Exercise: 12.2 (Page No: 206)

1. A park, in the shape of a quadrilateral ABCD, has ∠C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

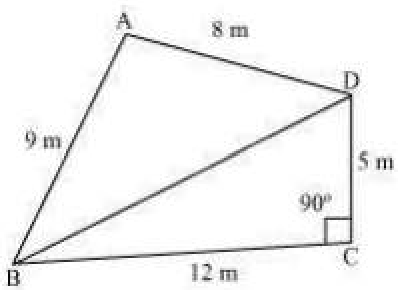
Solution:

First, construct a quadrilateral ABCD and join BD.

We know that

∠C = 90°, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m

The diagram is:



Now, apply Pythagoras theorem in ΔBCD

$BD^2 = BC^2 + CD^2$

⇒ $BD^2 = 12^2+5^2$

⇒ $BD^2 = 169$

⇒ BD = 13 m

Now, the area of ΔBCD = (½ ×12×5) = 30 m^2

The semi perimeter of ΔABD

(s) = (perimeter/2)

= (8+9+13)/2 m

= 30/2 m = 15 m

Using Heron's formula,

Area of ΔABD

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15(15 - 13)(15 - 9)(15 - 8)}\ m^2$$

$$= \sqrt{15 \times 2 \times 6 \times 7}\ m^2$$

= 6√35 m^2 = 35.5 m^2 (approximately)

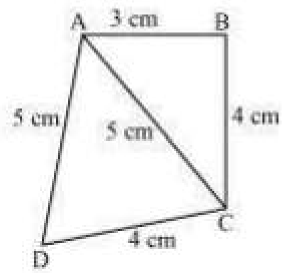
∴ The area of quadrilateral ABCD = Area of ΔBCD+Area of ΔABD

= 30 m^2+35.5m^2 = 65.5 m^2

2. Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Solution:

First, construct a diagram with the given parameter.



Now, apply Pythagorean theorem in ΔABC,

$AC^2 = AB^2+BC^2$

⇒ $5^2 = 3^2+4^2$

⇒ 25 = 25

Thus, it can be concluded that ΔABC is a right angled at B.

So, area of ΔBCD = (½ ×3×4) = 6 cm^2

The semi perimeter of ΔACD (s) = (perimeter/2) = (5+5+4)/2 cm = 14/2 cm = 7 m

Now, using Heron's formula,

Area of ΔABD

$$\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\left[\sqrt{7(7-5)(7-5)(7-4)}\right]\text{cm}^2$$
$$=\left(\sqrt{7\times2\times2\times3}\right)\text{cm}^2$$

= 2√21 cm^2 = 9.17 cm^2 (approximately)

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔABD = 6 cm^2 +9.17 cm^2 = 15.17 cm^2

3. Radha made a picture of an aeroplane with coloured paper as shown in Fig 12.15. Find the total area of the paper used.

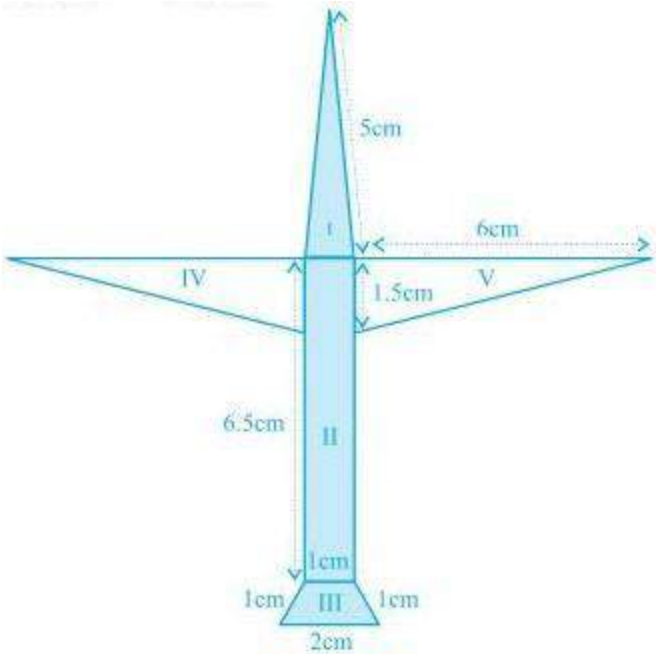
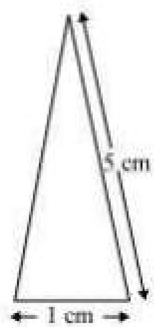


Fig. 12.15

Solution:

For the triangle I section:



It is an isosceles triangle and the sides are 5 cm, 1 cm and 5 cm

Perimeter = 5+5+1 = 11 cm

So, semi perimeter = 11/2 cm = 5.5 cm

Using Heron's formula,

Area = $\sqrt{[s(s-a)(s-b)(s-c)]}$

= $\sqrt{[5.5(5.5-5)(5.5-5)(5.5-1)]}$ cm^2

= $\sqrt{[5.5\times0.5\times0.5\times4.5]}$ cm^2

= $0.75\sqrt{11}$ cm^2

= $0.75 \times 3.317 cm^2$

= $2.488 cm^2$ (approx)

For the quadrilateral II section:

This quadrilateral is a rectangle with length and breadth as 6.5 cm and 1 cm respectively.

∴ Area = $6.5\times1\ cm^2=6.5\ cm^2$

For the quadrilateral III section:

It is a trapezoid with 2 sides as 1 cm each and the third side as 2 cm.

Area of the trapezoid = Area of the parallelogram + Area of the equilateral triangle

The perpendicular height of the parallelogram will be

$$\left(\sqrt{1^2-(0.5)^2}\right)$$

= 0.86 cm

And, the area of the equilateral triangle will be $(\sqrt{3}/4\times a^2)$ = 0.43

∴ Area of the trapezoid = 0.86+0.43 = 1.3 cm^2 (approximately).

For triangle IV and V:

These triangles are 2 congruent right angled triangles having the base as 6 cm and height 1.5 cm

Area triangles IV and V = $2\times(½\times6\times1.5)\ cm^2 = 9\ cm^2$

So, the total area of the paper used = $(2.488+6.5+1.3+9)\ cm^2 = 19.3\ cm^2$

4. A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Solution:

Given,

It is given that the parallelogram and triangle have equal areas.

The sides of the triangle are given as 26 cm, 28 cm and 30 cm.

So, the perimeter = 26+28+30 = 84 cm

And its semi perimeter = 84/2 cm = 42 cm

Now, by using Heron's formula, area of the triangle =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

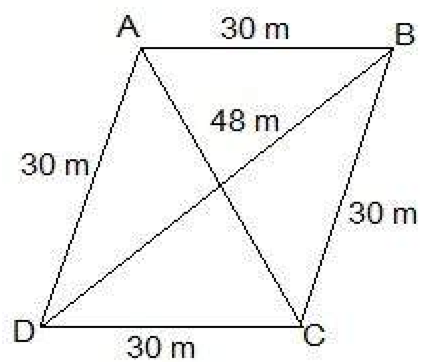
= $\sqrt{[42(42-26)(46-28)(46-30)]}\ cm^2$

= √[46×16×14×16] cm^2
= 336 cm^2

Now, let the height of parallelogram be h.
As the area of parallelogram = area of the triangle,
28 cm× h = 336 cm^2
∴ h = 336/28 cm
So, the height of the parallelogram is 12 cm.

5. A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Solution:
Draw a rhombus-shaped field first with the vertices as ABCD. The diagonal AC divides the rhombus into two congruent triangles which are having equal areas. The diagram is as follows.



Consider the triangle BCD,
Its semi-perimeter = (48 + 30 + 30)/2 m = 54 m
Using Heron's formula,
Area of the ΔBCD =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$\left(\sqrt{54(54-48)(54-30)(54-30)}\right) m^2$$

$$\left(\sqrt{54\times 6\times 24\times 24}\right) m^2$$

= 432 m^2

∴ Area of field = 2 × area of the ΔBCD = (2 × 432) m^2 = 864 m^2

Thus, the area of the grass field that each cow will be getting = (864/18) m^2 = 48 m^2

6. An umbrella is made by stitching 10 triangular pieces of cloth of two different colours (see Fig.12.16), each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?

Fig. 12.16

Solution:

For each triangular piece, The semi perimeter will be

s = (50+50+20)/2 cm = 120/2 cm = 60cm

Using Heron's formula,

Area of the triangular piece

= $\sqrt{s(s-a)(s-b)(s-c)}$

= √[60(60-50)(60-50)(60-20)] cm^2

= √[60×10×10×40] cm^2

= 200√6 cm^2

∴ The area of all the triangular pieces = 5 × 200√6 cm^2 = 1000√6 cm^2

7. A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in Fig. 12.17. How much paper of each shade has been used in it?

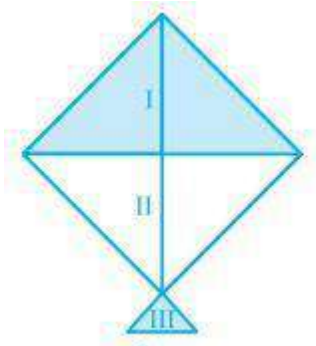


Fig. 12.17

Solution:

As the kite is in the shape of a square, its area will be

A = (½)×(diagonal)^2

⇒Area of the kite = (½)×32×32 = 512 cm^2.

The area of shade I = Area of shade II

⇒ 512/2 cm^2 = 256 cm^2

So, the total area of the paper that is required in each shade = 256 cm^2

For the triangle section (III),

The sides are given as 6 cm, 6 cm and 8 cm

Now, the semi perimeter of this isosceles triangle = (6+6+8)/2 cm = 10 cm

By using Heron's formula, the area of the III triangular piece will be

= $\sqrt{s(s-a)(s-b)(s-c)}$

= $\sqrt{10(10-6)(10-6)(10-8)}$ cm^2

= $\sqrt{10\times4\times4\times2}$ cm^2

= $8\sqrt{6}$ cm^2

8. A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm (see Fig. 12.18). Find the cost of polishing the tiles at the rate of 50p per cm^2 .

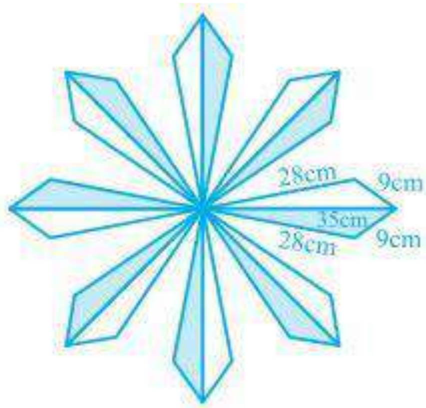


Fig. 12.18

Solution:

The semi perimeter of the each triangular shape = (28+9+35)/2 cm = 36 cm
By using Heron's formula,
The area of each triangular shape will be

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$\left(\sqrt{36\times(36-35)\times(36-28)\times(36-9)}\right)$$

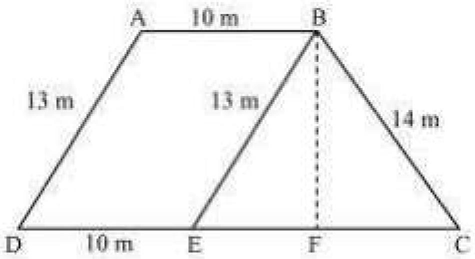
$$\left(\sqrt{36}\times 1\times 8\times 27\right) cm^2$$

= 36√6 cm^2 = 88.2 cm^2

Now, the total area of 16 tiles = 16×88.2 cm^2 = 1411.2 cm^2
It is given that the polishing cost of tiles = 50 paise/cm^2
∴ The total polishing cost of the tiles = Rs. (1411.2×0.5) = Rs. 705.6

9. A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Solution:
First, draw a line segment BE parallel to the line AD. Then, from B, draw a perpendicular on the line segment CD.



Now, it can be seen that the quadrilateral ABED is a parallelogram. So,

AB = ED = 10 m

AD = BE = 13 m

EC = 25-ED = 25-10 = 15 m

Now, consider the triangle BEC,

Its semi perimeter (s) = (13+14+15)/2 = 21 m

By using Heron's formula,

Area of ΔBEC =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

$$\left(\sqrt{21 \times (21-13) \times (21-14) \times (21-15)}\right) m^2$$

$$\left(\sqrt{21} \times 8 \times 7 \times 6\right) m^2$$

= 84 m^2

We also know that the area of ΔBEC = (½)×CE×BF

84 cm^2 = (½)×15×BF

⟹ BF = (168/15) cm = 11.2 cm

So, the total area of ABED will be BF×DE i.e. 11.2×10 = 112 m^2

∴ Area of the field = 84+112 = 196 m^2