Q.1: Answer the following :
(a) You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?
(b) An astronaut inside a small space ship orbiting around the earth cannot detect gravity. If the space station orbiting around the earth has a large size, can he hope to detect gravity?
(c) If you compare the gravitational force on the earth due to the sun to that due to the moon, you would find that the Sun's pull is greater than the moon's pull. (you can check this yourself using the data available in the succeeding exercises). However, the tidal effect of the moon's pull is greater than the tidal effect of sun. Why?

Sol:

(a). No, as of now, no method has been devised to shield a body from gravity because gravity is independent of medium and it is the virtue of each and every matter. So the shield would exert the gravitational forces.

(b). Yes, if the spaceship is large enough then the astronaut will definitely detect the Mars gravity.

(c). Gravitational force is inversely proportional to the square of the distance whereas, Tidal effects are inversely proportional to the cube of the distance. So as the distance between the earth and moon is smaller than the distance between earth and sun, the moon will have a greater influence on the earth's tidal waves.

Q2. Choose the correct alternative :
(a) Acceleration due to gravity increases/decreases with increasing altitude.
(b) Acceleration due to gravity increases/decreases with increasing depth (assume the earth to be a sphere of uniform density).
(c) Acceleration due to gravity is independent of mass of the earth/mass of the body.
(d) The formula –G M m(1/r 2 – 1/r1) is more/less accurate than the formula mg(r2 – r1) for the difference of potential energy between two points r2 and r1 distance away from the centre of the earth.

Sol:

(a). body.

(b). more.

(c). decreases.

(d). increases.

Q.3: Suppose there existed a planet that went around the Sun twice as fast as the earth. What would be its orbital size as compared to that of the earth?

Sol:

Time taken by the earth for one complete revolution, T_E = 1 Year

Radius of Earth's orbit, R_E = 1 AU

Thus, the time taken by the planet to complete one complete revolution:

$T_P = \frac{1}{2} T_E = \frac{1}{2}$ year

Let, the orbital radius of this planet = R_P

Now, according to the Kepler's third law of planetary motion:

$$\left(\frac{R_P}{R_E}\right)^3 = \left(\frac{T_P}{T_E}\right)^2$$

$$\frac{R_P}{R_E} = \left(\frac{T_P}{T_E}\right)^{\frac{2}{3}}$$

$$\frac{R_P}{R_E} = \left(\frac{T_P}{T_E}\right)^{\frac{2}{3}} = \left(\frac{\frac{1}{2}T_E}{T_E}\right)^{\frac{2}{3}} = \left(\frac{1}{2}\right)^{\frac{2}{3}} = 0.63$$

Therefore, radius of orbit of this planet is 0.63 times smaller than the radius of orbit of the Earth.

Q.4: Io, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. Show that the mass of Jupiter is about one-thousandth that of the sun.

Sol:

Given,

Orbital period of Io, T_{IO} = 1.769 days = 1.769 × 24 × 60 × 60 s

Orbital radius of Io, $R_{IO} = 4.22 \times 10^8$ m

We know mass of Jupiter:

$M_J = 4\pi^2 R_{IO}^3 / G T_{IO}^2$ .... (1)

Where;

M_J = Mass of Jupiter

G = Universal gravitational constant

Also,

Orbital period of the earth,

T_E = 365.25 days = 365.25 × 24 × 60 × 60 s

Orbital radius of the Earth, R_E = 1 AU = 1.496 × 10^{11} m

We know that the mass of sun is:

$M_S = \frac{4\pi^2 R_E^3}{G T_E^2}$ .... (2)

Therefore,

$$\frac{M_S}{M_J} = \frac{4\pi^2 R_E^3}{G T_E^2} \times \frac{G T_{10}^2}{4\pi^2 R_{10}^3}$$

$$\frac{R_E^3}{T_E^2} \times \frac{T_{10}^2}{R_{10}^3}$$

Now, on substituting the values, we will get:

$$\left[\frac{1.769\times24\times60\times60}{365.25\times24\times60\times60}\right]^2 \times \left[\frac{1.496\times10^{11}}{4.22\times10^8}\right]^3 = 1045\ 04$$

Therefore, $\frac{M_S}{M_J} \sim 1000$

$M_S \sim 1000 \times M_J$

[Which proves that, the Sun's mass is 1000 times that of Jupiter's)

Q.5: Let us assume that our galaxy consists of 2.5 × 10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution ? Take the diameter of the Milky Way to be 10^5 ly

Sol:

Mass of Sombrero Galaxy, M = **3 × 10^{11} solar mass**
Solar mass = Mass of Sun = **2.0 × 10^{36} kg**
Mass of the galaxy, **M = 3 × 10^{11} × 2 × 10^{36} = 6 × 10^{41} kg**
Diameter of Sombrero Galaxy, **d = 5 x 10^4 ly**
Radius of Sombrero Galaxy, **r = 2.5 × 10^4 ly**

We know that:

1 light year = 9.46 × 10^{15} m
Therefore, r = 2.5 × 10^4 × 9.46 × 10^{15}= **2.365 x 10^{20} m**

As this star revolves around the massive black hole in center of the Sombrero galaxy, its time period can be found with the relation:

$$T = \left[\frac{4\pi^2 r^3}{GM}\right]^{\frac{1}{2}}$$

$$\left[\frac{4\times3.14^2\times(2.365\times10^{20})^3}{(6.67\times10^{-11})\times(6\times10^{41})}\right]^{\frac{1}{2}}$$

= **4.246 x 10^{15} s**

Now we know, 1 year = 365 × 24 × 60 × 60 s

$1s = \frac{1}{365\times24\times60\times60}$

years

Therefore, 4.246 × 10^{15}s = $\frac{4.246\times10^{15}}{365\times24\times60\times60}$ 1.34 × 10^8 years.

Q.6: Choose the correct alternative:
(a) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.
(b) The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's

influence

Sol:

(a). more

(b). kinetic.

Q7. Does the escape speed of a body from the earth depend on

(a) the mass of the body,

(b)the location from where it is projected,

(c) the direction of projection,

(d) the height of the location from where the body is launched?

Sol: (b)

Explanation:

Escape velocity is independent of the direction of projection and the mass of the body. It depends upon the gravitational potential at the place from where the body is projected. Gravitational potential depends slightly on the altitude and the latitude of the place, thus escape velocity depends slightly upon the location from where it is projected.

Q.8: A comet orbits the sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

Sol:

An asteroid orbiting a star will have constant angular momentum and the constant value of total energy throughout its orbit.

Q9. Which of the following symptoms is likely to afflict an astronaut in space (a) swollen feet, (b) swollen face, (c) headache, (d) orientational problem

Sol:

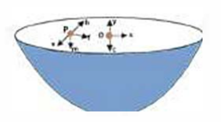
(a). In zero gravity the blood flow to the feet isn't increased so the astronaut does not get swollen feet.

(b). Due to zero gravity, the weight the bones have to bear is greatly reduced, this causes bone loss in astronauts spending greater amounts of time in space.

(c). Space has different orientations, so orientational problems can affect an astronaut.

(d). Due to increased blood supply to their faces, astronauts can be affected by headaches.

Q.10: In the following two exercises, choose the correct answer from among the given ones:
The gravitational intensity at the centre of a hemispherical shell of uniform mass density has the direction indicated by the arrow (see Fig 8.12) (i) a, (ii) b, (iii) c, (iv) 0



Sol: (i) c

Reason:

Inside a hollow sphere, gravitational forces on any particle at any point is symmetrically placed. However, in this case, the upper half of the sphere is removed. Since gravitational intensity is gravitational force per unit mass it will act in direction point downwards along 'c'.

Q.11: For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii) e, (iii) f, (iv) g.

Sol: (iii) m

Reason: Making use of the logic/explanation from the above answer we can conclude that the gravitational intensity at P is directed downwards along m.

Q12. A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

Sol:

Given:

Mass of the comet, $M_C = 7.4 \times 10^{24}$ kg

Mass of the Earth, $M_E = 6 \times 10^{24}$ kg
Orbital radius, $r = 3.84 \times 10^{10}$ m
Mass of the shuttle = m kg

Let 'x' be the distance from the center of the Earth where the gravitational force acting on the Shuttle 'S' becomes zero.

According to Newton's law of gravitation, we have:

$$\Rightarrow \frac{G\,m\,M_A}{(r-x)^2} = \frac{G\,m\,M_E}{x^2}$$

$$\Rightarrow \frac{M_A}{M_E} = \left(\frac{r-x}{x}\right)^2 \Rightarrow \frac{r}{x} - 1 = \left(\frac{7.4\times10^{21}}{6\times10^{24}}\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{r}{x} - 1 = 1.1106$$

$$\Rightarrow x = \frac{r}{2.1106} = \frac{3.84\times10^{10}}{2.1106} = 1.819 \times 10^{10}\ m$$

Q.13: How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5 × 10^8 km

Sol:

Given:
Earth's orbit, r = 1.5 × 10^{11} m
Time taken by the Earth for one complete revolution,
T = 1 year = **365.25 days**
i.e. **T = (365.25 × 24 × 60 × 60) seconds**

Since, Universal gravitational constant, G = 6.67 × 10^{-11} $Nm^2\ kg^{-2}$

Therefore, **mass of the Sun, M =** $\frac{4\pi^2 r^3}{G T^2}$

$$\Rightarrow M = \frac{4\times(3.14)^2\times(1.5\times10^{11})^3}{(6.67\times10^{-11})\times(365.25\times24\times60\times60)^2}$$

$$\Rightarrow M = \frac{4\times(3.14)^2\times(1.5\times10^{11})^3}{(6.67\times10^{-11})\times(365.25\times24\times60\times60)^2}$$

$$\Rightarrow M = \frac{1.331\times10^{35}}{66.425\times10^{3}} = 2.004 \times 10^{30}\ kg$$

Therefore, the estimated mass of the Sun is 2.004 × 10^{30} Kg

Q14. A saturn year is 29.5 times the earth year. How far is the saturn from the sun if the earth is 1.50 × 10^8 km away from the sun ?

Sol:

Given:

Distance between Earth and the Sun, $\mathbf{r_e}$ = 15 × 10^7 km **= 1.50 × 10^8 m**
Time period of the Earth **= T_e**
Time period of Uranus, **T_u = 84 T_e**

Let, the **distance between** the **Sun** and the Saturn be $\mathbf{r_s}$

Now, according to the **Kepler's third law of planetary motion**:

T = $\left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$

For **Uranus** and **Sun**, we can write:

$$\Rightarrow \frac{r_u^3}{r_e^3} = \frac{T_u^2}{T_e^2}$$

$$\Rightarrow r_u = r_e \left[\frac{T_u}{T_e}\right]^{\frac{2}{3}}$$

$$\Rightarrow r_u = 1.5 \times 10^{11} \left[\frac{84\ T_e}{T_e}\right]^{\frac{2}{3}} = \mathbf{1.5 \times 10^{11} \times (84)^{\frac{2}{3}}} \Rightarrow r_u = 28.77 \times 10^{12}$$

m

Therefore, Saturn is 28.77 × 10^{12} m away from the Sun.

Q.15: A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Sol:

Given:

Weight of the man, W = 63 N

We know that acceleration due to **gravity** at **height 'h'** from the **Earth's surface** is:

g' = $\frac{g}{\left[1+\left(\frac{h}{R_e}\right)\right]^2}$

Where, **g = Acceleration due to gravity on the Earth's surface**
And, **R_e = Radius of the Earth**

For h = $\frac{R_e}{2}$

g' =

$$\frac{g}{\left[1+\left(\frac{R_e}{2R_e}\right)\right]^2}$$

$\Rightarrow$ **g' =** $\frac{g}{\left[1+\left(\frac{1}{2}\right)\right]^2} = \frac{4}{9}\,g$

Also , the **weight of a body** of mass **'m' kg** at a height of **'h' meters** can be represented as :
W' = mg

= m × $\frac{4}{9}$ g = $\frac{4}{9}$

mg

=

$\frac{4}{9}$

W

=

$\frac{4}{9}$ × 63 **= 28 N.**

Q.16: Assuming the earth to be a sphere of uniform mass density, how much would a body weigh halfway down to the centre of the earth if it weighed 250 N on the surface?

Sol:

Given,

The weight of a body having **mass 'm'** at the surface of earth, **W** = mg = **250 N**

Body of **mass 'm'** is located at **depth, d =** $\frac{1}{3}$ **R_e**

Where, **R_e = Radius of the Earth**
Now, acceleration due to **gravity g'** at **depth (d)** is given by the relation:

g' = $1-\left(\frac{d}{R_e}\right)g$

$$\Rightarrow g' = 1-\left(\frac{R_e}{3R_e}\right)g = \frac{2}{3}\,g$$

Now, weight of the body at depth d,
W' = mg'

W' = m × $\frac{2}{3}$ g = $\frac{2}{3}$ mg

W' = $\frac{2}{3}$ W

$\Rightarrow$ W = $\frac{2}{3}$ × 250 **= 166.66 N**

Therefore, the weight of a body at one-third of the way down to the center of the earth = 166.66 N

Q.17: A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth ? Mass of the earth = 6.0×10^{24} kg; mean radius of the earth = 6.4×10^6 m; G = 6.67×10^{-11} N m^2 kg^{-2}

Sol:

Given:
Velocity of the **missile**, **v** = 5 km/s = **5×10^3 m/s**
Mass of the Earth, **$M_E = 6 \times 10^{24}$ kg**
Radius of the **Earth**, **$R_E = 6.4 \times 10^6$ m**

Let, the **height** reached by the **missile** be **'h'** and the mass of the **missile be** 'm'.
Now, at the **surface of the Earth**:
Total energy of the rocket at the surface of the Earth = Kinetic energy + Potential energy

T_{E1} =

$$\frac{1}{2}mv^2 + \frac{-GM_E\ m}{R_E}$$

Now, at **highest point 'h'**:

Kinetic Energy = 0 [Since, v = 0]

And, Potential energy =

$$\frac{-GM_E\ m}{R_E+h}$$

Therefore, total energy of the missile at highest point 'h':

$$T_{E2} = 0 + \frac{-GM_E\ m}{R_E+h}$$

$$\Rightarrow \mathbf{T_{E2}} = \frac{-GM_E\ m}{R_E+h}$$

According to the **law of conservation of energy**, we have :
Total energy of the rocket at the **Earth's surface T_{E1}** = **Total energy** at **height 'h' T_{E2}:**

$$\Rightarrow \frac{1}{2}mv^2 + \frac{-GM_E\ m}{R_E} = \frac{-GM_E\ m}{R_E+h}$$

$$\Rightarrow \frac{1}{2}v^2 + \frac{-GM_E}{R_E} = \frac{-GM_E}{R_E+h}$$

$$\Rightarrow v^2 = 2GM_E \times \left[\frac{1}{R_E} - \frac{1}{(R_E+h)}\right] = 2GM_E \left[\frac{h}{(R_E+h)R_E}\right]$$

$$\Rightarrow v^2 = \frac{g\ R_E\ h}{R_E+h}$$

Where,

$$g = \frac{GM}{R_E^2}$$

= 9.8 ms^{-2}

Therefore, $v^2 (R_E + h) = 2gR_Eh$

$\Rightarrow v^2R_E = h\ (2gR_E - v^2)$

$$\Rightarrow h = \frac{R_E\ v^2}{2gR_E - v^2} = \frac{(6.4\times10^6)\times(4\times10^3)^2}{2\times9.8\times6.4\times10^6-(4\times10^3)^2}$$

$\Rightarrow$ **h = 9.356 × 10^5 m**

Therefore, the **missile** reaches at height of 9.36 × 10^5 m from the surface.

Now, the **distance** from the **center** of the **earth = h + R_E** = 9.36 × 10^5 + 6.4 x 10^6 **= 7.33 x10^6 m**

Therefore, the greatest height the missile can attain before falling back to the earth is 9.356 × 10^5 m and the final distance of the missile from the center of the earth is 7.33 x10^6 m

Q.18: The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Sol:

Given,

Escape velocity of the Earth's surface, **v_{esc} = 11.2 km/s**
Projection velocity of the rocket, **v_P = 2v_{esc}**

Let, **mass of the body = m kg**

And, **the velocity** of the **rocket** at a **distance very far away** from the surface of earth = v_F

Total energy of the rocket on the surface = $\frac{1}{2}$ mv_p^2 – $\frac{1}{2}$$mv^2_{esp}$

Total energy of the rocket at a distance very far away from the Earth = $\frac{1}{2}$ mv_F^2

Now, **according** to the **law of conservation of energy**, we have:

$\frac{1}{2}$ mv_P^2 – $\frac{1}{2}$ mv_{ESC}^2 = $\frac{1}{2}$ mv_F^2

mv_{ESC}^2 = $\frac{1}{2}$ mv_F^2

$\sqrt{(v_p)^2 - (v_{esc})^2}$

[Since, v_P = $2v_{esp}$]

$\sqrt{(2v_{esp})^2 - (v_{esc})^2}$

$\sqrt{3}$ v_{esc}

⇒ $v_F = \sqrt{3} \times 11.2$ = **19.39 km/s**

Therefore, the speed of rocket at a distance far away from the surface of earth is 19.39 km/s.

Q.19: A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0×10^{24} kg; radius of the earth = 6.4×10^6 m; G = 6.67×10^{-11} N m^2 kg^{-2}

Sol:

Given:

Mass of the **satellite, m = 420000 kg**

Radius of **the Earth,** R_E = 6.4×10^6 **m**

Mass of the **Earth,** M_E = 6.0×10^{24} **kg**

Universal gravitational constant, **G = 6.67×10^{-11} Nm^2 kg^{-2}**

Height of the **satellite**, h = 400 km = 0.40×10^6 m

We know that the **Total energy** of the satellite at **height 'h'** $= \frac{1}{2}mv^2 + \left[\frac{-GM_E m}{R_E+h}\right]$

Also, **orbital velocity** of the satellite, **v** $= \left[\frac{GM_E}{R_E+h}\right]^{\frac{1}{2}}$

Total energy atheight, h $= \frac{1}{2}\left[\frac{GM_E}{R_E+h}\right] - \left[\frac{GM_E m}{R_E+h}\right]$

Therefore, $T_E = \frac{-1}{2}\left[\frac{GM_E m}{R_E+h}\right]$

This negative sign means that the ISS is bounded to Earth. This is the **bound energy** of the satellite.

Now, **Total Energy** required to send the satellite out of its orbit = **– (Bound energy)**

$= \frac{1}{2}\left[\frac{GM_E m}{R_E+h}\right]$

$$\frac{1}{2}\left[\frac{(6.67\times10^{-11})\times(6\times10^{24})\times(4.2\times10^{5})}{(6.4\times10^{6})+(0.380\times10^{6})}\right] = \frac{1}{2}\times\left[\frac{1.681\times10^{20}}{6.78\times10^{6}}\right]$$

$\Rightarrow$ **T_E = 1.2396 x 10^{13} J**

Therefore, **the amount of energy required to take ISS out from the gravitational influence of earth is 1.2396 x 10^{13} J**

Q.20: Two stars each of one solar mass (= 2×10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide ? The radius of each star is 10^4
km. Assume the stars to remain undistorted until they collide. (Use the known value of G)

Sol:

Given:
Radius of each planet, **R** = 10^3 km = **10^6 m**

Distance between the planet, **r** = 10^{10} km = **10^{13} m**

Mass of each planet, **M** = 2 × **10^{31} kg**

For negligible speeds, v = 0

So the **total energy** of two planets separated by **a distance 'r'**:

$T_E = \frac{-GMM}{r} + \frac{1}{2}mv^2$

Since, v = 0;

Therefore, $T_E = \frac{-GMM}{r}$ . . . . (1)

Now, when the planets are just about to collide:

Let, the **velocity** of the planets = **v**

The centers of the two planets are at a distance of = 2R

Total kinetic energy of the two planets = $\frac{1}{2}$ Mv^2 + $\frac{1}{2}$ Mv^2 = **Mv^2**

Total potential energy of the two planets = $\frac{GMM}{2R}$

Therefore, **Total energy** of the two stars = **Mv^2** -

$\frac{GMM}{2R}$ . . . . (2)

Now, according to **the law of conservation of energy** :

Mv^2 –

$\frac{GMM}{2R} = \frac{-GMM}{}$

$\frac{-GM}{r} + \frac{GM}{2R}$

v^2 = GM ×

$\left[\frac{-1}{r} + \frac{1}{2R}\right] \Rightarrow$ v^2 = 6.67 × 10^{-11} × 2 × 10^{31} × $\left[\frac{-1}{(10^{13})} + \frac{1}{(2\times10^{6})}\right]$

$\Rightarrow$ v^2 = 6.67 × 10^{14}

Therefore, v = (6.67 × 1014)1/2 = 2.583 × 107m/s.

Hence, the speed at which they will collide = 2.583 × 10^7m/s.

Q.21: Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the midpoint of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?

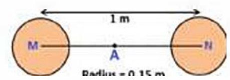
Sol:

Given:

Radius of spheres, R = 0.10 m

Distance between two spheres, r = 1.0 m

Mass of each sphere, M = 100 kg



Two spheres

From the above figure, 'A' is the mid-point and since each sphere will exert the gravitational force in the opposite direction. **Therefore, the gravitational force at this point will be zero.**

Gravitational potential at the midpoint (A) is:

$$U = \left[\frac{-GM}{\frac{r}{2}} + \frac{-GM}{\frac{r}{2}}\right]$$

$$U = \left[\frac{-4GM}{r}\right]$$

$$U = \left[\frac{-4\times(6.67\times10^{-11})\times(1000)}{1.0}\right]$$

⇒ **U= -2.668 x 10^{-7} J /kg**

Therefore, the gravitational potential and force at the mid-point of the line connecting the centers of the two spheres is = -2.668 x 10^{-7} J /kg

The net force on an object, placed at the mid-point is zero. However, if the object is displaced even a little towards any of the two bodies it will not return to its equilibrium position. Thus, the body is in unstable **equilibrium.**

Q.22: As you have learnt in the text, a geostationary satellite orbits the earth at a height of nearly 36,000 km from the surface of the earth. What is the potential due to earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero). Mass of the earth = 6.0×10^{24} kg, radius = 6400 km

Sol:

Given:

Radius of the Earth, R = 6400 km = 0.64 × 10^7 m

Mass of Earth, M = 6 x 10^{24} kg

Height of the geo-stationary satellite from earth's surface, h = 36000 km = 3.6 x 10^7 m

Therefore, gravitational potential at height 'h' on the geo-stationary satellite due to the earth's gravity:

$$G_P = \frac{-GM}{R+h}$$

$$\Rightarrow G_P = \frac{-(6.67\times10^{-11})\times(6\times10^{24})}{(0.64\times10^7)+(3.6\times10^7)}$$

$$\Rightarrow G_P = \frac{-40.02\times10^{13}}{4.24\times10^7} = -9.439 \times 10^6 \text{ J/Kg}$$

Therefore, the gravitational potential due to Earth's gravity on a geo-stationary satellite orbiting earth is -9.439 × 10^6 J/Kg

Q.23: A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2

rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (mass of the sun = 2×10^{30} kg).

Sol.

Any matter/ object will remain stuck to the surface if the **outward centrifugal force** is **lesser** than the **inward gravitational pull**.

Gravitational force, f_G = $\frac{GM_s m}{R^2}$ [Neglecting negative sign]

Here,
M = Mass of the star = $10 \times 2\times 10^{30}$ = **2×10^{31} kg**
m = Mass of the object
R = Radius of the star = 10 km = **1×10^4 m**

Therefore, f_G =

$$\frac{(6.67\times10^{-11})\times(2\times10^{31})\ m}{(1\times10^4)^2} = 1.334\times 10^{13} m\ N$$

Now, **Centrifugal force, $f_C = m r \omega^2$**
Here, ω = Angular speed = $2\pi v$
v = Angular frequency = 10 rev s^{-1}
$f_c = m R (2\pi v)^2$

$f_c = m \times (10^4) \times 4 \times (3.14)^2 \times (10)^2 = (3.9\times10^7 m)$ N

As $f_G > f_C$, the object will remain stuck to the surface of black hole.

Q.24: A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the space ship = 1000 kg; mass of the sun = 2×10^{30} kg; mass of mars = 6.4×10^{23} kg; radius of mars = 3395 km; the radius of the orbit of mars = 2.28×10^8 km; G = 6.67×10-11 N m^2 kg^{-2}

Sol:

Given,

Mass of the Sun, M = 2×10^{30} kg

Mass of the spaceship, m_S = 2000 kg

Radius of Mars, r = 3395 km = **3.395×10^6 m**

Mass of Mars, M_m = 6.4×10^{23} kg

Orbital radius of Mars, R = 2.28×10^8 km = **2.28×10^{11} m**

Universal gravitational constant, G = 6.67×10^{-11} m^2 kg^{-2}

Now,

Potential energy of the spaceship due to the **gravity of Sun** = $\frac{-GMm_s}{R}$

Potential energy of the spaceship due to **gravity of Mars** = $\frac{-GM_M\ m_s}{r}$

As the spaceship is stationed on Mars, its **velocity** is 'zero' and thus, its **kinetic energy is also 'zero'.**

Thus, **Total energy of the spaceship**

$$= \frac{-GM_M\ m_s}{r} - \frac{GMm_s}{R}$$

T_E

$$= -Gm_s\left[\frac{M}{R} + \frac{M_M}{r}\right]$$

The **negative sign** means that the system is inbound state.

Therefore, energy required to launch the spaceship out of the solar system:
T_E = − (bound energy)

T_E

$$= Gm_s\left[\frac{M}{R} + \frac{M_M}{r}\right]$$

Now, on **substituting the values of G, M, R, m_s, M_M and r** we will get:

$$T_E = (6.67\times10^{-11})\times(2\times10^3)\times\left[\frac{2\times10^{30}}{2.28\times10^{11}} + \frac{6.4\times10^{23}}{3.395\times10^6}\right]$$

$$\Rightarrow \; 13.34 \times 10^{-8} \times \left[87.7 \times 10^{17} + 1.885 \times 10^{17}\right] = \mathbf{1.195 \times 10^{12} J}$$

Therefore, the total amount of energy required to launch a spaceship stationed on Mars to out of the solar system = 1.195×10^{12} J

Q.25: A rocket is fired 'vertically' from the surface of mars with a speed of 2 km s^{-1}. If 20% of its initial energy is lost due to martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it ? Mass of mars = 6.4×10^{23} kg; radius of mars = 3395 km; G = 6.67×10^{-11} N m2 kg^{-2}

Sol.

Given:

Mass of Venus, M = 4.8×10^{24} kg

Initial velocity of the missile, **v = 2 km/s = 2×10^3 m/s**

Radius of Venus, R = 6052 km = **6.05×10^6 m**

Universal gravitational constant, G = 6.67×10^{-11} N m^2 kg^{-2}

Let the **mass** of the **missile** = **m kg**

Since, the Initial kinetic energy of the missile = $\frac{1}{2}$ mv^2

and the Initial potential energy of the missile = $\frac{-GMm}{R}$

Thus, total initial energy $= \frac{1}{2}mv^2 + \left(\frac{-GMm}{R}\right)$

It is given that **25 % of initial kinetic energy** is lost in overcoming the atmospheric resistance of Venus; **this means that only 75% of the total kinetic energy is available for propelling** it upwards.

Hence, the total available initial energy $= \left[\frac{75}{100} \times \frac{1}{2}mv^2\right] - \frac{GMm}{R} = 0.375mv^2 - \frac{GMm}{R}$

Let 'h' be the maximum height attained by the missile.
Now, at **height 'h' the final velocity = 0** and hence, the **kinetic energy = 0**

Therefore, the **total energy of the missile** at height 'h' = $\frac{-GMm}{R+h}$

Now, according to the law of conservation of energy:

$$\Rightarrow \quad 0.375mv^2 - \frac{GM}{R} = \frac{-GMm}{R+h}$$

$$\Rightarrow \quad 0.375\, v^2 = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\Rightarrow 0.375v^2 = GM \times \left[\frac{h}{R\,(R+h)}\right]$$

$$\Rightarrow \frac{R+h}{h} = \frac{GM}{0.375\, v^2 R}$$

$$\Rightarrow \frac{R}{h} = \frac{GM}{0.375\, v^2 R} - 1$$

$$\Rightarrow \frac{R}{h} = \frac{GM - 0.375\, v^2 R}{0.375\, v^2 R}$$

$$\Rightarrow \frac{1}{h} = \frac{GM - 0.375\, v^2 R}{0.375\, v^2 R^2}$$

$$\Rightarrow h = \frac{0.375\, v^2 R^2}{GM - 0.375\, v^2 R}$$

Now, on substituting the values of G, **M, v** and R we will get:

$$\Rightarrow h = \frac{0.375 \times (3 \times 10^3)^2 \times (6.05 \times 10^6)^2}{[(6.67 \times 10^{-11}) \times (4.8 \times 10^{24})] - [0.375 \times (3 \times 10^3)^2 \times 6.05 \times 10^6]}$$

$$\Rightarrow h = \frac{0.375 \times (3 \times 10^3)^2 \times (6.05 \times 10^6)^2}{[(6.67 \times 10^{-11}) \times (4.8 \times 10^{24})] - [0.375 \times (3 \times 10^3)^2 \times 6.05 \times 10^6]}$$

$$\Rightarrow \frac{1.235 \times 10^{20}}{3.2016 \times 10^{14} - 2.042 \times 10^{13}} = \mathbf{412023.75\; m = 4.12\; km}$$

Therefore, the maximum height that is achieved by the missile before returning back to the surface = 4.12 km.