1. Determine which of the following polynomials has \((x + 1)\) a factor:

(i) \(x^3 + x^2 + x + 1\)

Solution:
Let \(p(x) = x^3 + x^2 + x + 1\)
The zero of \(x + 1\) is -1. [\(x + 1 = 0 \text{ means } x = -1\)]
\[p(-1) = (-1)^3 + (-1)^2 + (-1) + 1\]
\[= -1 + 1 - 1 + 1\]
\[= 0\]
∴ By factor theorem, \(x + 1\) is a factor of \(x^3 + x^2 + x + 1\)

(ii) \(x^4 + x^3 + x^2 + x + 1\)

Solution:
Let \(p(x) = x^4 + x^3 + x^2 + x + 1\)
The zero of \(x + 1\) is -1. [\(x + 1 = 0 \text{ means } x = -1\)]
\[p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1\]
\[= 1 - 1 + 1 - 1 + 1\]
\[= 1 \neq 0\]
∴ By factor theorem, \(x + 1\) is not a factor of \(x^4 + x^3 + x^2 + x + 1\)

(iii) \(x^4 + 3x^3 + 3x^2 + x + 1\)

Solution:
Let \(p(x) = x^4 + 3x^3 + 3x^2 + x + 1\)
The zero of \(x + 1\) is -1.
\[p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1\]
\[= 1 - 3 + 3 - 1 + 1\]
\[= 1 \neq 0\]
∴ By factor theorem, \(x + 1\) is not a factor of \(x^4 + 3x^3 + 3x^2 + x + 1\)

(iv) \(x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}\)

Solution:
Let \(p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}\)
The zero of \(x + 1\) is -1.
\[p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}\]
\[= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}\]
\[= 2\sqrt{2} \neq 0\]
∴ By factor theorem, \(x + 1\) is not a factor of \(x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}\)
2. Use the Factor Theorem to determine whether \( g(x) \) is a factor of \( p(x) \) in each of the following cases:

(i) \( p(x) = 2x^3 + x^2 - 2x - 1 \), \( g(x) = x + 1 \)

Solution:

\[
p(x) = 2x^3 + x^2 - 2x - 1, \quad g(x) = x + 1
\]

\[
g(x) = 0
\]

\[
\Rightarrow x + 1 = 0
\]

\[
\Rightarrow x = -1
\]

\[\therefore\ \text{Zero of } g(x) \text{ is } -1.\]

Now,

\[
p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1
\]

\[
= -2 + 1 + 2 - 1
\]

\[
= 0
\]

\[\therefore\ \text{By factor theorem, } g(x) \text{ is a factor of } p(x).\]

(ii) \( p(x) = x^3 + 3x^2 + 3x + 1 \), \( g(x) = x + 2 \)

Solution:

\[
p(x) = x^3 + 3x^2 + 3x + 1, \quad g(x) = x + 2
\]

\[
g(x) = 0
\]

\[
\Rightarrow x + 2 = 0
\]

\[
\Rightarrow x = -2
\]

\[\therefore\ \text{Zero of } g(x) \text{ is } -2.\]

Now,

\[
p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1
\]

\[
= -8 + 12 - 6 + 1
\]

\[
= -1 \neq 0
\]

\[\therefore\ \text{By factor theorem, } g(x) \text{ is not a factor of } p(x).\]

(iii) \( p(x) = x^3 - 4x^2 + x + 6 \), \( g(x) = x - 3 \)

Solution:

\[
p(x) = x^3 - 4x^2 + x + 6, \quad g(x) = x - 3
\]

\[
g(x) = 0
\]

\[
\Rightarrow x - 3 = 0
\]

\[
\Rightarrow x = 3
\]

\[\therefore\ \text{Zero of } g(x) \text{ is } 3.\]

Now,

\[
p(3) = (3)^3 - 4(3)^2 + (3) + 6
\]

\[
= 27 - 36 + 3 + 6
\]

\[
= 0
\]

\[\therefore\ \text{By factor theorem, } g(x) \text{ is a factor of } p(x).\]

3. Find the value of \( k \), if \( x - 1 \) is a factor of \( p(x) \) in each of the following cases:

(i) \( p(x) = x^2 + x + k \)
Solution:
If \( x-1 \) is a factor of \( p(x) \), then \( p(1) = 0 \)

By Factor Theorem

\[ (1)^2+(1)+k = 0 \]
\[ 1+1+k = 0 \]
\[ 2+k = 0 \]
\[ k = -2 \]

(ii) \( p(x) = 2x^2+kx+\sqrt{2} \)

Solution:
If \( x-1 \) is a factor of \( p(x) \), then \( p(1)=0 \)

\[ 2(1)^2+k(1)+\sqrt{2} = 0 \]
\[ 2+k+\sqrt{2} = 0 \]
\[ k = -(2+\sqrt{2}) \]

(iii) \( p(x) = kx^2-\sqrt{2}x+1 \)

Solution:
If \( x-1 \) is a factor of \( p(x) \), then \( p(1)=0 \)

By Factor Theorem

\[ k(1)^2-\sqrt{2}(1)+1=0 \]
\[ k = \sqrt{2}-1 \]

(iv) \( p(x)=kx^2–3x+k \)

Solution:
If \( x-1 \) is a factor of \( p(x) \), then \( p(1) = 0 \)

By Factor Theorem

\[ k(1)^2–3(1)+k = 0 \]
\[ k–3+k = 0 \]
\[ 2k–3 = 0 \]
\[ k= \frac{3}{2} \]

4. Factorize:
(i) \( 12x^2–7x+1 \)

Solution:
Using the splitting the middle term method,
We have to find a number whose sum = -7 and product =1×12 = 12
We get -3 and -4 as the numbers

\[ 12x^2–7x+1 = 12x^2-4x-3x+1 \]
\[ = 4x(3x-1)-1(3x-1) \]
\[ = (4x-1)(3x-1) \]

(ii) \( 2x^2+7x+3 \)

Solution:
Using the splitting the middle term method,
We have to find a number whose sum = 7 and product = 2×3 = 6
We get 6 and 1 as the numbers [6+1 = 7 and 6×1 = 6]

\[ 2x^2+7x+3 = 2x^2+6x+1x+3 \]
\[ = 2x(x+3)+1(x+3) \]
\[ = (2x+1)(x+3) \]

(iii) \(6x^2+5x-6\)
Solution:
Using the splitting the middle term method,
We have to find a number whose sum = 5 and product = 6×(-6) = -36
We get -4 and 9 as the numbers [-4+9 = 5 and -4×9 = -36]

\[ 6x^2+5x-6 = 6x^2+9x–4x–6 \]
\[ = 3(2x+3)–2(2x+3) \]
\[ = (2x+3)(3x–2) \]

(iv) \(3x^2–x–4\)
Solution:
Using the splitting the middle term method,
We have to find a number whose sum = -1 and product = 3×(-4) = -12
We get -4 and 3 as the numbers [-4+3 = -1 and -4×3 = -12]

\[ 3x^2–x–4 = 3x^2–x–4 \]
\[ = 3x^2–x+3x–4 \]
\[ = x(3x–4)+1(3x–4) \]
\[ = (3x–4)(x+1) \]

5. Factorize:
(i) \(x^3–2x^2–x+2\)
Solution:
Let \(p(x) = x^3–2x^2–x+2\)
Factors of 2 are ±1 and ± 2
By trial method, we find that
\(p(1) = 0\)
So, \((x+1)\) is factor of \(p(x)\)
Now,
\(p(x) = x^3–2x^2–x+2\)
\(p(-1) = (-1)^3–2(-1)^2–(-1)+2\)
\[ = -1-1+1+2 \]
\[ = 0 \]
Therefore, \((x+1)\) is the factor of \(p(x)\)
Now, Dividend = Divisor \times Quotient + Remainder

\[(x+1)(x^2-3x+2) = (x+1)(x^2-x-2x+2) = (x+1)(x(x-1)-2(x-1)) = (x+1)(x-1)(x+2)\]

(ii) \(x^3-3x^2-9x-5\)

**Solution:**
Let \(p(x) = x^3-3x^2-9x-5\)
Factors of 5 are \(\pm1\) and \(\pm5\)
By trial method, we find that \(p(5) = 0\)
So, \((x-5)\) is factor of \(p(x)\)
Now, \(p(x) = x^3-3x^2-9x-5\)
\(p(5) = (5)^3-3(5)^2-9(5)-5\)
\[= 125-75-45-5\]
\[= 0\]
Therefore, \((x-5)\) is the factor of \(p(x)\)
Now, Dividend = Divisor × Quotient + Remainder

\[(x-5)(x^2+2x+1) = (x-5)(x^2+x+x+1)\]
\[= (x-5)(x(x+1)+1(x+1))\]
\[= (x-5)(x+1)(x+1)\]

(iii) \(x^3+13x^2+32x+20\)

Solution:
Let \(p(x) = x^3+13x^2+32x+20\)
Factors of 20 are \(\pm1, \pm2, \pm4, \pm5, \pm10\) and \(\pm20\)
By trial method, we find that
\(p(-1) = 0\)
So, \((x+1)\) is factor of \(p(x)\)
Now,
\(p(x)= x^3+13x^2+32x+20\)
\(p(-1) = (-1)^3+13(-1)^2+32(-1)+20\)
\[= -1+13-32+20\]
\[= 0\]
Therefore, \((x+1)\) is the factor of \(p(x)\)
Now, Dividend = Divisor \times Quotient + Remainder

(x+1)(x^2+12x+20) = (x+1)(x^2+2x+10x+20)
= (x-5)x(x+2)+10(x+2)
= (x-5)(x+2)(x+10)

(iv) \(2y^3+y^2-2y-1\)

Solution:
Let \(p(y) = 2y^3+y^2-2y-1\)
Factors = \(2\times(-1)=-2\) are \(\pm 1\) and \(\pm 2\)
By trial method, we find that
\(p(1) = 0\)
So, \((y-1)\) is factor of \(p(y)\)
Now,
\(p(y) = 2y^3+y^2-2y-1\)
\(p(1) = 2(1)^3+(1)^2-2(1)-1\)
\[= 2+1-2-1\]
\[= 0\]
Therefore, \((y-1)\) is the factor of \(p(y)\)
Now, Dividend = Divisor \times Quotient + Remainder

\[(y-1)(2y^2+3y+1) = (y-1)(2y^2+2y+y+1) = (y-1)(2y(y+1)+1(y+1)) = (y-1)(2y+1)(y+1)\]