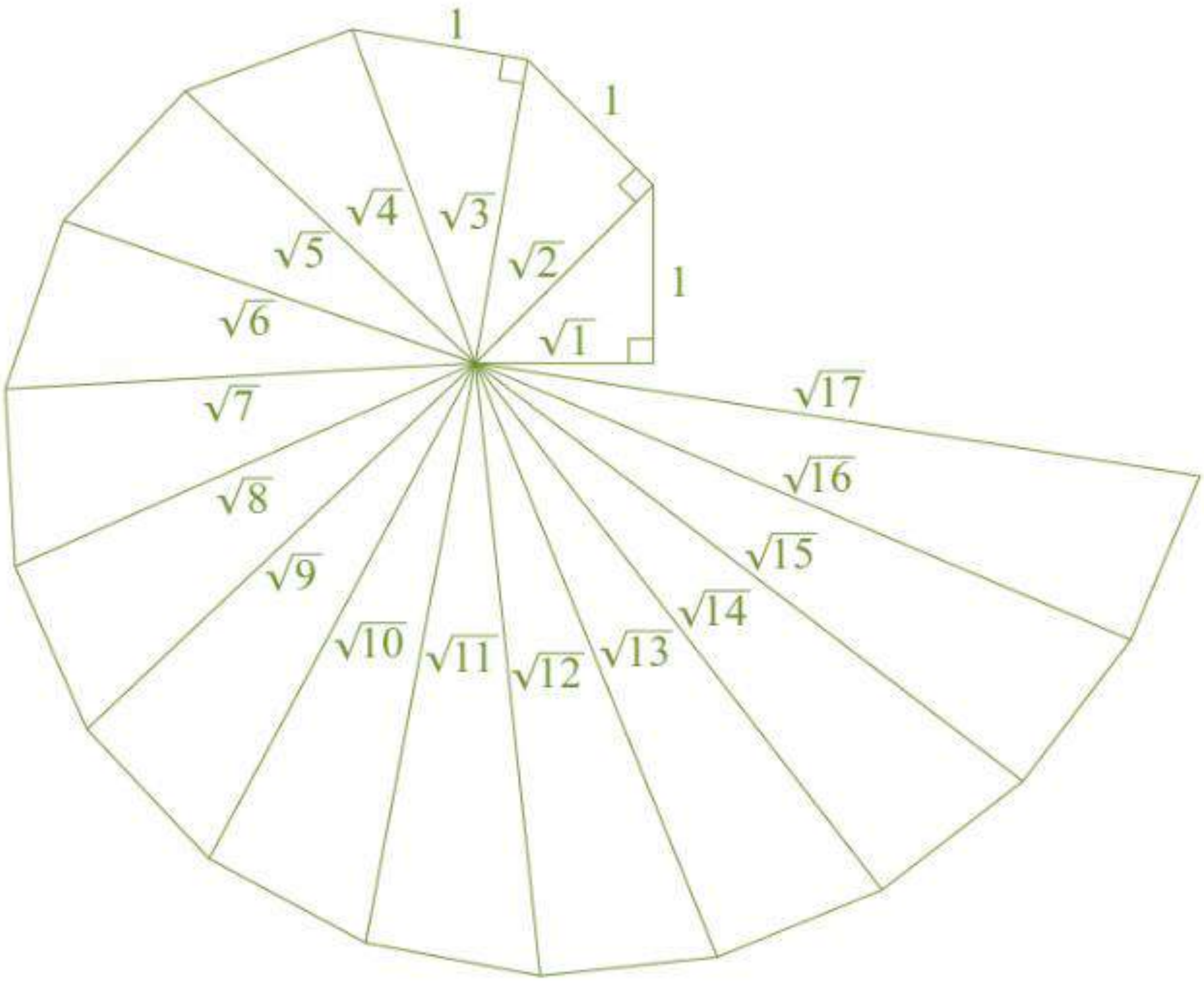


Fig. 1.9 : Constructing square root spiral

Constructing this manner, you can get the line segment P_{n-1}Pn by square root spiral drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points P_2, P_3,....,Pn,... ., and joined them to create a beautiful spiral depicting √2, √3, √4, ...

Solution:



Step 1: Mark a point O on the paper. Here, O will be the center of the square root spiral.
Step 2: From O, draw a straight line, OA, of 1cm horizontally.
Step 3: From A, draw a perpendicular line, AB, of 1 cm.
Step 4: Join OB. Here, OB will be of √2
Step 5: Now, from B, draw a perpendicular line of 1 cm and mark the end point C.
Step 6: Join OC. Here, OC will be of √3
Step 7: Repeat the steps to draw √4, √5, √6….

Exercise 1.3 Page: 14

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) 36/100

Solution:

```
        00.36
100 | 360-
    | 300
    |   600-
    |   600
    |    0
```

= 0.36 (Terminating)

(ii) 1/11

Solution:

```
     0.0909…
11 | 1
   | 0
   |  10
   |   0
   |  100
   |   99
   |    10
   |     0
   |    100
   |     99
   |       1
```

= 0.0909… = $0.\overline{\overline{09}}$ (Non terminating and repeating)

(iii) $4\frac{1}{8}$

Solution:

$$4\frac{1}{8} = \frac{33}{8}$$

4.125
8 | 33
32
10
8
20
16
40
40
0

= 4.125 (Terminating)

(iii) 3/13

Solution:

0.230769
13 | 30
26
40
39
10
0
100
91
90
78
120
117
3

$= 0.230769\ldots = 0.\overline{230769}$

(iv)2/11

Solution:

0.18
11 | 2
0
20
11
90
88
2

$= 0.181818181818\ldots = 0.\overline{18}$ (Non terminating and repeating)

(iv) 329/400

Solution:

$$
\begin{array}{r|l}
 & 0.8225 \\
400 & 329 \\
 & \underline{0} \\
 & 3290 \\
 & \underline{3200} \\
 & 900 \\
 & \underline{800} \\
 & 1000 \\
 & \underline{800} \\
 & 2000 \\
 & \underline{2000} \\
 & 0
\end{array}
$$

= 0.8225 (Terminating)

2. You know that 1/7 = $0.\overline{142857}$. Can you predict what the decimal expansions of 2/7, 3/7, 4/7, 5/7, 6/7 are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of 1/7 carefully.]

Solution:

$1/7 = 0.\overline{142857}$

$\therefore 2\times1/7 = 2\times0.\overline{142857} = 0.\overline{285714}$

$3\times1/7 = 3\times0.\overline{142857} = 0.\overline{428571}$

$4\times1/7 = 4\times0.\overline{142857} = 0.\overline{571428}$

$5\times1/7 = 5\times0.\overline{142857} = 0.\overline{714285}$

$6\times1/7 = 6\times0.\overline{142857} = 0.\overline{857142}$

3. Express the following in the form p/q, where p and q are integers and q ≠ 0.

(i) $0.\overline{6}$

Solution:

$0.\overline{6} = 0.666\ldots$

Assume that $x = 0.666\ldots$

Then, $10x = 6.666\ldots$

$10x = 6 + x$

$9x = 6$

$x = 2/3$

(ii) $0.4\overline{7}$

Solution:

$0.4\overline{7}$ $= 0.4777\ldots$

$0.4\overline{7}$

= (4/10) + (0.777…./10)
Assume that x = 0.777…
Then, 10x = 7.777…
10x = 7 + x
x = 7/9
(4/10)+(0.777../10) = (4/10)+(7/90) (∴ x = 7/9 and x = 0.777…⇒0.777…/10 = 7/(9×10) = 7/90)
= (36/90)+(7/90) = 43/90

(iii) $0.\overline{001}$
Solution:

$0.\overline{001} = 0.001001\ldots$
Assume that x = 0.001001…
Then, 1000x = 1.001001…
1000x = 1 + x
999x = 1
x = 1/999

4. Express 0.99999.... in the form p/q . Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solution:
Assume that x = 0.9999….. Eq (a)
Multiplying both sides by 10,
10x = 9.9999…. Eq. (b)
Eq.(b) – Eq.(a), we get
10x = 9.9999…
- x = - 0.9999…
9x = 9
x = 1

The difference between 1 and 0.999999 is 0.000001 which is negligible.
Hence, we can conclude that, 0.999 is too much near 1, therefore, 1 as the answer can be justified.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17? Perform the division to check your answer.

Solution:
1/17

Dividing 1 by 17:

```
        0.0588235294117647
17 |    1
        0
       10
        0
      100
       85
      150
      136
      140
      136
       40
       34
       60
       51
       90
       85
       50
       34
      160
      153
       70
       68
       20
       17
       30
       17
      130
      119
      110
      102
       80
       68
      120
      119
        1
```

$\frac{1}{17} = 0.\overline{0588235294117647}$

∴ There are 16 digits in the repeating block of the decimal expansion of 1/17.

6. Look at several examples of rational numbers in the form p/q (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Solution:

We observe that when q is 2, 4, 5, 8, 10… Then the decimal expansion is terminating. For example:

1/2 = 0. 5, denominator q = 2^1

7/8 = 0. 875, denominator q =2^3

4/5 = 0. 8, denominator q = 5^1

We can observe that the terminating decimal may be obtained in the situation where prime factorization of the denominator of the given fractions has the power of only 2 or only 5 or both.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Solution:

We know that all irrational numbers are non-terminating non-recurring. ∴ three numbers with decimal expansions that are non-terminating non-recurring are:

a) √3 = 1.732050807568

b) √26 = 5.099019513592

c) √101 = 10.04987562112

8. Find three different irrational numbers between the rational numbers 5/7 and 9/11.

Solution:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

∴ Three different irrational numbers are:

a) 0.73073007300073000073...

b) 0.75075007300075000075…

c) 0.76076007600076000076...

9. Classify the following numbers as rational or irrational according to their type:

(i)√23

Solution:

√23 = 4.79583152331…

Since the number is non-terminating non-recurring therefore, it is an irrational number.

(ii)√225

Solution:

$\sqrt{225} = 15 = 15/1$

Since the number can be represented in p/q form, it is a rational number.

(i) 0.3796

Solution:

Since the number,0.3796, is terminating, it is a rational number.

(ii) 7.478478

Solution:

The number,7.478478, is non-terminating but recurring, it is a rational number.

(iii) 1.101001000100001…

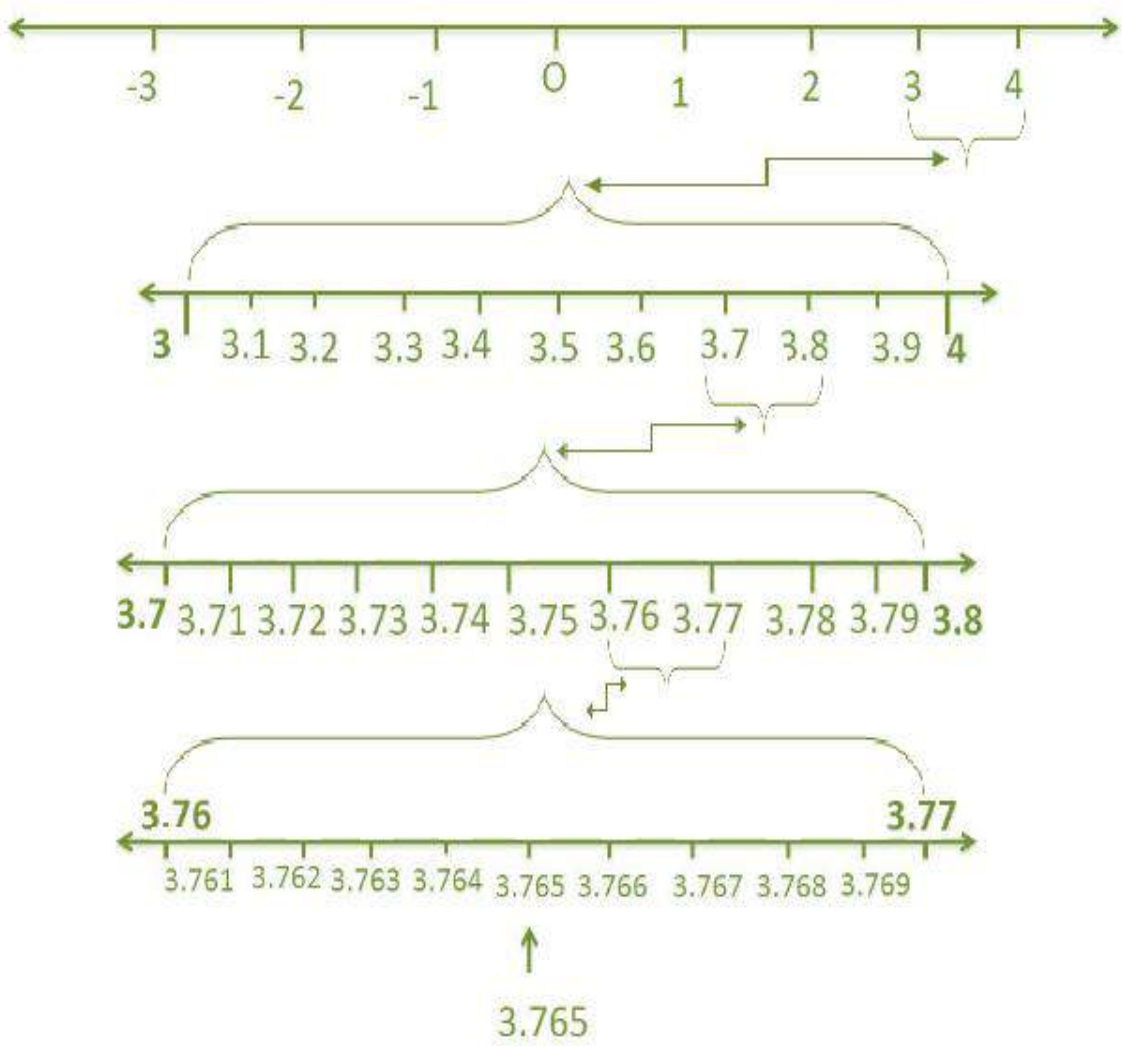
Solution:

Since the number,1.101001000100001…, is non-terminating non-repeating (non-recurring), it is an irrational number.

Exercise 1.4 Page: 18

1. Visualise 3.765 on the number line, using successive magnification.

Solution:

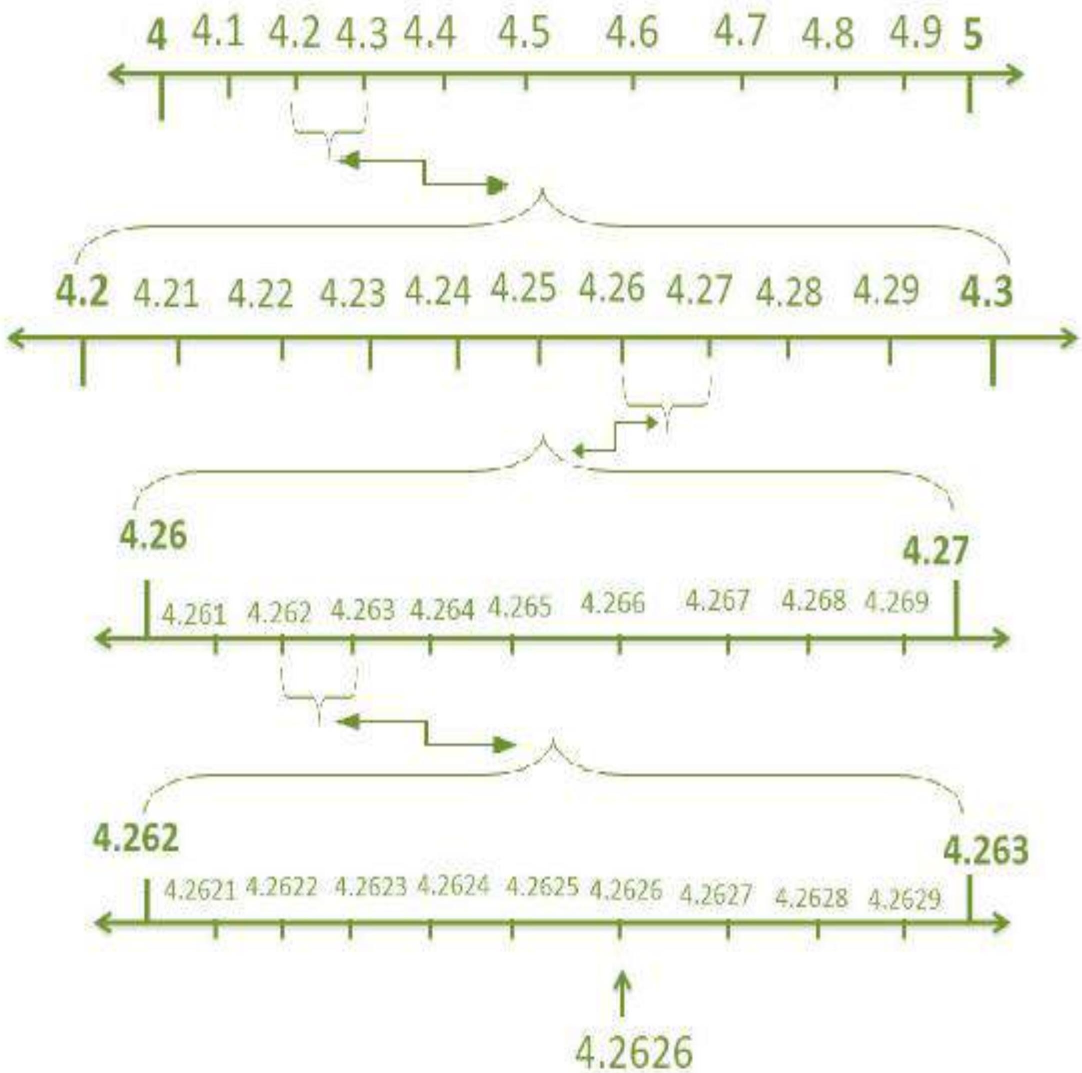


2. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Solution:

$4.\overline{26} = 4.26262626\ldots$

$4.\overline{26}$ up to 4 decimal places= 4.2626



Exercise 1.5 Page: 24

1. Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$

Solution:

We know that, $\sqrt{5} = 2.2360679\ldots$

Here, 2.2360679…is non-terminating and non-recurring.

Now, substituting the value of $\sqrt{5}$ in $2 - \sqrt{5}$, we get,

$2-\sqrt{5} = 2-2.2360679\ldots = -0.2360679$

Since the number, $-0.2360679\ldots$, is non-terminating non-recurring, $2 - \sqrt{5}$ is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

Solution:

$(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23}$

$= 3$

$= 3/1$

Since the number 3/1 is in p/q form, $(3 + \sqrt{23}) - \sqrt{23}$ is rational.

(iii) $2\sqrt{7}/7\sqrt{7}$

Solution:

$2\sqrt{7}/7\sqrt{7} = (2/7) \times (\sqrt{7}/\sqrt{7})$

We know that $(\sqrt{7}/\sqrt{7}) = 1$

Hence, $(2/7) \times (\sqrt{7}/\sqrt{7}) = (2/7) \times 1 = 2/7$

Since the number, 2/7 is in p/q form, $2\sqrt{7}/7\sqrt{7}$ is rational.

(iv) $1/\sqrt{2}$

Solution:

Multiplying and dividing numerator and denominator by $\sqrt{2}$ we get,

$(1/\sqrt{2}) \times (\sqrt{2}/\sqrt{2}) = \sqrt{2}/2$ (since $\sqrt{2} \times \sqrt{2} = 2$)

We know that, $\sqrt{2} = 1.4142\ldots$

Then, $\sqrt{2}/2 = 1.4142/2 = 0.7071..$

Since the number, 0.7071..is non-terminating non-recurring, $1/\sqrt{2}$ is an irrational number.

(v) 2π

Solution:

We know that, the value of $\pi = 3.1415$

Hence, $2\pi = 2 \times 3.1415.. = 6.2830\ldots$

Since the number, 6.2830…, is non-terminating non-recurring, 2π is an irrational number.

2. Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

Solution:

$(3+\sqrt{3})(2+\sqrt{2})$
Opening the brackets, we get, $(3\times2)+(3\times\sqrt{2})+(\sqrt{3}\times2)+(\sqrt{3}\times\sqrt{2})$
$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(2+\sqrt{2})$
Solution:

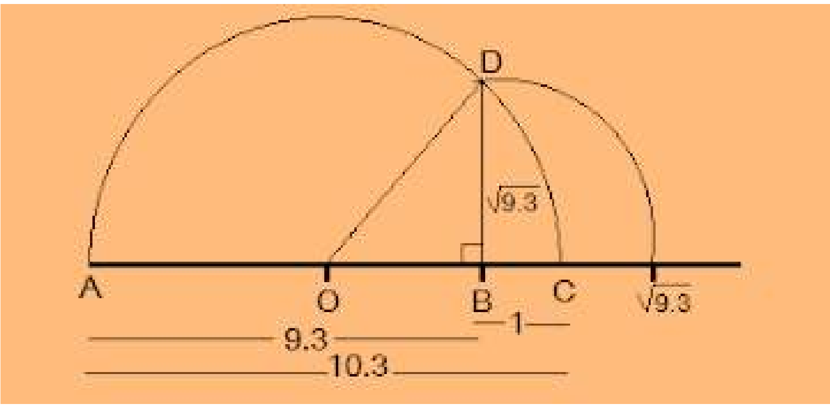
$(3+\sqrt{3})(2+\sqrt{2}) = 3^2-(\sqrt{3})^2 = 9-3$
$= 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$
Solution:
$(\sqrt{5}+\sqrt{2})^2 = \sqrt{5}^2+(2\times\sqrt{5}\times\sqrt{2})+\sqrt{2}^2$
$= 5+2\times\sqrt{10}+2 = 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$
Solution:
$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5}^2-\sqrt{2}^2) = 5-2 = 3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter, (say d). That is, π =c/d. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?
Solution:
There is no contradiction. When we measure a value with a scale, we only obtain an approximate value. We never obtain an exact value. Therefore, we may not realize whether c or d is irrational. The value of π is almost equal to 22/7 or 3.142857…

4. Represent $(\sqrt{9.3})$ on the number line.
Solution:
Step 1: Draw a 9.3 units long line segment, AB. Extend AB to C such that BC=1 unit.
Step 2: Now, AC = 10.3 units. Let the centre of AC be O.
Step 3: Draw a semi-circle of radius OC with centre O.
Step 4: Draw a BD perpendicular to AC at point B intersecting the semicircle at D. Join OD.
Step 5: OBD, obtained, is a right angled triangle.
Here, OD 10.3/2 (radius of semi-circle), OC = 10.3/2 , BC = 1
OB = OC – BC
$\Rightarrow (10.3/2)-1 = 8.3/2$
Using Pythagoras theorem,
We get,
$OD^2= BD^2+OB^2$
$\Rightarrow (10.3/2)^2 = BD^2+(8.3/2)^2$
$\Rightarrow BD^2 = (10.3/2)^2-(8.3/2)^2$
$\Rightarrow (BD)^2 = (10.3/2)-(8.3/2)(10.3/2)+(8.3/2)$
$\Rightarrow BD^2 = 9.3$
$\Rightarrow BD = \sqrt{9.3}$
Thus, the length of BD is $\sqrt{9.3}$ units.
Step 6: Taking BD as radius and B as centre draw an arc which touches the line segment. The point where it touches the line segment is at a distance of $\sqrt{9.3}$ from O as shown in the figure.



5. Rationalize the denominators of the following:

(i) 1/√7

Solution:

Multiply and divide 1/√7 by √7

(1×√7)/(√7×√7) = √7/7

(ii) 1/(√7-√6)

Solution:

Multiply and divide 1/(√7-√6) by (√7+√6)

[1/(√7-√6)]×(√7+√6)/(√7+√6) = (√7+√6)/(√7-√6)(√7+√6)

= (√7+√6)/$\sqrt{7}^2$-$\sqrt{6}^2$ [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√7+√6)/(7-6)

= (√7+√6)/1

= √7+√6

(iii) 1/(√5+√2)

Solution:

Multiply and divide 1/(√5+√2) by (√5-√2)

[1/(√5+√2)]×(√5-√2)/(√5-√2) = (√5-√2)/(√5+√2)(√5-√2)

= (√5-√2)/($\sqrt{5}^2$-$\sqrt{2}^2$) [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√5-√2)/(5-2)

= (√5-√2)/3

(iv) 1/(√7-2)

Solution:

Multiply and divide 1/(√7-2) by (√7+2)

1/(√7-2)×(√7+2)/(√7+2) = (√7+2)/(√7-2)(√7+2)

= (√7+2)/($\sqrt{7}^2$-2^2) [denominator is obtained by the property, (a+b)(a-b) = a^2-b^2]

= (√7+2)/(7-4)

= (√7+2)/3

Exercise 1.6 Page: 26

1. Find:

(i)$64^{1/2}$

Solution:

$64^{1/2} = (8\times8)^{1/2}$

$= (8^2)^{½}$

$= 8^1$ $(2\times1/2 = 2/2 =1)$

$= 8$

(ii)$32^{1/5}$

Solution:

$32^{1/5} = (2^5)^{1/5}$

$= (2^5)^{⅕}$

$= 2^1$ $[5\times1/5 = 1]$

$= 2$

(iii)$125^{1/3}$

Solution:

$(125)^{1/3} = (5\times5\times5)^{1/3}$

$= (5^3)^{⅓}$

$= 5^1$ $(3\times1/3 = 3/3 = 1)$

$= 5$

2. Find:

(i)$9^{3/2}$

Solution:

$9^{3/2} = (3\times3)^{3/2}$

$= (3^2)^{3/2}$

$= 3^3$ $[2\times3/2 = 3]$

$=27$

(ii)$32^{2/5}$

Solution:

$32^{2/5} = (2\times2\times2\times2\times2)^{2/5}$

$= (2^5)^{2/5}$

$= 2^2$ $[5\times2/5= 2]$

$= 4$

(iii)$16^{3/4}$

Solution:

$16^{3/4} = (2\times2\times2\times2)^{3/4}$

$= (2^4)^{3/4}$

$= 2^3$ $[4\times3/4 = 3]$

= 8

(iv) $125^{-1/3}$

$125^{-1/3} = (5\times5\times5)^{-1/3}$

$= (5^3)^{-1/3}$

$= 5^{-1}$ [3×-1/3 = -1]

$= 1/5$

3. Simplify:

(i) $2^{2/3}\times2^{1/5}$

Solution:

$2^{2/3}\times2^{1/5} = 2^{(2/3)+(1/5)}$ [Since, $a^m\times a^n=a^{m+n}$____ Laws of exponents]

$= 2^{13/15}$ [2/3 + 1/5 = (2×5+3×1)/(3×5) = 13/15]

(ii) $(1/3^3)^7$

Solution:

$(1/3^3)^7 = (3^{-3})^7$ [Since,$(a^m)^n = a^{m \times n}$____ Laws of exponents]

$= 3^{-27}$

(iii) $11^{1/2}/11^{1/4}$

Solution:

$11^{1/2}/11^{1/4} = 11^{(1/2)-(1/4)}$

$= 11^{1/4}$ [(1/2) - (1/4) = (1×4-2×1)/(2×4) = 4-2)/8 = 2/8 = ¼]

(iv) $7^{1/2}\times8^{1/2}$

Solution:

$7^{1/2}\times8^{1/2} = (7\times8)^{1/2}$ [Since, $(a^m\times b^m = (a\times b)^m$ ____ Laws of exponents

$= 56^{1/2}$