Exercise 1.3

1. Prove that √5 is irrational.
Solutions: Let us assume, that **√**5 is rational number.
i.e. **√**5 = x/y (where, x and y are co-primes)
y**√**5= x

Squaring both the sides, we get,
$(y\sqrt{5})^2 = x^2$

$\Rightarrow 5y^2 = x^2$………………………………. (1)

Thus, x^2 is divisible by 5, so x is also divisible by 5.

Let us say, x = 5k, for some value of k and substituting the value of x in equation (1), we get,
$5y^2 = (5k)^2$
$\Rightarrow y^2 = 5k^2$
y^2 is divisible by 5 it means y is divisible by 5.

Therefore, x and y are co-primes. Since, our assumption about $\sqrt{5}$ is rational is incorrect.

Hence, **√**5 is irrational number.

2. Prove that 3 + 2√5 + is irrational.

Solutions: Let us assume 3 + 2√5 is rational.
Then we can find co-prime x and y (y ≠ 0) such that 3 + 2√5 = x/y
Rearranging, we get,
$2\sqrt{5} = \frac{x}{y} - 3$
$\sqrt{5} = \frac{1}{2}(\frac{x}{y} - 3)$
Since, x and y are integers, thus,
$\frac{1}{2}(\frac{x}{y} - 3)$ is a rational number.

Therefore, √5 is also a rational number. But this contradicts the fact that √5 is irrational.

So, we conclude that 3 + 2√5 is irrational.

3. Prove that the following are irrationals:
(i) 1/√2
(ii) 7√5
(iii) 6 + √2

Solutions:

(i) $1/\sqrt{2}$

Let us assume $1/\sqrt{2}$ is rational.
Then we can find co-prime x and y ($y \neq 0$) such that $1/\sqrt{2} = x/y$
Rearranging, we get,
$\sqrt{2} = y/x$

Since, x and y are integers, thus, $\sqrt{2}$ is a rational number, which contradicts the fact that $\sqrt{2}$ is irrational.

Hence, we can conclude that $1/\sqrt{2}$ is irrational.

(ii) $7\sqrt{5}$

Let us assume $7\sqrt{5}$ is a rational number.
Then we can find co-prime a and b ($b \neq 0$) such that $7\sqrt{5} = x/y$
Rearranging, we get,

$\sqrt{5} = x/7y$

Since, x and y are integers, thus, $\sqrt{5}$ is a rational number, which contradicts the fact that $\sqrt{5}$ is irrational.

Hence, we can conclude that $7\sqrt{5}$ is irrational.

(iii) $6 + \sqrt{2}$

Let us assume $6 + \sqrt{2}$ is a rational number.
Then we can find co-primes x and y ($y \neq 0$) such that $6 + \sqrt{2} = x/y$·
Rearranging, we get,

$\sqrt{2} = (x/y) - 6$

Since, x and y are integers, thus $(x/y) - 6$ is a rational number and therefore, $\sqrt{2}$ is rational. This contradicts the fact that $\sqrt{2}$ is an irrational number.

Hence, we can conclude that $6 + \sqrt{2}$ is irrational.