

Exercise: 10.1

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1. Fill in the blanks:

- (i) The centre of a circle lies in _____ of the circle. (exterior/ interior)
- (ii) A point, whose distance from the centre of a circle is greater than its radius lies in _____ of the circle. (exterior/ interior)
- (iii) The longest chord of a circle is a _____ of the circle.
- (iv) An arc is a _____ when its ends are the ends of a diameter.
- (v) Segment of a circle is the region between an arc and _____ of the circle.
- (vi) A circle divides the plane, on which it lies, in _____ parts.

Solution:

- (i) The centre of a circle lies in **interior** of the circle.
- (ii) A point, whose distance from the centre of a circle is greater than its radius lies in **exterior** of the circle.
- (iii) The longest chord of a circle is a **diameter** of the circle.
- (iv) An arc is a **semicircle** when its ends are the ends of a diameter.
- (v) Segment of a circle is the region between an arc and **chord** of the circle.
- (vi) A circle divides the plane, on which it lies, in **3 (three)** parts.

2. Write True or False: Give reasons for your Solutions.

- (i) Line segment joining the centre to any point on the circle is a radius of the circle.
- (ii) A circle has only finite number of equal chords.
- (iii) If a circle is divided into three equal arcs, each is a major arc.
- (iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.
- (v) Sector is the region between the chord and its corresponding arc.
- (vi) A circle is a plane figure.

Solution:

- (i) **True.** Any line segment drawn from the centre of the circle to any point on it is the radius of the circle and will be of equal length.
- (ii) **False.** There can be infinite numbers of equal chords of a circle.
- (iii) **False.** For unequal arcs, there can be major and minor arcs. So, equal arcs on a circle cannot be said as a major arc or a minor arc.
- (iv) **True.** Any chord whose length is twice as long as the radius of the circle always passes through the centre of the circle and thus, it is known as the diameter of the circle.
- (v) **False.** A sector is a region of a circle between the arc and the two radii of the circle.
- (vi) **True.** A circle is a 2d figure and it can be drawn on a plane.

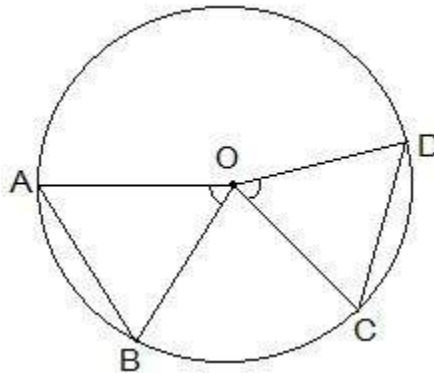
Exercise: 10.2

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1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.

Solution:

To recall, a circle is a collection of points whose every point is equidistant from its centre. So, two circles can be congruent only when the distance of every point of both the circles are equal from the centre.



For the second part of the question, it is given that $AB = CD$ i.e. two equal chords. Now, it is to be proven that angle AOB is equal to angle COD.

Proof:

Consider the triangles $\triangle AOB$ and $\triangle COD$,

$OA = OC$ and $OB = OD$ (Since they are the radii of the circle)

$AB = CD$ (As given in the question)

So, by SSS congruency, $\triangle AOB \cong \triangle COD$

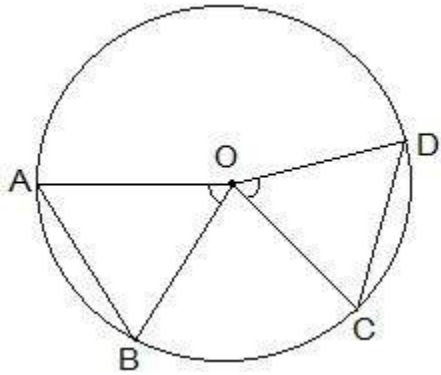
\therefore By CPCT we have,

$\angle AOB = \angle COD$. (Hence proved).

2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.

Solution:

Consider the following diagram-



Here, it is given that $\angle AOB = \angle COD$ i.e. they are equal angles.

Now, we will have to prove that the line segments AB and CD are equal i.e. $AB = CD$.

Proof:

In triangles AOB and COD,

$\angle AOB = \angle COD$ (as given in the question)

$OA = OC$ and $OB = OD$ (these are the radii of the circle)

So, by SAS congruency, $\triangle AOB \cong \triangle COD$.

\therefore By the rule of CPCT, we have

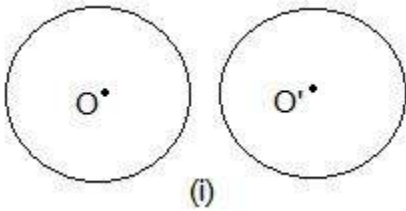
$AB = CD$. (Hence proved).

Exercise: 10.3

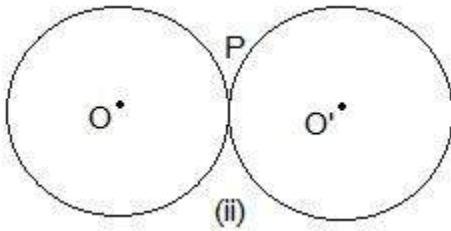
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1. Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

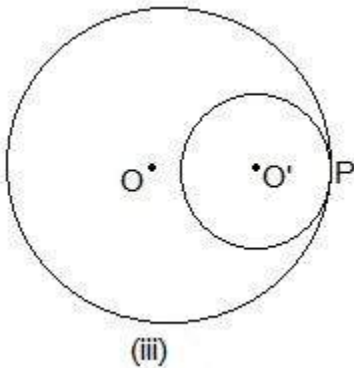
Solution:



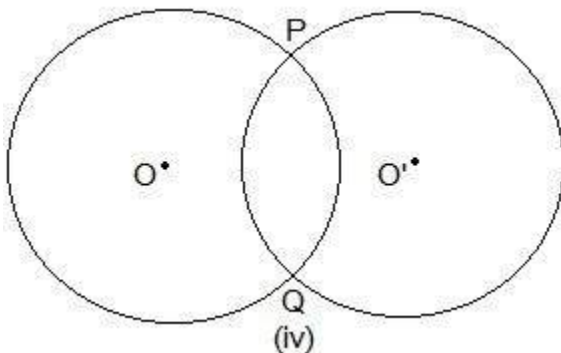
In these two circles, no point is common.



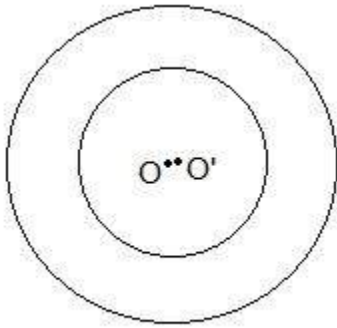
Here, only one point "P" is common.



Even here, P is the common point.



Here, two points are common which are P and Q.

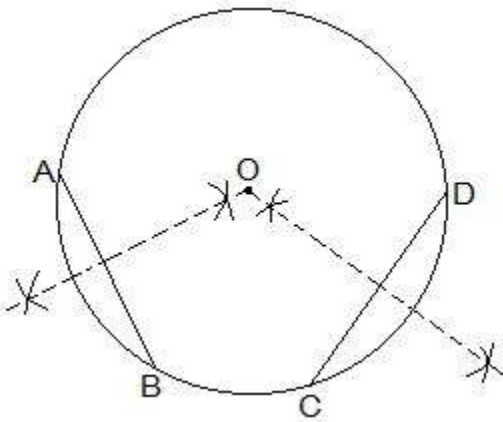


(v)

No point is common in the above circle.

2. Suppose you are given a circle. Give a construction to find its centre.

Solution:



The construction steps to find the center of the circle are:

Step I: Draw a circle first.

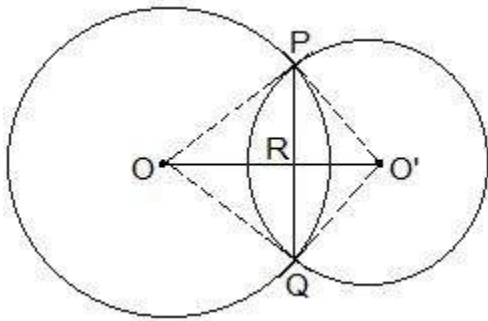
Step II: Draw 2 chords AB and CD in the circle.

Step III: Draw the perpendicular bisectors of AB and CD.

Step IV: Connect the two perpendicular bisectors at a point. This intersection point of the two perpendicular bisectors is the centre of the circle.

3. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.

Solution:



It is given that two circles intersect each other at P and Q.

To prove:

OO' is perpendicular bisector of PQ.

Proof:

Triangle $\Delta POO'$ and $\Delta QOO'$ are similar by SSS congruency since

$OP = OQ$ and $O'P = O'Q$ (Since they are also the radii)

$OO' = OO'$ (It is the common side)

So, It can be said that $\Delta POO' \cong \Delta QOO'$

$\therefore \angle POO' = \angle QOO'$ --- (i)

Even triangles ΔPOR and ΔQOR are similar by SAS congruency as

$OP = OQ$ (Radii)

$\angle POR = \angle QOR$ (As $\angle POO' = \angle QOO'$)

$OR = OR$ (Common arm)

So, $\Delta POR \cong \Delta QOR$

$\therefore \angle PRO = \angle QRO$

Also, we know that

$\angle PRO + \angle QRO = 180^\circ$

Hence, $\angle PRO = \angle QRO = 180^\circ / 2 = 90^\circ$

So, OO' is the perpendicular bisector of PQ.

Exercise: 10.4

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1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Solution:

Given parameters are:

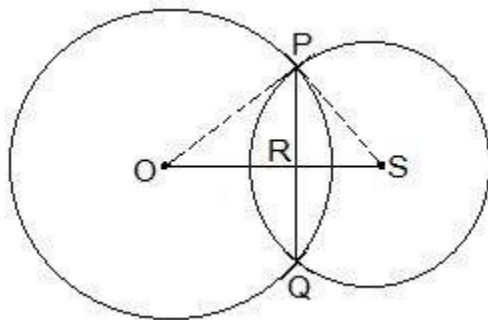
$$OP = 5\text{cm}$$

$$OS = 4\text{cm and}$$

$$PS = 3\text{cm}$$

$$\text{Also, } PQ = 2PR$$

Now, suppose $RS = x$. The diagram for the same is shown below.



Consider the ΔPOR ,

$$OP^2 = OR^2 + PR^2$$

$$\Rightarrow 5^2 = (4-x)^2 + PR^2$$

$$\Rightarrow 25 = 16 + x^2 - 8x + PR^2$$

$$\therefore PR^2 = 9 - x^2 + 8x \text{ --- (i)}$$

Now consider ΔPRS ,

$$PS^2 = PR^2 + RS^2$$

$$\Rightarrow 3^2 = PR^2 + x^2$$

$$\therefore PR^2 = 9 - x^2 \text{ --- (ii)}$$

By equating equation (i) and equation (ii) we get,

$$9 - x^2 + 8x = 9 - x^2$$

$$\Rightarrow 8x = 0$$

$$\Rightarrow x = 0$$

Now, put the value of x in equation (i)

$$PR^2 = 9 - 0^2$$

$$\Rightarrow PR = 3\text{cm}$$

∴ The length of the cord i.e. $PQ = 2PR$

So, $PQ = 2 \times 3 = 6\text{cm}$

2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Solution:

Let AB and CD be two equal cords (i.e. $AB = CD$). In the above question, it is given that AB and CD intersect at a point, say, E .

It is now to be proven that the line segments $AE = DE$ and $CE = BE$

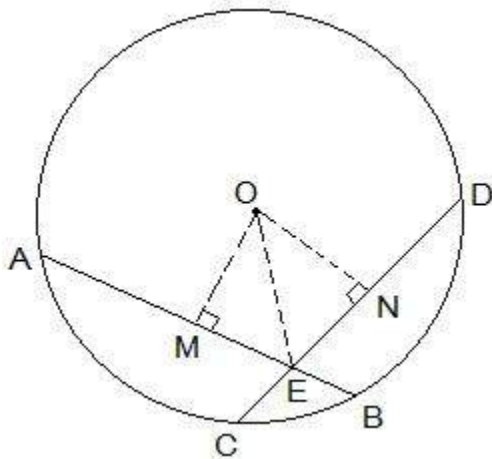
Construction Steps:

Step 1: From the center of the circle, draw a perpendicular to AB i.e. $OM \perp AB$

Step 2: Similarly, draw $ON \perp CD$.

Step 3: Join OE .

Now, the diagram is as follows-



Proof:

From the diagram, it is seen that OM bisects AB and so, $OM \perp AB$

Similarly, ON bisects CD and so, $ON \perp CD$

It is known that $AB = CD$. So,

$AM = ND$ --- (i)

and $MB = CN$ --- (ii)

Now, triangles $\triangle OME$ and $\triangle ONE$ are similar by RHS congruency since

$\angle OME = \angle ONE$ (They are perpendiculars)

$OE = OE$ (It is the common side)

$OM = ON$ (AB and CD are equal and so, they are equidistant from the centre)

∴ $\triangle OME \cong \triangle ONE$

$ME = EN$ (by CPCT) --- (iii)

Now, from equations (i) and (ii) we get,

$$AM + ME = ND + EN$$

$$\text{So, } AE = ED$$

Now from equations (ii) and (iii) we get,

$$MB - ME = CN - EN$$

$$\text{So, } EB = CE \text{ (Hence proved).}$$

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution:

From the question we know the following:

(i) AB and CD are 2 chords which are intersecting at point E.

(ii) PQ is the diameter of the circle.

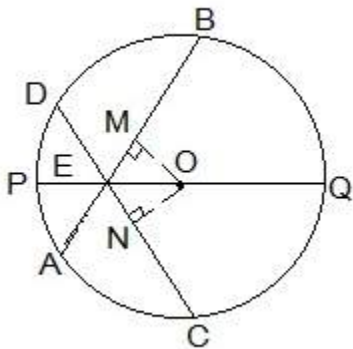
(iii) AB = CD.

Now, we will have to prove that $\angle BEQ = \angle CEQ$

For this, the following construction has to be done:

Construction:

Draw two perpendiculars are drawn as $OM \perp AB$ and $ON \perp D$. Now, join OE. The constructed diagram will look as follows:



Now, consider the triangles $\triangle OEM$ and $\triangle OEN$.

Here,

(i) $OM = ON$ [Since the equal chords are always equidistant from the centre]

(ii) $OE = OE$ [It is the common side]

(iii) $\angle OME = \angle ONE$ [These are the perpendiculars]

So, by RHS congruency criterion, $\triangle OEM \cong \triangle OEN$.

Hence, by CPCT rule, $\angle MEO = \angle NEO$

$\therefore \angle BEQ = \angle CEQ$ (Hence proved).

4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$ (see Fig. 10.25).

Solution:

The given image is as follows:

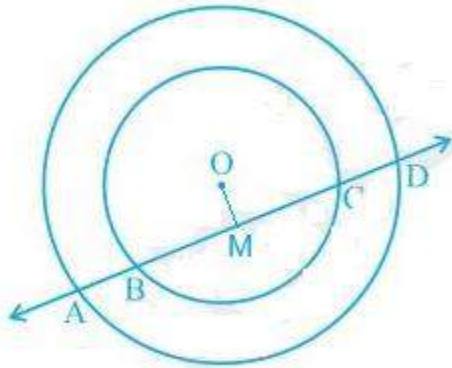


Fig. 10.25

First, draw a line segment from O to AD such that $OM \perp AD$.

So, now OM is bisecting AD since $OM \perp AD$.

Therefore, $AM = MD$ --- (i)

Also, since $OM \perp BC$, OM bisects BC.

Therefore, $BM = MC$ --- (ii)

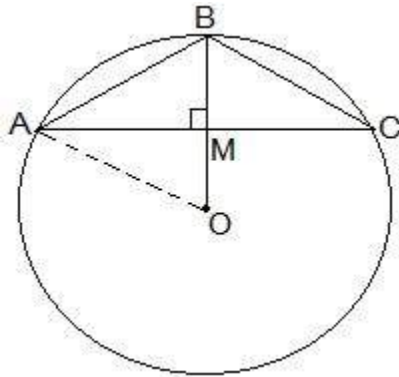
From equation (i) and equation (ii),

$$AM - BM = MD - MC$$

$$\therefore AB = CD$$

5. Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Solution:



Let the positions of Reshma, Salma and Mandip be represented as A, B and C respectively.

From the question, we know that $AB = BC = 6\text{cm}$.

So, the radius of the circle i.e. $OA = 5\text{cm}$

Now, draw a perpendicular $BM \perp AC$.

Since $AB = BC$, ABC can be considered as an isosceles triangle. M is mid-point of AC . BM is the perpendicular bisector of AC and thus it passes through the centre of the circle.

Now,

let $AM = y$ and

$OM = x$

So, BM will be $= (5-x)$.

By applying Pythagorean theorem in ΔOAM we get,

$$OA^2 = OM^2 + AM^2$$

$$\Rightarrow 5^2 = x^2 + y^2 \text{ --- (i)}$$

Again, by applying Pythagorean theorem in ΔAMB ,

$$AB^2 = BM^2 + AM^2$$

$$\Rightarrow 6^2 = (5-x)^2 + y^2 \text{ --- (ii)}$$

Subtracting equation (i) from equation (ii), we get

$$36 - 25 = (5-x)^2 - x^2 - y^2$$

Now, solving this equation we get the value of x as

$$x = 7/5$$

Substituting the value of x in equation (i), we get

$$y^2 + (49/25) = 25$$

$$\Rightarrow y^2 = 25 - (49/25)$$

Solving it we get the value of y as

$$y = 24/5$$

Thus,

$$AC = 2 \times AM$$

$$= 2 \times y$$

$$= 2 \times (24/5) \text{ m}$$

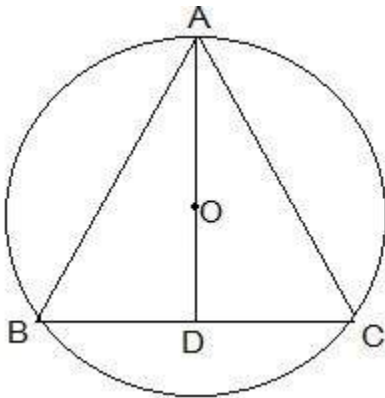
$AC = 9.6$ m

So, the distance between Reshma and Mandip is 9.6 m.

6. A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Solution:

First, draw a diagram according to the given statements. The diagram will look as follows.



Here the positions of Ankur, Syed and David are represented as A, B and C respectively. Since they are sitting at equal distances, the triangle ABC will form an equilateral triangle.

$AD \perp BC$ is drawn. Now, AD is median of ΔABC and it passes through the centre O.

Also, O is the centroid of the ΔABC . OA is the radius of the circle.

$$OA = \frac{2}{3} AD$$

Let the side of a triangle a metres then $BD = \frac{a}{2}$ m.

Applying Pythagoras theorem in ΔABD ,

$$AB^2 = BD^2 + AD^2$$

$$\Rightarrow AD^2 = AB^2 - BD^2$$

$$\Rightarrow AD^2 = a^2 - \left(\frac{a}{2}\right)^2$$

$$\Rightarrow AD^2 = \frac{3a^2}{4}$$

$$\Rightarrow AD = \frac{\sqrt{3}a}{2}$$

$$OA = \frac{2}{3} AD$$

$$\Rightarrow 20 \text{ m} = \frac{2}{3} \times \frac{\sqrt{3}a}{2}$$

$$\Rightarrow a = 20\sqrt{3} \text{ m}$$

So, the length of the string of the toy is $20\sqrt{3}$ m.

Exercise: 10.5

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1. In Fig. 10.36, A, B and C are three points on a circle with centre O such that $\angle BOC = 30^\circ$ and $\angle AOB = 60^\circ$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.

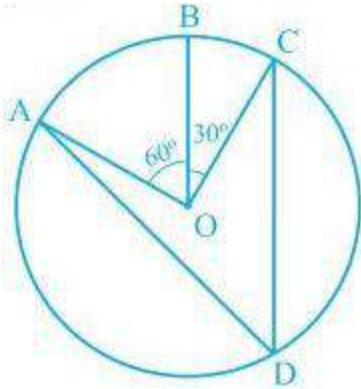


Fig. 10.36

Solution:

It is given that,

$$\angle AOC = \angle AOB + \angle BOC$$

$$\text{So, } \angle AOC = 60^\circ + 30^\circ$$

$$\therefore \angle AOC = 90^\circ$$

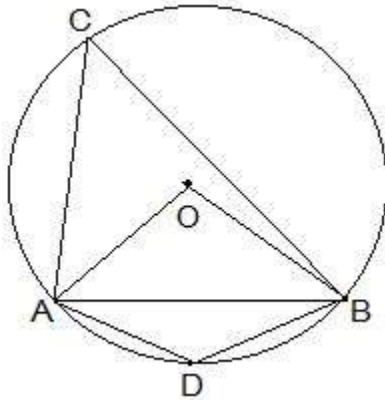
It is known that an angle which is subtended by an arc at the centre of the circle is double the angle subtended by that arc at any point on the remaining part of the circle.

So,

$$\begin{aligned} \angle ADC &= \left(\frac{1}{2}\right)\angle AOC \\ &= \left(\frac{1}{2}\right) \times 90^\circ = 45^\circ \end{aligned}$$

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution:



Here, the chord AB is equal to the radius of the circle. In the above diagram, OA and OB are the two radii of the circle.

Now, consider the ΔOAB . Here,

$AB = OA = OB = \text{radius of the circle}$.

So, it can be said that ΔOAB has all equal sides and thus, it is an equilateral triangle.

$\therefore \angle AOB = 60^\circ$

And, $\angle ACB = \frac{1}{2} \angle AOB$

So, $\angle ACB = \frac{1}{2} \times 60^\circ = 30^\circ$

Now, since ACBD is a cyclic quadrilateral,

$\angle ADB + \angle ACB = 180^\circ$ (Since they are the opposite angles of a cyclic quadrilateral)

So, $\angle ADB = 180^\circ - 30^\circ = 150^\circ$

So, the angle subtended by the chord at a point on the minor arc and also at a point on the major arc are 150° and 30° respectively.

3. In Fig. 10.37, $\angle PQR = 100^\circ$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.

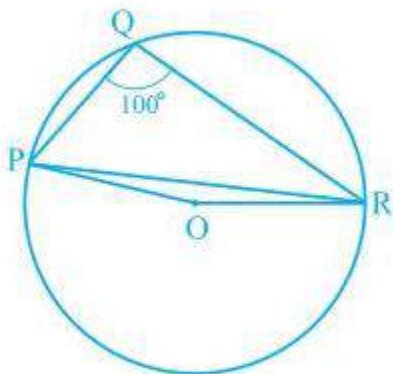


Fig. 10.37

Solution:

Since angle which is subtended by an arc at the centre of the circle is double the angle subtended by that arc at any point on the remaining part of the circle.

So, the reflex $\angle POR = 2 \times \angle PQR$

We know the values of angle PQR as 100°

So, $\angle POR = 2 \times 100^\circ = 200^\circ$

$\therefore \angle POR = 360^\circ - 200^\circ = 160^\circ$

Now, in $\triangle OPR$,

OP and OR are the radii of the circle

So, $OP = OR$

Also, $\angle OPR = \angle ORP$

Now, we know sum of the angles in a triangle is equal to 180 degrees

So,

$\angle POR + \angle OPR + \angle ORP = 180^\circ$

$\Rightarrow \angle OPR + \angle OPR = 180^\circ - 160^\circ$

As $\angle OPR = \angle ORP$

$\Rightarrow 2\angle OPR = 20^\circ$

Thus, $\angle OPR = 10^\circ$

4. In Fig. 10.38, $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$.

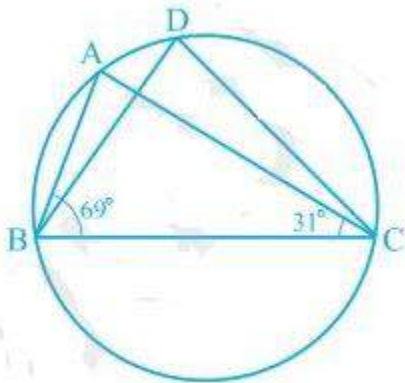


Fig. 10.38

Solution:

We know that angles in the segment of the circle are equal so,

$\angle BAC = \angle BDC$

Now in the in $\triangle ABC$, sum of all the interior angles will be 180°

So, $\angle ABC + \angle BAC + \angle ACB = 180^\circ$

Now, by putting the values,

$$\angle BAC = 180^\circ - 69^\circ - 31^\circ$$

So, $\angle BAC = 80^\circ$

5. In Fig. 10.39, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that $\angle BEC = 130^\circ$ and $\angle ECD = 20^\circ$. Find $\angle BAC$.

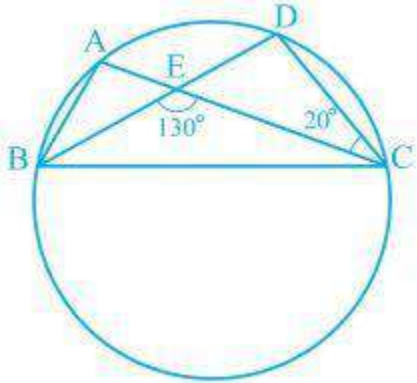


Fig. 10.39

Solution:

We know that the angles in the segment of the circle are equal.

So,

$$\angle BAC = \angle CDE$$

Now, by using the exterior angles property of the triangle In $\triangle CDE$ we get,

$$\angle CEB = \angle CDE + \angle DCE$$

We know that $\angle DCE$ is equal to 20°

$$\text{So, } \angle CDE = 110^\circ$$

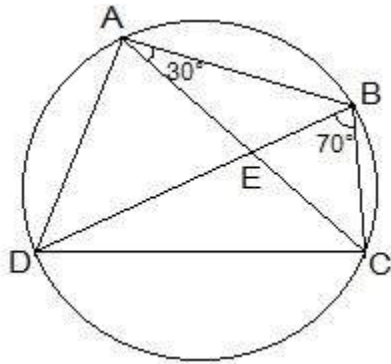
$\angle BAC$ and $\angle CDE$ are equal

$$\therefore \angle BAC = 110^\circ$$

6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC$ is 30° , find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Solution:

Consider the following diagram.



Consider the chord CD,

We know that angles in the same segment are equal.

So, $\angle CBD = \angle CAD$

$\therefore \angle CAD = 70^\circ$

Now, $\angle BAD$ will be equal to the sum of angles BAC and CAD.

So, $\angle BAD = \angle BAC + \angle CAD$
 $= 30^\circ + 70^\circ$

$\therefore \angle BAD = 100^\circ$

We know that the opposite angles of a cyclic quadrilateral sum up to 180 degrees.

So,

$\angle BCD + \angle BAD = 180^\circ$

It is known that $\angle BAD = 100^\circ$

So, $\angle BCD = 80^\circ$

Now consider the $\triangle ABC$.

Here, it is given that $AB = BC$

Also, $\angle BCA = \angle CAB$ (They are the angles opposite to equal sides of a triangle)

$\angle BCA = 30^\circ$

also, $\angle BCD = 80^\circ$

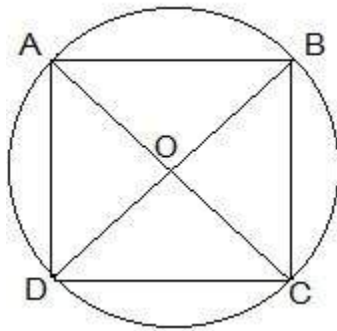
$\angle BCA + \angle ACD = 80^\circ$

Thus, $\angle ACD = 50^\circ$ and $\angle ECD = 50^\circ$

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution:

Draw a cyclic quadrilateral ABCD inside a circle with center O such that its diagonal AC and BD are two diameters of the circle.



We know that the angles in the semi-circle are equal.

So, $\angle ABC = \angle BCD = \angle CDA = \angle DAB = 90^\circ$

So, as each internal angle is 90° , it can be said that the quadrilateral ABCD is a rectangle.

8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution:

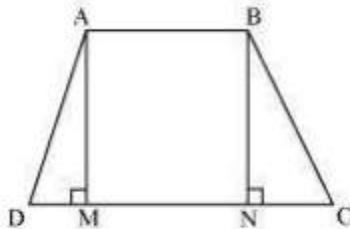
Construction:

Consider a trapezium ABCD with $AB \parallel CD$ and $BC = AD$.

Draw $AM \perp CD$ and $BN \perp CD$.

In $\triangle AMD$ and $\triangle BNC$,

The diagram will look as follows:



In $\triangle AMD$ and $\triangle BNC$,

$AD = BC$ (Given)

$\angle AMD = \angle BNC$ (By construction, each is 90°)

$AM = BN$ (Perpendicular distance between two parallel lines is same)

$\triangle AMD \cong \triangle BNC$ (RHS congruence rule)

$\angle ADC = \angle BCD$ (CPCT) ... (1)

$\angle BAD$ and $\angle ADC$ are on the same side of transversal AD.

$\angle BAD + \angle ADC = 180^\circ$... (2)

$\angle BAD + \angle BCD = 180^\circ$ [Using equation (1)]

This equation shows that the opposite angles are supplementary.

Therefore, ABCD is a cyclic quadrilateral.

9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see Fig. 10.40). Prove that $\angle ACP = \angle QCD$.

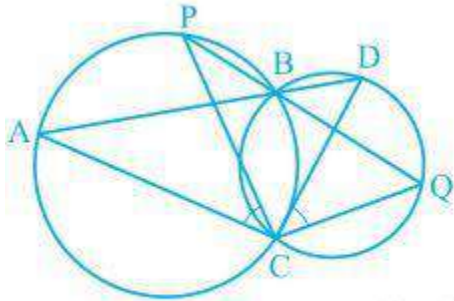


Fig. 10.40

Solution:

Construction:

Join the chords AP and DQ.

For chord AP, we know that angles in the same segment are equal.

So, $\angle PBA = \angle ACP$ --- (i)

Similarly for chord DQ,

$\angle DBQ = \angle QCD$ --- (ii)

It is known that ABD and PBQ are two line segments which are intersecting at B.

At B, the vertically opposite angles will be equal.

$\therefore \angle PBA = \angle DBQ$ --- (iii)

From equation (i), equation (ii) and equation (iii) we get,

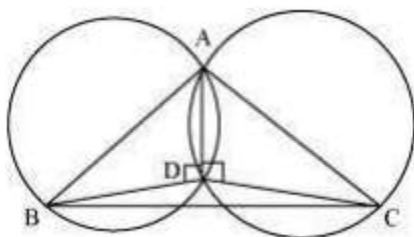
$\angle ACP = \angle QCD$

10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Solution:

First draw a triangle ABC and then two circles having diameter as AB and AC respectively.

We will have to now prove that D lies on BC and BDC is a straight line.



Proof:

We know that angle in the semi-circle are equal

So, $\angle ADB = \angle ADC = 90^\circ$

Hence, $\angle ADB + \angle ADC = 180^\circ$

$\therefore \angle BDC$ is straight line.

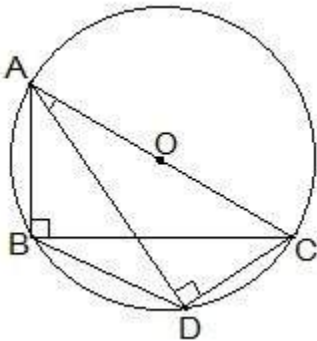
So, it can be said that D lies on the line BC.

11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle CAD = \angle CBD$.

Solution:

We know that AC is the common hypotenuse and $\angle B = \angle D = 90^\circ$.

Now, it has to be proven that $\angle CAD = \angle CBD$



Since, $\angle ABC$ and $\angle ADC$ are 90° , it can be said that They lie in the semi-circle.

So, triangles ABC and ADC are in the semi-circle and the points A, B, C and D are concyclic.

Hence, CD is the chord of the circle with center O.

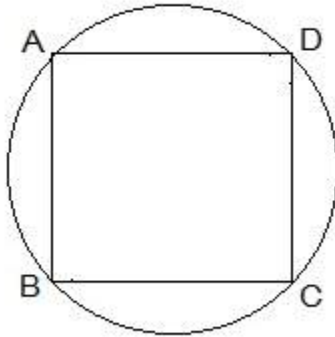
We know that the angles which are in the same segment of the circle are equal.

$\therefore \angle CAD = \angle CBD$

12. Prove that a cyclic parallelogram is a rectangle.

Solution:

It is given that ABCD is a cyclic parallelogram and we will have to prove that ABCD is a rectangle.

**Proof:**

Let ABCD be a cyclic parallelogram.

$$A + C = 180^\circ \text{ (Opposite angles of a cyclic quadrilateral) ... (1)}$$

We know that opposite angles of a parallelogram are equal.

$$A = C \text{ and } B = D$$

From equation (1),

$$A + C = 180^\circ$$

$$A + A = 180^\circ$$

$$2 A = 180^\circ$$

$$A = 90^\circ$$

Parallelogram ABCD has one of its interior angles as 90° .

Thus, ABCD is a rectangle.

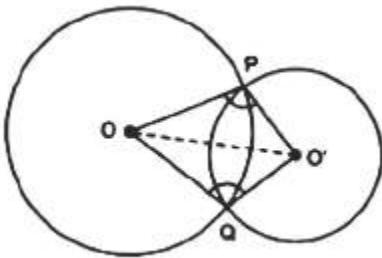
Exercise: 10.6

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1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Solution:

Consider the following diagram



In $\Delta POO'$ and $\Delta QOO'$

$OP = OQ$ (Radius of circle 1)

$O'P = O'Q$ (Radius of circle 2)

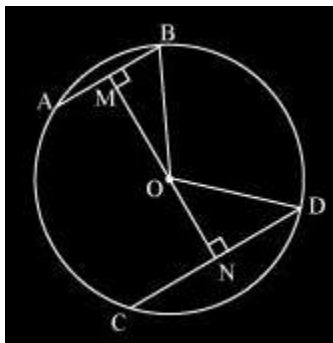
$OO' = OO'$ (Common arm)

So, by SSS congruency, $\Delta POO' \cong \Delta QOO'$

Thus, $\angle OPO' = \angle OQO'$ (proved).

2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 , find the radius of the circle.

Solution:



Here, $OM \perp AB$ and $ON \perp CD$. is drawn and OB and OD are joined.

We know that AB bisects BM as the perpendicular from the centre bisects chord.

Since $AB = 5$ so,

$$BM = AB/2$$

Similarly, $ND = CD/2 = 11/2$

Now, let ON be x.

So, $OM = 6 - x$.

Consider $\triangle MOB$,

$$OB^2 = OM^2 + MB^2$$

Or,

$$OB^2 = 36 + x^2 - 12x + \frac{25}{4} \quad \dots (1)$$

Consider $\triangle NOD$,

$$OD^2 = ON^2 + ND^2$$

Or

$$OD^2 = x^2 + \frac{121}{4} \quad \dots (2)$$

We know, $OB = OD$ (radii)

From equation 1 and equation 2 we get

$$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$12x = 36 + \frac{25}{4} - \frac{121}{4}$$

$$= \frac{144 + 25 - 121}{4}$$

$$12x = \frac{48}{4} = 12$$

$$x = 1$$

Now, from equation (2) we have,

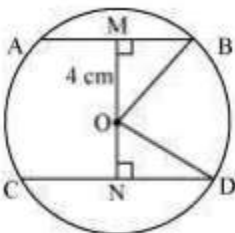
$$OD^2 = 1^2 + (121/4)$$

$$\text{Or } OD = (5/2) \times \sqrt{5} \text{ cm}$$

3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance 4 cm from the centre, what is the distance of the other chord from the centre?

Solution:

Consider the following diagram



Here AB and CD are 2 parallel chords. Now, join OB and OD.

Distance of smaller chord AB from the centre of the circle = 4 cm

So, $OM = 4$ cm

$MB = AB/2 = 3$ cm

Consider $\triangle OMB$

$$OB^2 = OM^2 + MB^2$$

$$\text{Or, } OB = 5 \text{ cm}$$

Now, consider $\triangle OND$,

$OB = OD = 5$ (since they are the radii)

$$\Rightarrow ND = CD/2 = 4 \text{ cm}$$

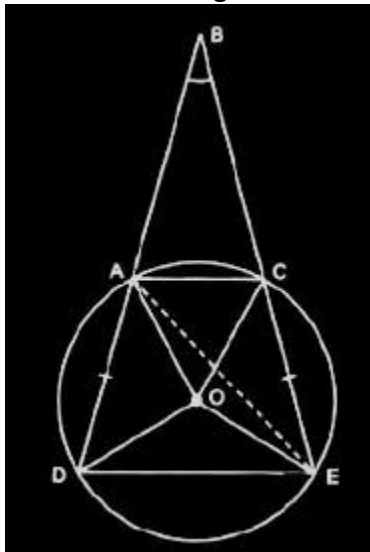
Now, $OD^2 = ON^2 + ND^2$

$$\text{Or, } ON = 3 \text{ cm.}$$

4. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Solution:

Consider the diagram



Here $AD = CE$

We know, any exterior angle of a triangle is equal to the sum of interior opposite angles.

So,

$$\angle DAE = \angle ABC + \angle AEC \text{ (in } \triangle BAE) \text{ -----(i)}$$

DE subtends $\angle DOE$ at the centre and $\angle DAE$ in the remaining part of the circle.

So,

$$\angle DAE = (\frac{1}{2})\angle DOE \text{ -----(ii)}$$

$$\text{Similarly, } \angle AEC = (\frac{1}{2})\angle AOC \text{ -----(iii)}$$

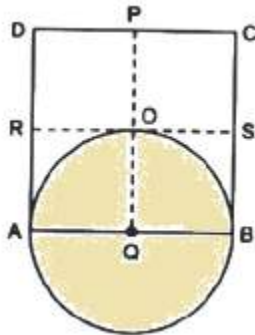
Now, from equation (i), (ii), and (iii) we get,

$$\angle DOE = \angle ABC + (\frac{1}{2})\angle AOC$$

$$\text{Or, } \angle ABC = (\frac{1}{2})[\angle DOE - \angle AOC] \text{ (hence proved).}$$

5. Prove that the circle drawn with any side of a rhombus as diameter, passes through the point of intersection of its diagonals.

Solution:



To prove: A circle drawn with Q as centre, will pass through A, B and O (i.e. $QA = QB = QO$)

Since all sides of a rhombus are equal,

$$AB = DC$$

Now, multiply $(\frac{1}{2})$ on both sides

$$(\frac{1}{2})AB = (\frac{1}{2})DC$$

So, $AQ = DP$

$$\Rightarrow BQ = DP$$

Since Q is the midpoint of AB,

$$AQ = BQ$$

Similarly,

$$RA = SB$$

Again, as PQ is drawn parallel to AD,

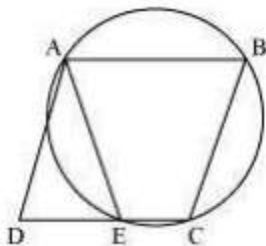
$$RA = QO$$

Now, as $AQ = BQ$ and $RA = QO$ we get,

$$QA = QB = QO \text{ (hence proved).}$$

6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that $AE = AD$.

Solution:



Here, ABCE is a cyclic quadrilateral. In a cyclic quadrilateral, the sum of the opposite angles is 180° .

$$\text{So, } \angle AEC + \angle CBA = 180^\circ$$

As $\angle AEC$ and $\angle AED$ are linear pair,

$$\angle AEC + \angle AED = 180^\circ$$

Or, $\angle AED = \angle CBA \dots (1)$

We know in a parallelogram; opposite angles are equal.

So, $\angle ADE = \angle CBA \dots (2)$

Now, from equations (1) and (2) we get,

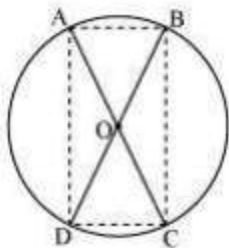
$\angle AED = \angle ADE$

Now, AD and AE are angles opposite to equal sides of a triangle,

$\therefore AD = AE$ (proved).

7. AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.

Solution:



Here chords AB and CD intersect each other at O.

Consider $\triangle AOB$ and $\triangle COD$,

$\angle AOB = \angle COD$ (They are vertically opposite angles)

$OB = OD$ (Given in the question)

$OA = OC$ (Given in the question)

So, by SAS congruency, $\triangle AOB \cong \triangle COD$

Also, $AB = CD$ (By CPCT)

Similarly, $\triangle AOD \cong \triangle COB$

Or, $AD = CB$ (By CPCT)

In quadrilateral ACBD, opposite sides are equal.

So, ACBD is a parallelogram.

We know that opposite angles of a parallelogram are equal.

So, $\angle A = \angle C$

Also, as ABCD is a cyclic quadrilateral,

$\angle A + \angle C = 180^\circ$

$\Rightarrow \angle A + \angle A = 180^\circ$

Or, $\angle A = 90^\circ$

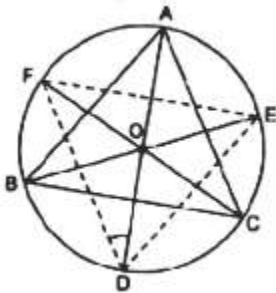
As ACBD is a parallelogram and one of its interior angles is 90° , so, it is a rectangle.

$\angle A$ is the angle subtended by chord BD. And as $\angle A = 90^\circ$, therefore, BD should be the diameter of the circle. Similarly, AC is the diameter of the circle.

8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90^\circ - (\frac{1}{2})A$, $90^\circ - (\frac{1}{2})B$ and $90^\circ - (\frac{1}{2})C$.

Solution:

Consider the following diagram



Here, ABC is inscribed in a circle with center O and the bisectors of $\angle A$, $\angle B$ and $\angle C$ intersect the circumcircle at D, E and F respectively.

Now, join DE, EF and FD

As angles in the same segment are equal, so,

$$\angle FDA = \angle FCA \text{ -----(i)}$$

$$\angle FDA = \angle EBA \text{ -----(ii)}$$

By adding equations (i) and (ii) we get,

$$\angle FDA + \angle EDA = \angle FCA + \angle EBA$$

$$\text{Or, } \angle FDE = \angle FCA + \angle EBA = (\frac{1}{2})\angle C + (\frac{1}{2})\angle B$$

We know, $\angle A + \angle B + \angle C = 180^\circ$

$$\text{So, } \angle FDE = (\frac{1}{2})[\angle C + \angle B] = (\frac{1}{2})[180^\circ - \angle A]$$

$$\Rightarrow \angle FDE = [90^\circ - (\frac{\angle A}{2})]$$

In a similar way,

$$\angle FED = [90^\circ - (\frac{\angle B}{2})]^\circ$$

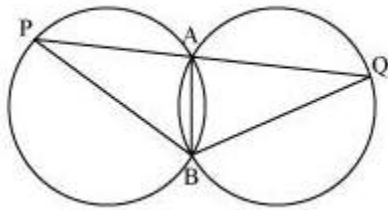
And,

$$\angle EFD = [90^\circ - (\frac{\angle C}{2})]^\circ$$

9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Solution:

The diagram will be



Here, $\angle APB = \angle AQB$ (as AB is the common chord in both the congruent circles.)

Now, consider $\triangle BPQ$,

$$\angle APB = \angle AQB$$

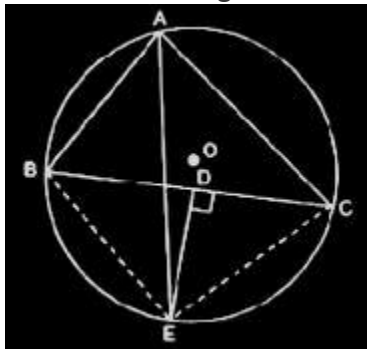
So, the angles opposite to equal sides of a triangle.

$$\therefore BQ = BP$$

10. In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the triangle ABC.

Solution:

Consider this diagram



Here, join BE and CE.

Now, since AE is the bisector of $\angle BAC$,

$$\angle BAE = \angle CAE$$

Also,

$$\therefore \text{arc BE} = \text{arc EC}$$

This implies, chord BE = chord EC

Now, consider triangles $\triangle BDE$ and $\triangle CDE$,

$$DE = DE \quad (\text{It is the common side})$$

$$BD = CD \quad (\text{It is given in the question})$$

$$BE = CE \quad (\text{Already proved})$$

So, by SSS congruency, $\triangle BDE \cong \triangle CDE$.

$$\text{Thus, } \therefore \angle BDE = \angle CDE$$

$$\text{We know, } \angle BDE = \angle CDE = 180^\circ$$

$$\text{Or, } \angle BDE = \angle CDE = 90^\circ$$

$$\therefore DE \perp BC \text{ (hence proved).}$$