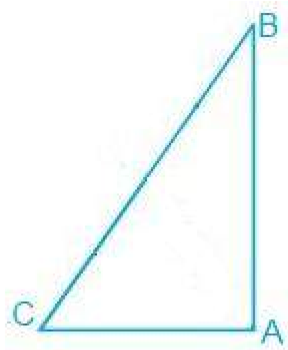
Exercise: 7.4 **(Page No: 132)**

1. Show that in a right-angled triangle, the hypotenuse is the longest side.



Solution:

It is known that ABC is a triangle right angled at B.

We know that,

$\angle A + \angle B + \angle C = 180°$

Now, if $\angle B + \angle C = 90°$ then $\angle A$ has to be 90°.

Since A is the largest angle of the triangle, the side opposite to it must be the largest.

So, AB is the hypotenuse which will be the largest side of the above right-angled triangle i.e. ΔABC.

2. In Fig. 7.48, sides AB and AC of ΔABC are extended to points P and Q respectively. Also, ∠PBC < ∠QCB. Show that AC > AB.

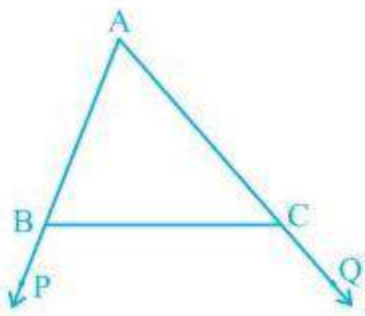


Fig. 7.48

Solution:

It is given that $\angle PBC < \angle QCB$

We know that $\angle ABC + \angle PBC = 180°$

So, $\angle ABC = 180° - \angle PBC$

Also,

$\angle ACB + \angle QCB = 180°$

Therefore $\angle ACB = 180° - \angle QCB$

Now, since $\angle PBC < \angle QCB$,

$\therefore \angle ABC > \angle ACB$

Hence, AC > AB as sides opposite to the larger angle is always larger.

3. In Fig. 7.49, $\angle B < \angle A$ and $\angle C < \angle D$. Show that AD < BC.

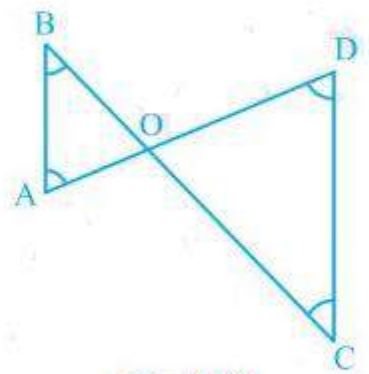


Fig. 7.49

Solution:

In the question, it is mentioned that angles B and angle C is smaller than angles A and D respectively i.e. $\angle B < \angle A$ and $\angle C < \angle D$.

Now,

Since the side opposite to the smaller angle is always smaller

AO < BO --- (i)

And OD < OC ---(ii)

By adding equation (i) and equation (ii) we get

AO+OD < BO + OC

So, AD < BC

4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig. 7.50).

Show that $\angle A > \angle C$ and $\angle B > \angle D$.

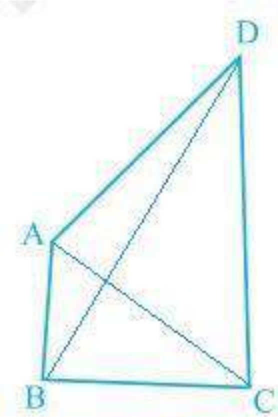


Fig. 7.50

Solution:

In ΔABD, we see that

AB < AD < BD

So, ∠ADB < ∠ABD --- (i) (Since angle opposite to longer side is always larger)

Now, in ΔBCD,

BC < DC < BD

Hence, it can be concluded that

∠BDC < ∠CBD --- (ii)

Now, by adding equation (i) and equation (ii) we get,

∠ADB + ∠BDC < ∠ABD + ∠CBD

⇒ ∠ADC < ∠ABC

⇒ ∠B > ∠D

Similarly, In triangle ABC,

∠ACB < ∠BAC --- (iii) (Since the angle opposite to the longer side is always larger)

Now, In ΔADC,

∠DCA < ∠DAC --- (iv)

By adding equation (iii) and equation (iv) we get,

∠ACB + ∠DCA < ∠BAC+∠DAC

⇒ ∠BCD < ∠BAD

∴ ∠A > ∠C

5. In Fig 7.51, PR > PQ and PS bisect ∠QPR. Prove that ∠PSR > ∠PSQ.

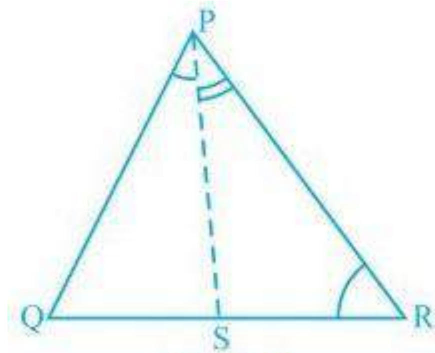


Fig. 7.51

Solution:

It is given that PR > PQ and PS bisects ∠QPR

Now we will have to prove that angle PSR is smaller than PSQ i.e. ∠PSR > ∠PSQ

Proof:

∠QPS = ∠RPS --- (ii) (As PS bisects ∠QPR)

∠PQR > ∠PRQ --- (i) (Since PR > PQ as angle opposite to the larger side is always larger)

∠PSR = ∠PQR + ∠QPS --- (iii) (Since the exterior angle of a triangle equals to the sum of opposite interior angles)

∠PSQ = ∠PRQ + ∠RPS --- (iv) (As the exterior angle of a triangle equals to the sum of opposite interior angles)

By adding (i) and (ii)

PQR +∠QPS > ∠PRQ +∠RPS

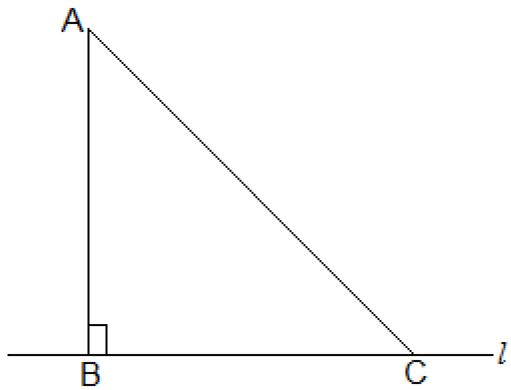
Thus, from (i), (ii), (iii) and (iv), we get

∠PSR > ∠PSQ

6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Solution:

First, let "l" be a line segment and "B" be a point lying on it. A line AB perpendicular to l is now drawn. Also, let C be any other point on l. The diagram will be as follows:



To prove:

AB < AC

Proof:

In ΔABC, ∠B = 90°

Now, we know that

∠A+∠B+∠C = 180°

∴ ∠A +∠C = 90°

Hence, ∠C must be an acute angle which implies ∠C < ∠B

So, AB < AC (As the side opposite to the larger angle is always larger)