## Miscellaneous Exercise



**1. Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of *x*-axis.**

**Solution:**

Let us consider,

If $\vec{r}$ is a unit vector in the XY-plane, then $\vec{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$.

Here, $\theta$ is the angle made by the unit vector with the positive direction of the $x$-axis.

Hence, for $\theta = 30°$ we have:

$$\vec{r} = \cos 30°\hat{i} + \sin 30°\hat{j} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$$

Therefore, the required unit vector is $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

**2. Find the scalar components and magnitude of the vector joining the points P $(x_1, y_1, z_1)$ and Q $(x_2, y_2, z_2)$.**

**Solution:**

Firstly let us consider,

The vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ can be found out by:

$$\overrightarrow{PQ} = \text{Position vector of Q} - \text{Position vector of P}$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Therefore, the scalar components and the magnitude of the vector joining the given points are respectively $\{(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)\}$ and $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

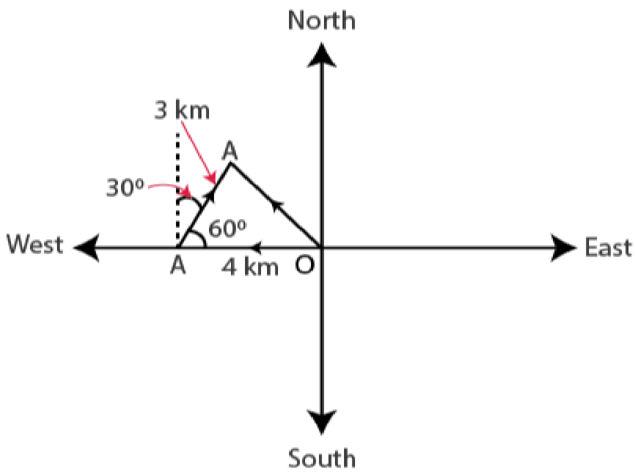
**3. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.**

**Solution:**

It is given that,

Let O and B be the initial and final positions of the girl respectively.

Then, the girl's position can be shown as:




$$\overrightarrow{OA} = -4\hat{i}$$
$$\overrightarrow{AB} = \hat{i}\left|\overrightarrow{AB}\right|\cos 60° + \hat{j}\left|\overrightarrow{AB}\right|\sin 60°$$
$$= \hat{i}3\times\frac{1}{2} + \hat{j}3\times\frac{\sqrt{3}}{2}$$
$$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

By the Triangle law of vector addition, we have

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$
$$= \left(-4\hat{i}\right) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$$
$$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$
$$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$
$$= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

Therefore, the girl's displacement from her initial point of departure is

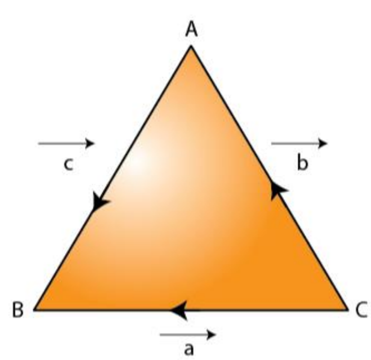
$\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$.

**4. If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.**

**Solution:**

It is given that,

In $\Delta ABC$, let $\overrightarrow{CB} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, and $\overrightarrow{AB} = \vec{c}$ (as shown in the following figure).



So, by the Triangle law of vector addition, we have $\vec{a} = \vec{b} + \vec{c}$ .

And, we know that $|\vec{a}|$, $|\vec{b}|$, and $|\vec{c}|$ represent the sides of $\Delta ABC$.
Also, it is known that the sum of the lengths of any two sides of a triangle is greater than the third side.

$\therefore |\vec{a}| < |\vec{b}| + |\vec{c}|$

Therefore, it is not true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$ .

**5. Find the value of *x* for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.**

**Solution:**

We know,

Given $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

So, $|x(\hat{i} + \hat{j} + \hat{k})| = 1$ .

Now,

$$|x(\hat{i} + \hat{j} + \hat{k})| = 1$$

$$\sqrt{x^2 + x^2 + x^2} = 1$$

$$\sqrt{3x^2} = 1$$

$$\sqrt{3}\,x = 1$$

$$x = \pm\frac{1}{\sqrt{3}}$$

Therefore, the required value of $x$ is $\pm\frac{1}{\sqrt{3}}$

**6. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.**

**Solution:**

Let us consider the,

Given vectors,

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Let $\vec{c}$ be the resultant of $\vec{a}$ and $\vec{b}$.

Then,

$$\vec{c} = \vec{a} + \vec{b} = (2+1)\hat{i} + (3-2)\hat{j} + (-1+1)\hat{k} = 3\hat{i} + \hat{j}$$

$$|\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{(3\hat{i} + \hat{j})}{\sqrt{10}}$$

Therefore, the vector of magnitude 5 units and parallel to the resultant of vectors $\vec{a}$ and $\vec{b}$ is

$$\pm 5 \cdot \hat{c} = \pm 5 \cdot \frac{1}{\sqrt{10}}(3\hat{i} + \hat{j}) = \pm \frac{3\sqrt{10}\hat{i}}{2} \pm \frac{\sqrt{10}}{2}\hat{j}.$$

**7. If $\vec{a} = \hat{i} + \hat{j} + \hat{k},\ \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.**

**Solution:**

Let us consider the given vectors,

Given,

$\vec{a} = \hat{i} + \hat{j} + \hat{k},\ \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$$\begin{aligned} 2\vec{a} - \vec{b} + 3\vec{c} &= 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k}) \\ &= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k} \\ &= 3\hat{i} - 3\hat{j} + 2\hat{k} \end{aligned}$$

$$|2\vec{a} - \vec{b} + 3\vec{c}| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9+9+4} = \sqrt{22}$$

Therefore, the unit vector along $2\vec{a} - \vec{b} + 3\vec{c}$ is

$$\frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} = \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}.$$

**8. Show that the points A (1, –2, –8), B (5, 0, –2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.**

**Solution:**

Firstly let us consider,

Given points are: A (1, –2, –8), B (5, 0, –2), and C (11, 3, 7).

$$\therefore \overrightarrow{AB} = (5-1)\hat{i} + (0+2)\hat{j} + (-2+8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\overrightarrow{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7+2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\overrightarrow{AC} = (11-1)\hat{i} + (3+2)\hat{j} + (7+8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{4^2 + 2^2 + 6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{10^2 + 5^2 + 15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$$

$$\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$$

Therefore, the given points A, B, and C are collinear.

Now, let point B divide AC in the ratio $\lambda : 1$. So, we have:

$$\overrightarrow{OB} = \frac{\lambda\overrightarrow{OC} + \overrightarrow{OA}}{(\lambda+1)}$$

$$5\hat{i} - 2\hat{k} = \frac{\lambda\left(11\hat{i} + 3\hat{j} + 7\hat{k}\right) + \left(\hat{i} - 2\hat{j} - 8\hat{k}\right)}{\lambda+1}$$

$$(\lambda+1)\left(5\hat{i} - 2\hat{k}\right) = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - 2\hat{j} - 8\hat{k}$$

$$5(\lambda+1)\hat{i} - 2(\lambda+1)\hat{k} = (11\lambda+1)\hat{i} + (3\lambda-2)\hat{j} + (7\lambda-8)\hat{k}$$

On equating the corresponding components, we have:

$$5(\lambda+1) = 11\lambda + 1$$

$$5\lambda + 5 = 11\lambda + 1$$

$$6\lambda = 4$$

$$\lambda = \frac{4}{6} = \frac{2}{3}$$

Therefore, point B divides AC in the ratio $2:3$.

**9. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are** $\left(2\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - 3\vec{b}\right)$ **externally in the ratio 1: 2. Also, show that P is the midpoint of the line segment RQ.**

**Solution:**

We know,

Given, $\overrightarrow{OP} = 2\vec{a} + \vec{b},\ \overrightarrow{OQ} = \vec{a} - 3\vec{b}$.
Also, given that point R divides a line segment joining two points P and Q externally in the ratio 1: 2. So, on using the section formula, we have

$$\overrightarrow{OR} = \frac{2\left(2\vec{a}+\vec{b}\right)-\left(\vec{a}-3\vec{b}\right)}{2-1} = \frac{4\vec{a}+2\vec{b}-\vec{a}+3\vec{b}}{1} = 3\vec{a}+5\vec{b}$$

Hence, the position vector of point R is $3\vec{a}+5\vec{b}$.
Now,

$$\text{Position vector of the mid-point of RQ} = \frac{\overrightarrow{OQ}+\overrightarrow{OR}}{2}$$
$$= \frac{\left(\vec{a}-3\vec{b}\right)+\left(3\vec{a}+5\vec{b}\right)}{2}$$
$$= 2\vec{a}+\vec{b}$$
$$= \overrightarrow{OP}$$

Therefore, P is the mid-point of the line segment RQ.

**10. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$.**
**Find the unit vector parallel to its diagonal. Also, find its area.**
**Solution:**
Firstly let us consider,

Adjacent sides of a parallelogram are given as: $\vec{a} = 2\hat{i}-4\hat{j}+5\hat{k}$ and $\vec{b} = \hat{i}-2\hat{j}-3\hat{k}$

We know that, the diagonal of a parallelogram is given by $\vec{a}+\vec{b}$.

$$\vec{a}+\vec{b} = (2+1)\hat{i}+(-4-2)\hat{j}+(5-3)\hat{k} = 3\hat{i}-6\hat{j}+2\hat{k}$$

Hence, the unit vector parallel to the diagonal is

$$\frac{\vec{a}+\vec{b}}{\left|\vec{a}+\vec{b}\right|} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{3^2+(-6)^2+2^2}} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{\sqrt{9+36+4}} = \frac{3\hat{i}-6\hat{j}+2\hat{k}}{7} = \frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}.$$

So, the area of parallelogram ABCD = $\left|\vec{a}\times\vec{b}\right|$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}$$

$$=\hat{i}(12+10)-\hat{j}(-6-5)+\hat{k}(-4+4)$$
$$=22\hat{i}+11\hat{j}$$
$$=11\left(2\hat{i}+\hat{j}\right)$$
$$\therefore\left|\vec{a}\times\vec{b}\right|=11\sqrt{2^2+1^2}=11\sqrt{5}$$

Therefore, the area of the parallelogram is $11\sqrt{5}$ square units.

**11. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$.**

**Solution:**

Firstly,

Let's assume a vector to be equally inclined to axes OX, OY, and OZ at angle $\alpha$.

Then, the direction cosines of the vector are cos $\alpha$, cos $\alpha$, and cos $\alpha$.

Now, we know that

$$\cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$$
$$3\cos^2\alpha=1$$
$$\cos\alpha=\frac{1}{\sqrt{3}}$$

Therefore, the direction cosines of the vector which are equally inclined to the axes are

$\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$

Hence proved.

**12. Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k},\ \vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c}.\vec{d}=15$.**

**Solution:**

Assume,

Let $\vec{d}=d_1\hat{i}+d_2\hat{j}+d_3\hat{k}$.

As $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:

$$\vec{d}\cdot\vec{a}=0$$
$$d_1+4d_2+2d_3=0 \qquad ...(i)$$

And,

$\vec{d}\cdot\vec{b}=0$

$3d_1-2d_2+7d_3=0 \quad ...(ii)$

Also, given that:

$\vec{c}\cdot\vec{d}=15$

$2d_1-d_2+4d_3=15 \quad ...(iii)$

On solving (i), (ii), and (iii), we obtain

$d_1=\frac{160}{3}, d_2=-\frac{5}{3}$ and $d_3=-\frac{70}{3}$

$\therefore \vec{d}=\frac{160}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{70}{3}\hat{k}=\frac{1}{3}\left(160\hat{i}-5\hat{j}-70\hat{k}\right)$

Therefore, the required vector is $\frac{1}{3}\left(160\hat{i}-5\hat{j}-70\hat{k}\right)$

**13. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of $\lambda$ .**

**Solution:**

Let's consider the

Sum of the given vectors is given by,

$\left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)$

$=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

Hence, unit vector along $\left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)$ is given as:

$$\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{4+4\lambda+\lambda^2+36+4}}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$$

Scalar product of $\left(\hat{i}+\hat{j}+\hat{k}\right)$ with this unit vector is 1.

$$\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\sqrt{\lambda^2+4\lambda+44}=\lambda+6$$

$$\lambda^2+4\lambda+44=(\lambda+6)^2$$

$$\lambda^2+4\lambda+44=\lambda^2+12\lambda+36$$

$$8\lambda=8$$

$$\lambda=1$$

Therefore, the value of A is 1.

**14. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.**

**Solution:**

Lets assume,

As $\vec{a}, \vec{b}$, and $\vec{c}$ are mutually perpendicular vectors, we have

$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0.$

Given that:

$|\vec{a}|=|\vec{b}|=|\vec{c}|$

Let vector $\vec{a}+\vec{b}+\vec{c}$ be inclined to $\vec{a}, \vec{b}$, and $\vec{c}$ at angles $\theta_1$, $\theta_2$, and $\theta_3$ respectively.

So, we have

$$\cos\theta_1=\frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}=\frac{\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$=\frac{|\vec{a}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|} \qquad [\vec{b}\cdot\vec{a}=\vec{c}\cdot\vec{a}=0]$$

$$=\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\theta_2=\frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}=\frac{\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{b}|}$$

$$=\frac{|\vec{b}|^2}{|\vec{a}+\vec{b}+\vec{c}|\cdot|\vec{b}|} \qquad [\vec{a}\cdot\vec{b}=\vec{c}\cdot\vec{b}=0]$$

$$=\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

$$\cos\theta_3=\frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}=\frac{\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}$$

$$=\frac{|\vec{c}|^2}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|} \qquad [\vec{a}\cdot\vec{c}=\vec{b}\cdot\vec{c}=0]$$

$$=\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}$$

Now, as $|\vec{a}|=|\vec{b}|=|\vec{c}|$, $\cos\theta_1=\cos\theta_2=\cos\theta_3$.

$\therefore \theta_1=\theta_2=\theta_3$

Therefore, the vector $(\vec{a}+\vec{b}+\vec{c})$ is equally inclined to $\vec{a}, \vec{b}$, and $\vec{c}$.

Hence proved.

**15. Prove that $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2$, if and only if $\vec{a}, \vec{b}$ are perpendicular, given $\vec{a}\neq\vec{0}, \vec{b}\neq\vec{0}$.**

**Solution:**

It is given that

Required to prove:

$(\vec{a}+\vec{b}).(\vec{a}+\vec{b})=|\vec{a}|^2+|\vec{b}|^2$

$\vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}=|\vec{a}|^2+|\vec{b}|^2$ [Distributivity of scalar products over addition]

$|\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2$ [$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ (Scalar product is commutative)]

$2\vec{a}\cdot\vec{b}=0$

$\vec{a}\cdot\vec{b}=0$ [$\vec{a}\neq\vec{0}, \vec{b}\neq\vec{0}$ (Given)]

Therefore, $\vec{a}$ and $\vec{b}$ are perpendicular.

Hence proved.

**16. If $\theta$ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b}\geq 0$ only when**

**(A)** $0<\theta<\frac{\pi}{2}$ **(B)** $0\leq\theta\leq\frac{\pi}{2}$

**(C)** $0<\theta<\pi$ **(D)** $0\leq\theta\leq\pi$

**Solution:**

**Explanation:**

Let's assume $\theta$ to be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors so that $|\vec{a}|$ and $|\vec{b}|$ are positive

We also know, $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

So,

$\vec{a}\cdot\vec{b}\geq 0$

$|\vec{a}||\vec{b}|\cos\theta\geq 0$

$\cos\theta\geq 0$ [$|\vec{a}|$ and $|\vec{b}|$ are positive]

$0\leq\theta\leq\frac{\pi}{2}$

Therefore, $\vec{a}.\vec{b}\geq 0$ when $0\leq\theta\leq\frac{\pi}{2}$.

The correct answer is B.

**17. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and $\theta$ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if**

**(A)** $\theta = \frac{\pi}{4}$ **(B)** $\theta = \frac{\pi}{3}$ **(C)** $\theta = \frac{\pi}{2}$ **(D)** $\theta = \frac{2\pi}{3}$

**Solution:**
**Explanation:**
Let $\vec{a}$ and $\vec{b}$ be two unit vectors and $\theta$ be the angle between them.
Then, $|\vec{a}| = |\vec{b}| = 1$.
Now, $\vec{a}+\vec{b}$ is a unit vector if $|\vec{a}+\vec{b}| = 1$.

$$|\vec{a}+\vec{b}| = 1$$
$$(\vec{a}+\vec{b})^2 = 1$$
$$(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = 1$$
$$\vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b} = 1$$
$$|\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = 1$$
$$1^2 + 2|\vec{a}||\vec{b}|\cos\theta + 1^2 = 1$$
$$1 + 2.1.1\cos\theta + 1 = 1$$
$$\cos\theta = -\frac{1}{2}$$
$$\theta = \frac{2\pi}{3}$$

Therefore, $\vec{a}+\vec{b}$ is a unit vector if $\theta = \frac{2\pi}{3}$.
Hence the correct answer is D.

**18. The value of** $\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{i}\times\hat{k}) + \hat{k}.(\hat{i}\times\hat{j})$ **is**
**(A) 0** **(B) –1** **(C) 1** **(D) 3**
**Solution:**
**Explanation:**
It is given that,

$$\hat{i}.(\hat{j}\times\hat{k}) + \hat{j}.(\hat{i}\times\hat{k}) + \hat{k}.(\hat{i}\times\hat{j})$$
$$= \hat{i}\cdot\hat{i} + \hat{j}\cdot(-\hat{j}) + \hat{k}\cdot\hat{k}$$
$$= 1 - \hat{j}\cdot\hat{j} + 1$$
$$= 1 - 1 + 1$$
$$= 1$$

Hence the correct answer is C.

**19. If $\theta$ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$ when $\theta$ is equal to**

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\pi$

**Solution:**

**Explanation:**

Let $\theta$ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$$|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$$

$$|\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$

$$\cos\theta = \sin\theta \qquad \left[|\vec{a}| \text{ and } |\vec{b}| \text{ are positive}\right]$$

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}$$

Thus, $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$ when $\theta$ is equal to $\frac{\pi}{4}$

So, the correct answer is B.