

## Exercise 2.1

Page: 32

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i)  $4x^2 - 3x + 7$

**Solution:**

The equation  $4x^2 - 3x + 7$  can be written as  $4x^2 - 3x^1 + 7x^0$

Since  $x$  is the only variable in the given equation and the powers of  $x$  (i.e., 2, 1 and 0) are whole numbers, we can say that the expression  $4x^2 - 3x + 7$  is a polynomial in one variable.

(ii)  $y^2 + \sqrt{2}$

**Solution:**

The equation  $y^2 + \sqrt{2}$  can be written as  $y^2 + \sqrt{2}y^0$

Since  $y$  is the only variable in the given equation and the powers of  $y$  (i.e., 2 and 0) are whole numbers, we can say that the expression  $y^2 + \sqrt{2}$  is a polynomial in one variable.

(iii)  $3\sqrt{t} + t\sqrt{2}$

**Solution:**

The equation  $3\sqrt{t} + t\sqrt{2}$  can be written as  $3t^{1/2} + \sqrt{2}t$

Though,  $t$  is the only variable in the given equation, the powers of  $t$  (i.e.,  $1/2$ ) is not a whole number. Hence, we can say that the expression  $3\sqrt{t} + t\sqrt{2}$  is **not** a polynomial in one variable.

(iv)  $y + 2/y$

**Solution:**

The equation  $y + 2/y$  can be written as  $y + 2y^{-1}$

Though,  $y$  is the only variable in the given equation, the powers of  $y$  (i.e.,  $-1$ ) is not a whole number. Hence, we can say that the expression  $y + 2/y$  is **not** a polynomial in one variable.

(v)  $x^{10} + y^3 + t^{50}$

**Solution:**

Here, in the equation  $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression  $x^{10} + y^3 + t^{50}$ . Hence, it is **not** a polynomial in one variable.

2. Write the coefficients of  $x^2$  in each of the following:

(i)  $2 + x^2 + x$

**Solution:**

The equation  $2 + x^2 + x$  can be written as  $2 + (1)x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable  $x^2$  is 1

$\therefore$ , the coefficients of  $x^2$  in  $2 + x^2 + x$  is 1.

**(ii)  $2-x^2+x^3$**

**Solution:**The equation  $2-x^2+x^3$  can be written as  $2+(-1)x^2+x^3$ 

We know that, coefficient is the number (along with its sign, i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable  $x^2$  is -1 $\therefore$  the coefficients of  $x^2$  in  $2-x^2+x^3$  is -1.

**(iii)  $(\pi/2)x^2+x$**

**Solution:**The equation  $(\pi/2)x^2+x$  can be written as  $(\pi/2)x^2 + x$ 

We know that, coefficient is the number (along with its sign, i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable  $x^2$  is  $\pi/2$ . $\therefore$  the coefficients of  $x^2$  in  $(\pi/2)x^2+x$  is  $\pi/2$ .

**(iii)  $\sqrt{2x-1}$**

**Solution:**The equation  $\sqrt{2x-1}$  can be written as  $0x^2+\sqrt{2x-1}$  [Since  $0x^2$  is 0]

We know that, coefficient is the number (along with its sign, i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable  $x^2$  is 0 $\therefore$ , the coefficients of  $x^2$  in  $\sqrt{2x-1}$  is 0.**3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.****Solution:**

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg.,  $3x^{35}+5$ 

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100

Eg.,  $4x^{100}$ **4. Write the degree of each of the following polynomials:**

**(i)  $5x^3+4x^2+7x$**

**Solution:**

The highest power of the variable in a polynomial is the degree of the polynomial.

Here,  $5x^3+4x^2+7x = 5x^3+4x^2+7x^1$ The powers of the variable  $x$  are: 3, 2, 1 $\therefore$  the degree of  $5x^3+4x^2+7x$  is 3 as 3 is the highest power of  $x$  in the equation.

**(ii)  $4-y^2$**

**Solution:**

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in  $4-y^2$ ,

The power of the variable  $y$  is 2

$\therefore$  the degree of  $4-y^2$  is 2 as 2 is the highest power of  $y$  in the equation.

**(iii)  $5t-\sqrt{7}$**

**Solution:**

The highest power of the variable in a polynomial is the degree of the polynomial.

Here, in  $5t-\sqrt{7}$ ,

The power of the variable  $y$  is: 1

$\therefore$  the degree of  $5t-\sqrt{7}$  is 1 as 1 is the highest power of  $y$  in the equation.

**(iv) 3**

**Solution:**

The highest power of the variable in a polynomial is the degree of the polynomial.

Here,  $3 = 3 \times 1 = 3 \times x^0$

The power of the variable here is: 0

$\therefore$  the degree of 3 is 0.

### 5. Classify the following as linear, quadratic and cubic polynomials:

**Solution:**

We know that,

Linear polynomial: A polynomial of degree one is called a linear polynomial.

Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Cubic polynomial: A polynomial of degree three is called a cubic polynomial.

**(i)  $x^2+x$**

**Solution:**

The highest power of  $x^2+x$  is 2

$\therefore$  the degree is 2

Hence,  $x^2+x$  is a quadratic polynomial

**(ii)  $x-x^3$**

**Solution:**

The highest power of  $x-x^3$  is 3

$\therefore$  the degree is 3

Hence,  $x-x^3$  is a cubic polynomial

**(iii)  $y+y^2+4$**

**Solution:**

The highest power of  $y+y^2+4$  is 2

$\therefore$  the degree is 2

Hence,  $y+y^2+4$  is a quadratic polynomial

**(iv)  $1+x$**

**Solution:**

The highest power of  $1+x$  is 1

$\therefore$  the degree is 1

Hence,  $1+x$  is a linear polynomial.

(v)  $3t$

**Solution:**

The highest power of  $3t$  is 1

$\therefore$  the degree is 1

Hence,  $3t$  is a linear polynomial.

(vi)  $r^2$

**Solution:**

The highest power of  $r^2$  is 2

$\therefore$  the degree is 2

Hence,  $r^2$  is a quadratic polynomial.

(vii)  $7x^3$

**Solution:**

The highest power of  $7x^3$  is 3

$\therefore$  the degree is 3

Hence,  $7x^3$  is a cubic polynomial.

Exercise 2.2

Page: 34

**1. Find the value of the polynomial  $(x)=5x-4x^2+3$** **(i)  $x = 0$** **(ii)  $x = -1$** **(iii)  $x = 2$** **Solution:**

Let  $f(x) = 5x - 4x^2 + 3$

**(iii) When  $x = 0$** 

$$\begin{aligned} f(0) &= 5(0) - 4(0)^2 + 3 \\ &= 3 \end{aligned}$$

**(ii) When  $x = -1$** 

$$\begin{aligned} f(x) &= 5x - 4x^2 + 3 \\ f(-1) &= 5(-1) - 4(-1)^2 + 3 \\ &= -5 - 4 + 3 \\ &= -6 \end{aligned}$$

**(iii) When  $x = 2$** 

$$\begin{aligned} f(x) &= 5x - 4x^2 + 3 \\ f(2) &= 5(2) - 4(2)^2 + 3 \\ &= 10 - 16 + 3 \\ &= -3 \end{aligned}$$

**2. Find  $p(0)$ ,  $p(1)$  and  $p(2)$  for each of the following polynomials:****(i)  $p(y) = y^2 - y + 1$** **Solution:**

$$\begin{aligned} p(y) &= y^2 - y + 1 \\ \therefore p(0) &= (0)^2 - (0) + 1 = 1 \\ p(1) &= (1)^2 - (1) + 1 = 1 \\ p(2) &= (2)^2 - (2) + 1 = 3 \end{aligned}$$

**(ii)  $p(t) = 2 + t + 2t^2 - t^3$** **Solution:**

$$\begin{aligned} p(t) &= 2 + t + 2t^2 - t^3 \\ \therefore p(0) &= 2 + 0 + 2(0)^2 - (0)^3 = 2 \\ p(1) &= 2 + 1 + 2(1)^2 - (1)^3 = 2 + 1 + 2 - 1 = 4 \\ p(2) &= 2 + 2 + 2(2)^2 - (2)^3 = 2 + 2 + 8 - 8 = 4 \end{aligned}$$

**(iii)  $p(x) = x^3$** **Solution:**

$$\begin{aligned} p(x) &= x^3 \\ \therefore p(0) &= (0)^3 = 0 \end{aligned}$$

$$p(1) = (1)^3 = 1$$
$$p(2) = (2)^3 = 8$$

**(iv)  $P(x) = (x-1)(x+1)$**

**Solution:**

$$p(x) = (x-1)(x+1)$$
$$\therefore p(0) = (0-1)(0+1) = (-1)(1) = -1$$
$$p(1) = (1-1)(1+1) = 0(2) = 0$$
$$p(2) = (2-1)(2+1) = 1(3) = 3$$

**3. Verify whether the following are zeroes of the polynomial, indicated against them.**

**(i)  $p(x)=3x+1$ ,  $x=-1/3$**

**Solution:**

$$\text{For, } x = -1/3, p(x) = 3x+1$$
$$\therefore p(-1/3) = 3(-1/3)+1 = -1+1 = 0$$
$$\therefore -1/3 \text{ is a zero of } p(x).$$

**(ii)  $p(x)=5x-\pi$ ,  $x = 4/5$**

**Solution:**

$$\text{For, } x = 4/5, p(x) = 5x-\pi$$
$$\therefore p(4/5) = 5(4/5)-\pi = 4-\pi$$
$$\therefore 4/5 \text{ is not a zero of } p(x).$$

**(iii)  $p(x)=x^2-1$ ,  $x=1$ ,  $-1$**

**Solution:**

$$\text{For, } x = 1, -1;$$
$$p(x) = x^2-1$$
$$\therefore p(1)=1^2-1=1-1 = 0$$
$$p(-1)=(-1)^2-1 = 1-1 = 0$$
$$\therefore 1, -1 \text{ are zeros of } p(x).$$

**(iv)  $p(x) = (x+1)(x-2)$ ,  $x = -1, 2$**

**Solution:**

$$\text{For, } x = -1, 2;$$
$$p(x) = (x+1)(x-2)$$
$$\therefore p(-1) = (-1+1)(-1-2)$$
$$= (0)(-3) = 0$$
$$p(2) = (2+1)(2-2) = (3)(0) = 0$$
$$\therefore -1, 2 \text{ are zeros of } p(x).$$

**(v)  $p(x) = x^2$ ,  $x = 0$**

**Solution:**

$$\text{For, } x = 0 p(x) = x^2$$
$$p(0) = 0^2 = 0$$
$$\therefore 0 \text{ is a zero of } p(x).$$

(vi)  $p(x) = lx+m$ ,  $x = -m/l$

Solution:

For,  $x = -m/l$  ;  $p(x) = lx+m$

$\therefore p(-m/l) = l(-m/l)+m = -m+m = 0$

$\therefore -m/l$  is a zero of  $p(x)$ .

(vii)  $p(x) = 3x^2-1$ ,  $x = -1/\sqrt{3}$ ,  $2/\sqrt{3}$

Solution:

For,  $x = -1/\sqrt{3}$ ,  $2/\sqrt{3}$  ;  $p(x) = 3x^2-1$

$\therefore p(-1/\sqrt{3}) = 3(-1/\sqrt{3})^2-1 = 3(1/3)-1 = 1-1 = 0$

$\therefore p(2/\sqrt{3}) = 3(2/\sqrt{3})^2-1 = 3(4/3)-1 = 4-1=3 \neq 0$

$\therefore -1/\sqrt{3}$  is a zero of  $p(x)$  but  $2/\sqrt{3}$  is not a zero of  $p(x)$ .

(viii)  $p(x) = 2x+1$ ,  $x = 1/2$

Solution:

For,  $x = 1/2$   $p(x) = 2x+1$

$\therefore p(1/2) = 2(1/2)+1 = 1+1 = 2 \neq 0$

$\therefore 1/2$  is not a zero of  $p(x)$ .

#### 4. Find the zero of the polynomials in each of the following cases:

(i)  $p(x) = x+5$

Solution:

$p(x) = x+5$

$\Rightarrow x+5 = 0$

$\Rightarrow x = -5$

$\therefore -5$  is a zero polynomial of the polynomial  $p(x)$ .

(ii)  $p(x) = x-5$

Solution:

$p(x) = x-5$

$\Rightarrow x-5 = 0$

$\Rightarrow x = 5$

$\therefore 5$  is a zero polynomial of the polynomial  $p(x)$ .

(iii)  $p(x) = 2x+5$

Solution:

$p(x) = 2x+5$

$\Rightarrow 2x+5 = 0$

$\Rightarrow 2x = -5$

$\Rightarrow x = -5/2$

$\therefore x = -5/2$  is a zero polynomial of the polynomial  $p(x)$ .

(iv)  $p(x) = 3x - 2$

Solution:

$$p(x) = 3x - 2$$

$$\Rightarrow 3x - 2 = 0$$

$$\Rightarrow 3x = 2$$

$$\Rightarrow x = 2/3$$

$\therefore x = 2/3$  is a zero polynomial of the polynomial  $p(x)$ .

(v)  $p(x) = 3x$

Solution:

$$p(x) = 3x$$

$$\Rightarrow 3x = 0$$

$$\Rightarrow x = 0$$

$\therefore 0$  is a zero polynomial of the polynomial  $p(x)$ .

(vi)  $p(x) = ax, a \neq 0$

Solution:

$$p(x) = ax$$

$$\Rightarrow ax = 0$$

$$\Rightarrow x = 0$$

$\therefore x = 0$  is a zero polynomial of the polynomial  $p(x)$ .

(vii)  $p(x) = cx + d, c \neq 0, c, d$  are real numbers.

Solution:

$$p(x) = cx + d$$

$$\Rightarrow cx + d = 0$$

$$\Rightarrow x = -d/c$$

$\therefore x = -d/c$  is a zero polynomial of the polynomial  $p(x)$ .



Exercise 2.3

Page: 40

**1. Find the remainder when  $x^3+3x^2+3x+1$  is divided by****(i)  $x+1$** **Solution:**

$$x+1=0$$

$$\Rightarrow x = -1$$

 **$\therefore$  Remainder:**

$$\begin{aligned} p(-1) &= (-1)^3+3(-1)^2+3(-1)+1 \\ &= -1+3-3+1 \\ &= 0 \end{aligned}$$

**(ii)  $x-1/2$** **Solution:**

$$x-1/2=0$$

$$\Rightarrow x = 1/2$$

 **$\therefore$  Remainder:**

$$\begin{aligned} p(1/2) &= (1/2)^3+3(1/2)^2+3(1/2)+1 \\ &= (1/8)+(3/4)+(3/2)+1 \\ &= 27/8 \end{aligned}$$

**(iii)  $x$** **Solution:**

$$x=0$$

 **$\therefore$  Remainder:**

$$\begin{aligned} p(0) &= (0)^3+3(0)^2+3(0)+1 \\ &= 1 \end{aligned}$$

**(iv)  $x+\pi$** **Solution:**

$$x+\pi=0$$

$$\Rightarrow x = -\pi$$

 **$\therefore$  Remainder:**

$$\begin{aligned} p(-\pi) &= (-\pi)^3+3(-\pi)^2+3(-\pi)+1 \\ &= -\pi^3+3\pi^2-3\pi+1 \end{aligned}$$

**(v)  $5+2x$** **Solution:**

$$5+2x=0$$

$$\Rightarrow 2x = -5$$

$$\Rightarrow x = -5/2$$

 **$\therefore$  Remainder:**

$$\begin{aligned} (-5/2)^3+3(-5/2)^2+3(-5/2)+1 &= (-125/8)+(75/4)-(15/2)+1 \\ &= -27/8 \end{aligned}$$

2. Find the remainder when  $x^3 - ax^2 + 6x - a$  is divided by  $x - a$ .

Solution:

$$\text{Let } p(x) = x^3 - ax^2 + 6x - a$$

$$x - a = 0$$

$$\therefore x = a$$

Remainder:

$$\begin{aligned} p(a) &= (a)^3 - a(a^2) + 6(a) - a \\ &= a^3 - a^3 + 6a - a = 5a \end{aligned}$$

3. Check whether  $7 + 3x$  is a factor of  $3x^3 + 7x$ .

Solution:

$$7 + 3x = 0$$

$$\Rightarrow 3x = -7$$

$$\Rightarrow x = -7/3$$

$\therefore$  Remainder:

$$\begin{aligned} 3(-7/3)^3 + 7(-7/3) &= -(343/9) + (-49/3) \\ &= (-343 - (49)3)/9 \\ &= (-343 - 147)/9 \\ &= -490/9 \neq 0 \end{aligned}$$

$\therefore 7 + 3x$  is not a factor of  $3x^3 + 7x$

Exercise 2.4

1. Determine which of the following polynomials has  $(x + 1)$  a factor:

(i)  $x^3+x^2+x+1$

Solution:

Let  $p(x) = x^3+x^2+x+1$

The zero of  $x+1$  is  $-1$ . [ $x+1 = 0$  means  $x = -1$ ]

$$\begin{aligned} p(-1) &= (-1)^3+(-1)^2+(-1)+1 \\ &= -1+1-1+1 \\ &= 0 \end{aligned}$$

∴By factor theorem,  $x+1$  is a factor of  $x^3+x^2+x+1$

(ii)  $x^4+x^3+x^2+x+1$

Solution:

Let  $p(x) = x^4+x^3+x^2+x+1$

The zero of  $x+1$  is  $-1$ . [ $x+1 = 0$  means  $x = -1$ ]

$$\begin{aligned} p(-1) &= (-1)^4+(-1)^3+(-1)^2+(-1)+1 \\ &= 1-1+1-1+1 \\ &= 1 \neq 0 \end{aligned}$$

∴By factor theorem,  $x+1$  is not a factor of  $x^4 + x^3 + x^2 + x + 1$

(iii)  $x^4+3x^3+3x^2+x+1$

Solution:

Let  $p(x) = x^4+3x^3+3x^2+x+1$

The zero of  $x+1$  is  $-1$ .

$$\begin{aligned} p(-1) &= (-1)^4+3(-1)^3+3(-1)^2+(-1)+1 \\ &= 1-3+3-1+1 \\ &= 1 \neq 0 \end{aligned}$$

∴By factor theorem,  $x+1$  is not a factor of  $x^4+3x^3+3x^2+x+1$

(iv)  $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Solution:

Let  $p(x) = x^3-x^2-(2+\sqrt{2})x + \sqrt{2}$

The zero of  $x+1$  is  $-1$ .

$$\begin{aligned} p(-1) &= (-1)^3-(-1)^2-(2+\sqrt{2})(-1) + \sqrt{2} = -1-1+2+\sqrt{2}+\sqrt{2} \\ &= 2\sqrt{2} \neq 0 \end{aligned}$$

∴By factor theorem,  $x+1$  is not a factor of  $x^3-x^2-(2+\sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether  $g(x)$  is a factor of  $p(x)$  in each of the following cases:

(i)  $p(x) = 2x^3 + x^2 - 2x - 1$ ,  $g(x) = x + 1$

Solution:

$$p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$$

$$g(x) = 0$$

$$\Rightarrow x + 1 = 0$$

$$\Rightarrow x = -1$$

$\therefore$  Zero of  $g(x)$  is  $-1$ .

Now,

$$\begin{aligned} p(-1) &= 2(-1)^3 + (-1)^2 - 2(-1) - 1 \\ &= -2 + 1 + 2 - 1 \\ &= 0 \end{aligned}$$

$\therefore$  By factor theorem,  $g(x)$  is a factor of  $p(x)$ .

(ii)  $p(x) = x^3 + 3x^2 + 3x + 1$ ,  $g(x) = x + 2$

Solution:

$$p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$$

$$g(x) = 0$$

$$\Rightarrow x + 2 = 0$$

$$\Rightarrow x = -2$$

$\therefore$  Zero of  $g(x)$  is  $-2$ .

Now,

$$\begin{aligned} p(-2) &= (-2)^3 + 3(-2)^2 + 3(-2) + 1 \\ &= -8 + 12 - 6 + 1 \\ &= -1 \neq 0 \end{aligned}$$

$\therefore$  By factor theorem,  $g(x)$  is not a factor of  $p(x)$ .

(iii)  $p(x) = x^3 - 4x^2 + x + 6$ ,  $g(x) = x - 3$

Solution:

$$p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$$

$$g(x) = 0$$

$$\Rightarrow x - 3 = 0$$

$$\Rightarrow x = 3$$

$\therefore$  Zero of  $g(x)$  is  $3$ .

Now,

$$\begin{aligned} p(3) &= (3)^3 - 4(3)^2 + (3) + 6 \\ &= 27 - 36 + 3 + 6 \\ &= 0 \end{aligned}$$

$\therefore$  By factor theorem,  $g(x)$  is a factor of  $p(x)$ .

3. Find the value of  $k$ , if  $x - 1$  is a factor of  $p(x)$  in each of the following cases:

(i)  $p(x) = x^2 + x + k$

**Solution:**

If  $x-1$  is a factor of  $p(x)$ , then  $p(1) = 0$

By Factor Theorem

$$\Rightarrow (1)^2 + (1) + k = 0$$

$$\Rightarrow 1 + 1 + k = 0$$

$$\Rightarrow 2 + k = 0$$

$$\Rightarrow k = -2$$

**(ii)  $p(x) = 2x^2 + kx + \sqrt{2}$**

**Solution:**

If  $x-1$  is a factor of  $p(x)$ , then  $p(1) = 0$

$$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$$

$$\Rightarrow 2 + k + \sqrt{2} = 0$$

$$\Rightarrow k = -(2 + \sqrt{2})$$

**(iii)  $p(x) = kx^2 - \sqrt{2}x + 1$**

**Solution:**

If  $x-1$  is a factor of  $p(x)$ , then  $p(1) = 0$

By Factor Theorem

$$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$$

$$\Rightarrow k = \sqrt{2} - 1$$

**(iv)  $p(x) = kx^2 - 3x + k$**

**Solution:**

If  $x-1$  is a factor of  $p(x)$ , then  $p(1) = 0$

By Factor Theorem

$$\Rightarrow k(1)^2 - 3(1) + k = 0$$

$$\Rightarrow k - 3 + k = 0$$

$$\Rightarrow 2k - 3 = 0$$

$$\Rightarrow k = 3/2$$

#### 4. Factorize:

**(i)  $12x^2 - 7x + 1$**

**Solution:**

Using the splitting the middle term method,

We have to find a number whose sum =  $-7$  and product =  $1 \times 12 = 12$

We get  $-3$  and  $-4$  as the numbers [ $-3 + -4 = -7$  and  $-3 \times -4 = 12$ ]

$$12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$$

$$= 4x(3x - 1) - 1(3x - 1)$$

$$= (4x - 1)(3x - 1)$$

**(ii)  $2x^2 + 7x + 3$**

**Solution:**

Using the splitting the middle term method,

We have to find a number whose sum = 7 and product =  $2 \times 3 = 6$

We get 6 and 1 as the numbers [6+1 = 7 and  $6 \times 1 = 6$ ]

$$\begin{aligned} 2x^2+7x+3 &= 2x^2+6x+1x+3 \\ &= 2x(x+3)+1(x+3) \\ &= (2x+1)(x+3) \end{aligned}$$

**(iii)  $6x^2+5x-6$**

**Solution:**

Using the splitting the middle term method,

We have to find a number whose sum = 5 and product =  $6 \times -6 = -36$

We get -4 and 9 as the numbers [-4+9 = 5 and  $-4 \times 9 = -36$ ]

$$\begin{aligned} 6x^2+5x-6 &= 6x^2+9x-4x-6 \\ &= 3x(2x+3)-2(2x+3) \\ &= (2x+3)(3x-2) \end{aligned}$$

**(iv)  $3x^2-x-4$**

**Solution:**

Using the splitting the middle term method,

We have to find a number whose sum = -1 and product =  $3 \times -4 = -12$

We get -4 and 3 as the numbers [-4+3 = -1 and  $-4 \times 3 = -12$ ]

$$\begin{aligned} 3x^2-x-4 &= 3x^2-x-4 \\ &= 3x^2-4x+3x-4 \\ &= x(3x-4)+1(3x-4) \\ &= (3x-4)(x+1) \end{aligned}$$

## 5. Factorize:

**(i)  $x^3-2x^2-x+2$**

**Solution:**

Let  $p(x) = x^3-2x^2-x+2$

Factors of 2 are  $\pm 1$  and  $\pm 2$

By trial method, we find that

$$p(1) = 0$$

So,  $(x+1)$  is factor of  $p(x)$

Now,

$$p(x) = x^3-2x^2-x+2$$

$$p(-1) = (-1)^3-2(-1)^2-(-1)+2$$

$$= -1-1+1+2$$

$$= 0$$

Therefore,  $(x+1)$  is the factor of  $p(x)$

$$\begin{array}{r}
 x^2 - 3x + 2 \\
 \hline
 x+1 \overline{) \begin{array}{r} x^3 - 2x^2 - x + 2 \\ x^3 + x^2 \\ \hline -3x^2 - x + 2 \\ -3x^2 - 3x \\ \hline 2x + 2 \\ 2x + 2 \\ \hline 0 \end{array} }
 \end{array}$$

Now, Dividend = Divisor  $\times$  Quotient + Remainder

$$\begin{aligned}
 (x+1)(x^2-3x+2) &= (x+1)(x^2-x-2x+2) \\
 &= (x+1)(x(x-1)-2(x-1)) \\
 &= (x+1)(x-1)(x+2)
 \end{aligned}$$

(ii)  $x^3-3x^2-9x-5$

**Solution:**

$$\text{Let } p(x) = x^3 - 3x^2 - 9x - 5$$

Factors of 5 are  $\pm 1$  and  $\pm 5$

By trial method, we find that

$$p(5) = 0$$

So,  $(x-5)$  is factor of  $p(x)$

Now,

$$p(x) = x^3 - 3x^2 - 9x - 5$$

$$p(5) = (5)^3 - 3(5)^2 - 9(5) - 5$$

$$= 125 - 75 - 45 - 5$$

$$= 0$$

Therefore,  $(x-5)$  is the factor of  $p(x)$

$$\begin{array}{r}
 x^2 + 2x + 1 \\
 \hline
 x-5 \overline{) \begin{array}{l} x^3 - 3x^2 - 9x - 5 \\ x^3 - 5x^2 \\ \hline - \quad + \\ \hline 2x^2 - 9x - 5 \\ 2x^2 - 10x \\ \hline - \quad + \\ \hline x - 5 \\ x - 5 \\ \hline - \quad + \\ \hline 0 \end{array} }
 \end{array}$$

Now, Dividend = Divisor  $\times$  Quotient + Remainder

$$\begin{aligned}
 (x-5)(x^2+2x+1) &= (x-5)(x^2+x+x+1) \\
 &= (x-5)(x(x+1)+1(x+1)) \\
 &= (x-5)(x+1)(x+1)
 \end{aligned}$$

**(iii)  $x^3+13x^2+32x+20$**

**Solution:**

Let  $p(x) = x^3+13x^2+32x+20$

Factors of 20 are  $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10$  and  $\pm 20$

By trial method, we find that

$p(-1) = 0$

So,  $(x+1)$  is factor of  $p(x)$

Now,

$p(x) = x^3+13x^2+32x+20$

$$\begin{aligned}
 p(-1) &= (-1)^3+13(-1)^2+32(-1)+20 \\
 &= -1+13-32+20 \\
 &= 0
 \end{aligned}$$

Therefore,  $(x+1)$  is the factor of  $p(x)$



$$\begin{array}{r}
 x^2 + 12x + 20 \\
 \hline
 x+1 \quad x^3 + 13x^2 + 32x + 20 \\
 \quad x^3 + x^2 \\
 \quad \hline
 \quad \quad 12x^2 + 32x + 20 \\
 \quad \quad 12x^2 + 12x \\
 \quad \quad \hline
 \quad \quad \quad 20x + 20 \\
 \quad \quad \quad 20x + 20 \\
 \quad \quad \quad \hline
 \quad \quad \quad \quad 0
 \end{array}$$

Now, Dividend = Divisor  $\times$  Quotient + Remainder

$$\begin{aligned}
 (x+1)(x^2+12x+20) &= (x+1)(x^2+2x+10x+20) \\
 &= (x-5)x(x+2)+10(x+2) \\
 &= (x-5)(x+2)(x+10)
 \end{aligned}$$

(iv)  $2y^3+y^2-2y-1$

**Solution:**

$$\text{Let } p(y) = 2y^3+y^2-2y-1$$

Factors =  $2 \times (-1) = -2$  are  $\pm 1$  and  $\pm 2$

By trial method, we find that

$$p(1) = 0$$

So,  $(y-1)$  is factor of  $p(y)$

Now,

$$p(y) = 2y^3+y^2-2y-1$$

$$p(1) = 2(1)^3+(1)^2-2(1)-1$$

$$= 2+1-2$$

$$= 0$$

Therefore,  $(y-1)$  is the factor of  $p(y)$

$$\begin{array}{r}
 2y^2 + 3y + 1 \\
 \hline
 y-1 \overline{) 2y^3 + y^2 - 2y - 1} \\
 \underline{2y^3 - 2y^2} \phantom{- 1} \\
 3y^2 - 2y - 1 \\
 \underline{3y^2 - 3y} \phantom{- 1} \\
 y - 1 \\
 \underline{y - 1} \\
 0
 \end{array}$$

Now, Dividend = Divisor  $\times$  Quotient + Remainder

$$\begin{aligned}
 (y-1)(2y^2+3y+1) &= (y-1)(2y^2+2y+y+1) \\
 &= (y-1)(2y(y+1)+1(y+1)) \\
 &= (y-1)(2y+1)(y+1)
 \end{aligned}$$

## Exercise 2.5

Page: 48

1. Use suitable identities to find the following products:

(i)  $(x+4)(x+10)$

**Solution:**

Using the identity,  $(x+a)(x+b) = x^2 + (a+b)x + ab$

[Here,  $a = 4$  and  $b = 10$ ]

We get,

$$\begin{aligned}(x+4)(x+10) &= x^2 + (4+10)x + (4 \times 10) \\ &= x^2 + 14x + 40\end{aligned}$$

(ii)  $(x+8)(x-10)$

**Solution:**

Using the identity,  $(x+a)(x+b) = x^2 + (a+b)x + ab$

[Here,  $a = 8$  and  $b = -10$ ]

We get,

$$\begin{aligned}(x+8)(x-10) &= x^2 + (8+(-10))x + (8 \times (-10)) \\ &= x^2 + (8-10)x - 80 \\ &= x^2 - 2x - 80\end{aligned}$$

(iii)  $(3x+4)(3x-5)$

**Solution:**

Using the identity,  $(x+a)(x+b) = x^2 + (a+b)x + ab$

[Here,  $x = 3x$ ,  $a = 4$  and  $b = -5$ ]

We get,

$$\begin{aligned}(3x+4)(3x-5) &= (3x)^2 + 4 + (-5)3x + 4 \times (-5) \\ &= 9x^2 + 3x(4-5) - 20 \\ &= 9x^2 - 3x - 20\end{aligned}$$

(iv)  $(y^2+3/2)(y^2-3/2)$

**Solution:**

Using the identity,  $(x+y)(x-y) = x^2 - y^2$

[Here,  $x = y^2$  and  $y = 3/2$ ]

We get,

$$\begin{aligned}(y^2+3/2)(y^2-3/2) &= (y^2)^2 - (3/2)^2 \\ &= y^4 - 9/4\end{aligned}$$

2. Evaluate the following products without multiplying directly:

(i)  $103 \times 107$

**Solution:**

$$103 \times 107 = (100+3) \times (100+7)$$

Using identity,  $[(x+a)(x+b) = x^2 + (a+b)x + ab$

Here,  $x = 100$

$$a = 3$$

$$b = 7$$

$$\begin{aligned}\text{We get, } 103 \times 107 &= (100+3) \times (100+7) \\ &= (100)^2 + (3+7)100 + (3 \times 7) \\ &= 10000 + 1000 + 21 \\ &= 11021\end{aligned}$$

**(ii)  $95 \times 96$** 

**Solution:**

$$95 \times 96 = (100-5) \times (100-4)$$

Using identity,  $[(x-a)(x-b) = x^2 - (a+b)x + ab]$

Here,  $x = 100$

$$a = -5$$

$$b = -4$$

$$\begin{aligned}\text{We get, } 95 \times 96 &= (100-5) \times (100-4) \\ &= (100)^2 + 100(-5+(-4)) + (-5 \times -4) \\ &= 10000 - 900 + 20 \\ &= 9120\end{aligned}$$

**(iii)  $104 \times 96$** 

**Solution:**

$$104 \times 96 = (100+4) \times (100-4)$$

Using identity,  $[(a+b)(a-b) = a^2 - b^2]$

Here,  $a = 100$

$$b = 4$$

$$\begin{aligned}\text{We get, } 104 \times 96 &= (100+4) \times (100-4) \\ &= (100)^2 - (4)^2 \\ &= 10000 - 16 \\ &= 9984\end{aligned}$$

**3. Factorize the following using appropriate identities:****(i)  $9x^2 + 6xy + y^2$** 

**Solution:**

$$9x^2 + 6xy + y^2 = (3x)^2 + (2 \times 3x \times y) + y^2$$

Using identity,  $x^2 + 2xy + y^2 = (x+y)^2$

Here,  $x = 3x$

$$y = y$$

$$\begin{aligned}9x^2 + 6xy + y^2 &= (3x)^2 + (2 \times 3x \times y) + y^2 \\ &= (3x+y)^2 \\ &= (3x+y)(3x+y)\end{aligned}$$

(ii)  $4y^2 - 4y + 1$

Solution:

$$4y^2 - 4y + 1 = (2y)^2 - (2 \times 2y \times 1) + 1^2$$

Using identity,  $x^2 - 2xy + y^2 = (x - y)^2$

Here,  $x = 2y$

$y = 1$

$$\begin{aligned} 4y^2 - 4y + 1 &= (2y)^2 - (2 \times 2y \times 1) + 1^2 \\ &= (2y - 1)^2 \\ &= (2y - 1)(2y - 1) \end{aligned}$$

(iii)  $x^2 - y^2/100$

Solution:

$$x^2 - y^2/100 = x^2 - (y/10)^2$$

Using identity,  $x^2 - y^2 = (x - y)(x + y)$

Here,  $x = x$

$y = y/10$

$$\begin{aligned} x^2 - y^2/100 &= x^2 - (y/10)^2 \\ &= (x - y/10)(x + y/10) \end{aligned}$$

**4. Expand each of the following, using suitable identities:**

(i)  $(x + 2y + 4z)^2$

(ii)  $(2x - y + z)^2$

(iii)  $(-2x + 3y + 2z)^2$

(iv)  $(3a - 7b - c)^2$

(v)  $(-2x + 5y - 3z)^2$

(vi)  $((1/4)a - (1/2)b + 1)^2$

Solution:

(i)  $(x + 2y + 4z)^2$

Using identity,  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here,  $x = x$

$y = 2y$

$z = 4z$

$$\begin{aligned} (x + 2y + 4z)^2 &= x^2 + (2y)^2 + (4z)^2 + (2 \times x \times 2y) + (2 \times 2y \times 4z) + (2 \times 4z \times x) \\ &= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz \end{aligned}$$

(ii)  $(2x - y + z)^2$

Using identity,  $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Here,  $x = 2x$

$y = -y$

$z = z$

$$\begin{aligned}(2x-y+z)^2 &= (2x)^2+(-y)^2+z^2+(2 \times 2x \times -y)+(2 \times -y \times z)+(2 \times z \times 2x) \\ &= 4x^2+y^2+z^2-4xy-2yz+4xz\end{aligned}$$

**(iii)  $(-2x+3y+2z)^2$**

**Solution:**

Using identity,  $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here,  $x = -2x$

$y = 3y$

$z = 2z$

$$\begin{aligned}(-2x+3y+2z)^2 &= (-2x)^2+(3y)^2+(2z)^2+(2 \times -2x \times 3y)+(2 \times 3y \times 2z)+(2 \times 2z \times -2x) \\ &= 4x^2+9y^2+4z^2-12xy+12yz-8xz\end{aligned}$$

**(iv)  $(3a-7b-c)^2$**

**Solution:**

Using identity  $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here,  $x = 3a$

$y = -7b$

$z = -c$

$$\begin{aligned}(3a-7b-c)^2 &= (3a)^2+(-7b)^2+(-c)^2+(2 \times 3a \times -7b)+(2 \times -7b \times -c)+(2 \times -c \times 3a) \\ &= 9a^2+49b^2+c^2-42ab+14bc-6ca\end{aligned}$$

**(v)  $(-2x+5y-3z)^2$**

**Solution:**

Using identity,  $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here,  $x = -2x$

$y = 5y$

$z = -3z$

$$\begin{aligned}(-2x+5y-3z)^2 &= (-2x)^2+(5y)^2+(-3z)^2+(2 \times -2x \times 5y)+(2 \times 5y \times -3z)+(2 \times -3z \times -2x) \\ &= 4x^2+25y^2+9z^2-20xy-30yz+12zx\end{aligned}$$

**(vi)  $((1/4)a-(1/2)b+1)^2$**

**Solution:**

Using identity,  $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here,  $x = (1/4)a$

$y = (-1/2)b$

$z = 1$

$$\begin{aligned} ((1/4)a - (1/2)b + 1)^2 &= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + \left(2 \times \frac{1}{4}a \times -\frac{1}{2}b\right) + \left(2 \times -\frac{1}{2}b \times 1\right) + \left(2 \times 1 \times \frac{1}{4}a\right) \\ &= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1^2 - \frac{2}{8}ab - \frac{2}{2}b + \frac{2}{4}a \\ &= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a \end{aligned}$$

### 5. Factorize:

(i)  $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii)  $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

**Solution:**

(i)  $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

Using identity,  $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

We can say that,  $x^2+y^2+z^2+2xy+2yz+2zx = (x+y+z)^2$

$$\begin{aligned} 4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz &= (2x)^2 + (3y)^2 + (-4z)^2 + (2 \times 2x \times 3y) + (2 \times 3y \times -4z) + (2 \times -4z \times 2x) \\ &= (2x + 3y - 4z)^2 \\ &= (2x + 3y - 4z)(2x + 3y - 4z) \end{aligned}$$

(iii)  $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Using identity,  $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

We can say that,  $x^2+y^2+z^2+2xy+2yz+2zx = (x+y+z)^2$

$$\begin{aligned} 2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz &= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + (2 \times -\sqrt{2}x \times y) + (2 \times y \times 2\sqrt{2}z) + (2 \times 2\sqrt{2}z \times -\sqrt{2}x) \\ &= (-\sqrt{2}x + y + 2\sqrt{2}z)^2 \\ &= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z) \end{aligned}$$

### 6. Write the following cubes in expanded form:

(i)  $(2x+1)^3$

(ii)  $(2a-3b)^3$

(iii)  $\left(\frac{3}{2}x+1\right)^3$

(iv)  $\left(x-\frac{2}{3}y\right)^3$

**Solution:**

(i)  $(2x+1)^3$

Using identity,  $(x+y)^3 = x^3+y^3+3xy(x+y)$   
 $(2x+1)^3 = (2x)^3+1^3+(3 \times 2x \times 1)(2x+1)$   
 $= 8x^3+1+6x(2x+1)$   
 $= 8x^3+12x^2+6x+1$

(ii)  $(2a-3b)^3$

Using identity,  $(x-y)^3 = x^3-y^3-3xy(x-y)$   
 $(2a-3b)^3 = (2a)^3-(3b)^3-(3 \times 2a \times 3b)(2a-3b)$   
 $= 8a^3-27b^3-18ab(2a-3b)$   
 $= 8a^3-27b^3-36a^2b+54ab^2$

(iii)  $((3/2)x+1)^3$

Using identity,  $(x+y)^3 = x^3+y^3+3xy(x+y)$   
 $((3/2)x+1)^3 = ((3/2)x)^3+1^3+(3 \times (3/2)x \times 1)((3/2)x + 1)$   
 $= \frac{27}{8}x^3+1+\frac{9}{2}x(\frac{3}{2}x+1)$   
 $= \frac{27}{8}x^3+1+\frac{27}{4}x^2+\frac{9}{2}x$   
 $= \frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$

(iv)  $(x-(2/3)y)^3$

Using identity,  $(x-y)^3 = x^3-y^3-3xy(x-y)$   
 $(x-\frac{2}{3}y)^3 = (x)^3-(\frac{2}{3}y)^3-(3 \times x \times \frac{2}{3}y)(x-\frac{2}{3}y)$   
 $= (x)^3-\frac{8}{27}y^3-2xy(x-\frac{2}{3}y)$   
 $= (x)^3-\frac{8}{27}y^3-2x^2y+\frac{4}{3}xy^2$

7. Evaluate the following using suitable identities:

(i)  $(99)^3$

(ii)  $(102)^3$

(iii)  $(998)^3$

Solutions:

(i)  $(99)^3$

Solution:

We can write 99 as 100-1  
Using identity,  $(x-y)^3 = x^3-y^3-3xy(x-y)$   
 $(99)^3 = (100-1)^3$   
 $= (100)^3-1^3-(3 \times 100 \times 1)(100-1)$   
 $= 1000000-1-300(100-1)$   
 $= 1000000-1-30000+300$   
 $= 970299$



(ii)  $(102)^3$

**Solution:**

We can write 102 as  $100+2$

Using identity,  $(x+y)^3 = x^3+y^3+3xy(x+y)$

$$\begin{aligned} (100+2)^3 &= (100)^3+2^3+(3 \times 100 \times 2)(100+2) \\ &= 1000000 + 8 + 600(100 + 2) \\ &= 1000000 + 8 + 60000 + 1200 \\ &= 1061208 \end{aligned}$$

(iii)  $(998)^3$

**Solution:**

We can write 99 as  $1000-2$

Using identity,  $(x-y)^3 = x^3-y^3-3xy(x-y)$

$$\begin{aligned} (998)^3 &= (1000-2)^3 \\ &= (1000)^3-2^3-(3 \times 1000 \times 2)(1000-2) \\ &= 1000000000-8-6000(1000-2) \\ &= 1000000000-8-6000000+12000 \\ &= 994011992 \end{aligned}$$

**8. Factorise each of the following:**

(i)  $8a^3+b^3+12a^2b+6ab^2$

(ii)  $8a^3-b^3-12a^2b+6ab^2$

(iii)  $27-125a^3-135a+225a^2$

(iv)  $64a^3-27b^3-144a^2b+108ab^2$

(v)  $27p^3-(1/216)-(9/2)p^2+(1/4)p$

**Solutions:**

(i)  $8a^3+b^3+12a^2b+6ab^2$

**Solution:**

The expression,  $8a^3+b^3+12a^2b+6ab^2$  can be written as  $(2a)^3+b^3+3(2a)^2b+3(2a)(b)^2$

$$\begin{aligned} 8a^3+b^3+12a^2b+6ab^2 &= (2a)^3+b^3+3(2a)^2b+3(2a)(b)^2 \\ &= (2a+b)^3 \\ &= (2a+b)(2a+b)(2a+b) \end{aligned}$$

Here, the identity,  $(x+y)^3 = x^3+y^3+3xy(x+y)$  is used.

(ii)  $8a^3-b^3-12a^2b+6ab^2$

**Solution:**

The expression,  $8a^3-b^3-12a^2b+6ab^2$  can be written as  $(2a)^3-b^3-3(2a)^2b+3(2a)(b)^2$

$$\begin{aligned} 8a^3-b^3-12a^2b+6ab^2 &= (2a)^3-b^3-3(2a)^2b+3(2a)(b)^2 \\ &= (2a-b)^3 \\ &= (2a-b)(2a-b)(2a-b) \end{aligned}$$

Here, the identity,  $(x-y)^3 = x^3-y^3-3xy(x-y)$  is used.

(iii)  $27-125a^3-135a+225a^2$

**Solution:**

The expression,  $27-125a^3-135a+225a^2$  can be written as  $3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$

$$\begin{aligned} 27-125a^3-135a+225a^2 &= 3^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2 \\ &= (3-5a)^3 \\ &= (3-5a)(3-5a)(3-5a) \end{aligned}$$

Here, the identity,  $(x-y)^3 = x^3-y^3-3xy(x-y)$  is used.

(iv)  $64a^3-27b^3-144a^2b+108ab^2$

**Solution:**

The expression,  $64a^3-27b^3-144a^2b+108ab^2$  can be written as  $(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$

$$\begin{aligned} 64a^3-27b^3-144a^2b+108ab^2 &= (4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2 \\ &= (4a-3b)^3 \\ &= (4a-3b)(4a-3b)(4a-3b) \end{aligned}$$

Here, the identity,  $(x-y)^3 = x^3-y^3-3xy(x-y)$  is used.

(v)  $7p^3-(1/216)-(9/2)p^2+(1/4)p$

**Solution:**

The expression,  $27p^3-(1/216)-(9/2)p^2+(1/4)p$

can be written as  $(3p)^3-(1/6)^3-3(3p)^2(1/6)+3(3p)(1/6)^2$

$$\begin{aligned} 27p^3-(1/216)-(9/2)p^2+(1/4)p &= (3p)^3-(1/6)^3-3(3p)^2(1/6)+3(3p)(1/6)^2 \\ &= (3p-1/6)^3 \\ &= (3p-1/6)(3p-1/6)(3p-1/6) \end{aligned}$$

## 9. Verify:

(i)  $x^3+y^3 = (x+y)(x^2-xy+y^2)$

(ii)  $x^3-y^3 = (x-y)(x^2+xy+y^2)$

**Solutions:**

(i)  $x^3+y^3 = (x+y)(x^2-xy+y^2)$

We know that,  $(x+y)^3 = x^3+y^3+3xy(x+y)$

$$\Rightarrow x^3+y^3 = (x+y)^3-3xy(x+y)$$

$$\Rightarrow x^3+y^3 = (x+y)[(x+y)^2-3xy]$$

Taking  $(x+y)$  common  $\Rightarrow x^3+y^3 = (x+y)[(x^2+y^2+2xy)-3xy]$

$$\Rightarrow x^3+y^3 = (x+y)(x^2+y^2-xy)$$

(ii)  $x^3-y^3 = (x-y)(x^2+xy+y^2)$

We know that,  $(x-y)^3 = x^3-y^3-3xy(x-y)$

$$\Rightarrow x^3-y^3 = (x-y)^3+3xy(x-y)$$

$$\Rightarrow x^3-y^3 = (x-y)[(x-y)^2+3xy]$$

Taking  $(x-y)$  common  $\Rightarrow x^3-y^3 = (x-y)[(x^2+y^2-2xy)+3xy]$

$$\Rightarrow x^3-y^3 = (x-y)(x^2+y^2+xy)$$

**10. Factorize each of the following:**

(i)  $27y^3+125z^3$

(ii)  $64m^3-343n^3$

**Solutions:**

(i)  $27y^3+125z^3$

The expression,  $27y^3+125z^3$  can be written as  $(3y)^3+(5z)^3$

$$27y^3+125z^3 = (3y)^3+(5z)^3$$

We know that,  $x^3+y^3 = (x+y)(x^2-xy+y^2)$

$$\begin{aligned} \therefore 27y^3+125z^3 &= (3y)^3+(5z)^3 \\ &= (3y+5z)[(3y)^2-(3y)(5z)+(5z)^2] \\ &= (3y+5z)(9y^2-15yz+25z^2) \end{aligned}$$

(ii)  $64m^3-343n^3$

The expression,  $64m^3-343n^3$  can be written as  $(4m)^3-(7n)^3$

$$64m^3-343n^3 = (4m)^3-(7n)^3$$

We know that,  $x^3-y^3 = (x-y)(x^2+xy+y^2)$

$$\begin{aligned} \therefore 64m^3-343n^3 &= (4m)^3-(7n)^3 \\ &= (4m-7n)[(4m)^2+(4m)(7n)+(7n)^2] \\ &= (4m-7n)(16m^2+28mn+49n^2) \end{aligned}$$

**11. Factorise:  $27x^3+y^3+z^3-9xyz$**

**Solution:**

The expression  $27x^3+y^3+z^3-9xyz$  can be written as  $(3x)^3+y^3+z^3-3(3x)(y)(z)$

$$27x^3+y^3+z^3-9xyz = (3x)^3+y^3+z^3-3(3x)(y)(z)$$

We know that,  $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$$\begin{aligned} \therefore 27x^3+y^3+z^3-9xyz &= (3x)^3+y^3+z^3-3(3x)(y)(z) \\ &= (3x+y+z)(3x)^2+y^2+z^2-3xy-yz-3xz \\ &= (3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz) \end{aligned}$$

**12. Verify that:**

$$x^3+y^3+z^3-3xyz = (1/2)(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$$

**Solution:**

We know that,

$$\begin{aligned} x^3+y^3+z^3-3xyz &= (x+y+z)(x^2+y^2+z^2-xy-yz-zx) \\ \Rightarrow x^3+y^3+z^3-3xyz &= (1/2)(x+y+z)[2(x^2+y^2+z^2-xy-yz-zx)] \\ &= (1/2)(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2xz) \\ &= (1/2)(x+y+z)[(x^2+y^2-2xy)+(y^2+z^2-2yz)+(x^2+z^2-2xz)] \\ &= (1/2)(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2] \end{aligned}$$

**13. If  $x+y+z = 0$ , show that  $x^3+y^3+z^3 = 3xyz$ .**

**Solution:**

We know that,

$$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-xz)$$

Now, according to the question, let  $(x+y+z) = 0$ ,

then,  $x^3+y^3+z^3-3xyz = (0)(x^2+y^2+z^2-xy-yz-xz)$

$$\Rightarrow x^3+y^3+z^3-3xyz = 0$$

$$\Rightarrow x^3+y^3+z^3 = 3xyz$$

Hence Proved

**14. Without actually calculating the cubes, find the value of each of the following:**

(i)  $(-12)^3+(7)^3+(5)^3$

(ii)  $(28)^3+(-15)^3+(-13)^3$

(i)  $(-12)^3+(7)^3+(5)^3$

**Solution:**

$$(-12)^3+(7)^3+(5)^3$$

Let  $a = -12$

$$b = 7$$

$$c = 5$$

We know that if  $x+y+z = 0$ , then  $x^3+y^3+z^3=3xyz$ .

Here,  $-12+7+5=0$

$$\begin{aligned} \therefore (-12)^3+(7)^3+(5)^3 &= 3xyz \\ &= 3 \times -12 \times 7 \times 5 \\ &= -1260 \end{aligned}$$

(ii)  $(28)^3+(-15)^3+(-13)^3$

**Solution:**

$$(28)^3+(-15)^3+(-13)^3$$

Let  $a = 28$

$$b = -15$$

$$c = -13$$

We know that if  $x+y+z = 0$ , then  $x^3+y^3+z^3 = 3xyz$ .

Here,  $x+y+z = 28-15-13 = 0$

$$\begin{aligned} \therefore (28)^3+(-15)^3+(-13)^3 &= 3xyz \\ &= 0+3(28)(-15)(-13) \\ &= 16380 \end{aligned}$$

**15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:**

(i) Area :  $25a^2-35a+12$

(ii) Area :  $35y^2+13y-12$

**Solution:**

(i) Area :  $25a^2-35a+12$

Using the splitting the middle term method,

We have to find a number whose sum = -35 and product =  $25 \times 12 = 300$

We get -15 and -20 as the numbers [-15 + -20 = -35 and -15 × -20 = 300]

$$\begin{aligned} 25a^2-35a+12 &= 25a^2-15a-20a+12 \\ &= 5a(5a-3)-4(5a-3) \\ &= (5a-4)(5a-3) \end{aligned}$$

Possible expression for length =  $5a-4$

Possible expression for breadth =  $5a-3$

(ii) Area :  $35y^2+13y-12$

Using the splitting the middle term method,

We have to find a number whose sum = 13 and product =  $35 \times -12 = 420$

We get -15 and 28 as the numbers [-15 + 28 = 13 and -15 × 28 = 420]

$$\begin{aligned} 35y^2+13y-12 &= 35y^2-15y+28y-12 \\ &= 5y(7y-3)+4(7y-3) \\ &= (5y+4)(7y-3) \end{aligned}$$

Possible expression for length =  $(5y+4)$

Possible expression for breadth =  $(7y-3)$

**16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?**

(i) Volume :  $3x^2-12x$

(ii) Volume :  $12ky^2+8ky-20k$

**Solution:**

(i) Volume :  $3x^2-12x$

$3x^2-12x$  can be written as  $3x(x-4)$  by taking  $3x$  out of both the terms.

Possible expression for length = 3

Possible expression for breadth =  $x$

Possible expression for height =  $(x-4)$

(ii) Volume:  $12ky^2+8ky-20k$

$12ky^2+8ky-20k$  can be written as  $4k(3y^2+2y-5)$  by taking  $4k$  out of both the terms.

$$12ky^2+8ky-20k = 4k(3y^2+2y-5)$$

[Here,  $3y^2+2y-5$  can be written as  $3y^2+5y-3y-5$  using splitting the middle term method.]

$$= 4k(3y^2+5y-3y-5)$$

$$= 4k[y(3y+5)-1(3y+5)]$$

$$= 4k(3y+5)(y-1)$$

Possible expression for length =  $4k$

Possible expression for breadth =  $(3y + 5)$

Possible expression for height =  $(y - 1)$