

Exercise: 6.3

(Page No: 107)

1. In Fig. 6.39, sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$, find $\angle PRQ$.

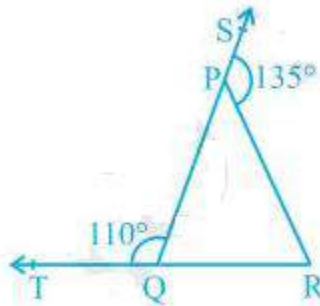


Fig. 6.39

Solution:

It is given the TQR is a straight line and so, the linear pairs (i.e. $\angle TQP$ and $\angle PQR$) will add up to 180°

So, $\angle TQP + \angle PQR = 180^\circ$

Now, putting the value of $\angle TQP = 110^\circ$ we get,

$$\angle PQR = 70^\circ$$

Consider the ΔPQR ,

Here, the side QP is extended to S and so, $\angle SPR$ forms the exterior angle.

Thus, $\angle SPR$ ($\angle SPR = 135^\circ$) is equal to the sum of interior opposite angles. (Triangle property)

$$\text{Or, } \angle PQR + \angle PRQ = 135^\circ$$

Now, putting the value of $\angle PQR = 70^\circ$ we get,

$$\angle PRQ = 135^\circ - 70^\circ$$

Hence, $\angle PRQ = 65^\circ$

2. In Fig. 6.40, $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ , find $\angle OZY$ and $\angle YOZ$.

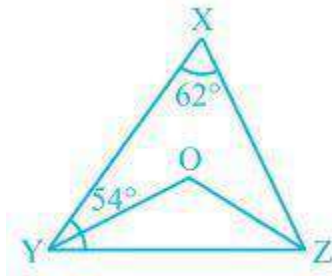


Fig. 6.40

Solution:

We know that the sum of the interior angles of the triangle.

So, $\angle X + \angle XYZ + \angle XZY = 180^\circ$

Putting the values as given in the question we get,

$62^\circ + 54^\circ + \angle XZY = 180^\circ$

Or, $\angle XZY = 64^\circ$

Now, we know that ZO is the bisector so,

$\angle OZY = \frac{1}{2} \angle XZY$

$\therefore \angle OZY = 32^\circ$

Similarly, YO is a bisector and so,

$\angle OYZ = \frac{1}{2} \angle XYZ$

Or, $\angle OYZ = 27^\circ$ (As $\angle XYZ = 54^\circ$)

Now, as the sum of the interior angles of the triangle,

$\angle OZY + \angle OYZ + \angle O = 180^\circ$

Putting their respective values, we get,

$\angle O = 180^\circ - 32^\circ - 27^\circ$

Hence, $\angle O = 121^\circ$

3. In Fig. 6.41, if $AB \parallel DE$, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, find $\angle DCE$.

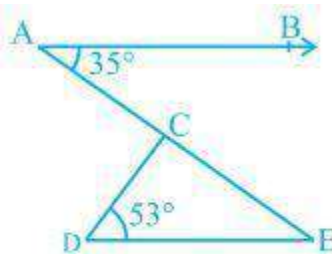


Fig. 6.41

Solution:

We know that

AE is a transversal since $AB \parallel DE$

Here $\angle BAC$ and $\angle AED$ are alternate interior angles.

Hence, $\angle BAC = \angle AED$

It is given that $\angle BAC = 35^\circ$

$\Rightarrow \angle AED = 35^\circ$

Now consider the triangle CDE. We know that the sum of the interior angles of a triangle is 180° .

$\therefore \angle DCE + \angle CED + \angle CDE = 180^\circ$

Putting the values, we get

$\angle DCE + 35^\circ + 53^\circ = 180^\circ$

Hence, $\angle DCE = 92^\circ$

4. In Fig. 6.42, if lines PQ and RS intersect at point T, such that $\angle PRT = 40^\circ$, $\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$, find $\angle SQT$.

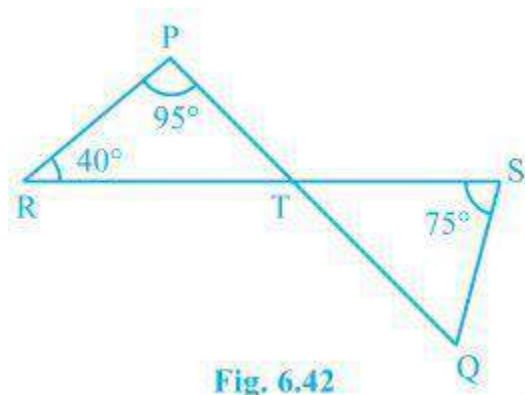


Fig. 6.42

Solution:

Consider triangle PRT.

$\angle PRT + \angle RPT + \angle PTR = 180^\circ$

So, $\angle PTR = 45^\circ$

Now $\angle PTR$ will be equal to $\angle STQ$ as they are vertically opposite angles.

So, $\angle PTR = \angle STQ = 45^\circ$

Again, in triangle STQ,

$\angle TSQ + \angle PTR + \angle SQT = 180^\circ$

Solving this we get,

$\angle SQT = 60^\circ$

5. In Fig. 6.43, if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y .

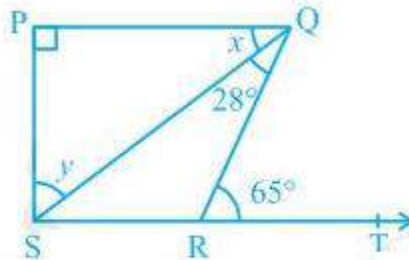


Fig. 6.43

Solution:

$$x + \angle SQR = \angle QRT \quad (\text{As they are alternate angles since } QR \text{ is transversal})$$

$$\text{So, } x + 28^\circ = 65^\circ$$

$$\therefore x = 37^\circ$$

It is also known that alternate interior angles are same and so,

$$\angle QSR = x = 37^\circ$$

Also, Now,

$$\angle QRS + \angle QRT = 180^\circ \quad (\text{As they are a Linear pair})$$

$$\text{Or, } \angle QRS + 65^\circ = 180^\circ$$

$$\text{So, } \angle QRS = 115^\circ$$

Now, we know that the sum of the angles in a quadrilateral is 360° . So,

$$\angle P + \angle Q + \angle R + \angle S = 360^\circ$$

Putting their respective values, we get,

$$\angle S = 360^\circ - 90^\circ - 65^\circ - 115^\circ$$

$$\text{Or, } \angle QS + y = 360^\circ$$

$$\Rightarrow y = 360^\circ - 90^\circ - 65^\circ - 115^\circ - 37^\circ$$

$$\text{Hence, } y = 53^\circ$$

6. In Fig. 6.44, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2} \angle QPR$.

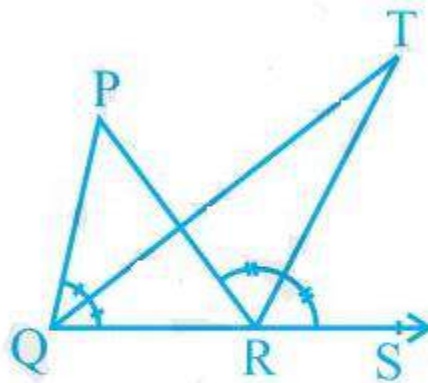


Fig. 6.44

Solution:

Consider the ΔPQR . $\angle PRS$ is the exterior angle and $\angle QPR$ and $\angle PQR$ are interior angles.

So, $\angle PRS = \angle QPR + \angle PQR$ (According to triangle property)

Or, $\angle PRS - \angle PQR = \angle QPR$ -----(i)

Now, consider the ΔQRT ,

$\angle TRS = \angle TQR + \angle QTR$

Or, $\angle QTR = \angle TRS - \angle TQR$

We know that QT and RT bisect $\angle PQR$ and $\angle PRS$ respectively.

So, $\angle PRS = 2 \angle TRS$ and $\angle PQR = 2 \angle TQR$

Now, $\angle QTR = \frac{1}{2} \angle PRS - \frac{1}{2} \angle PQR$

Or, $\angle QTR = \frac{1}{2} (\angle PRS - \angle PQR)$

From (i) we know that $\angle PRS - \angle PQR = \angle QPR$

So, $\angle QTR = \frac{1}{2} \angle QPR$ (hence proved).