

Exercise: 10.2

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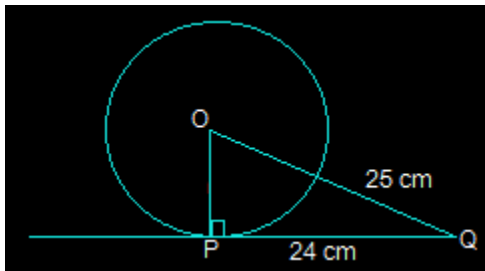
In Q.1 to 3, choose the correct option and give justification.

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

- (A) 7 cm
- (B) 12 cm
- (C) 15 cm
- (D) 24.5 cm

Answer:

First, draw a perpendicular from the center O of the circle to a point P on the circle which is touching the tangent. This line will be perpendicular to the tangent of the circle.



So, OP is perpendicular to PQ i.e. $OP \perp PQ$

From the above figure, it is also seen that $\triangle OPQ$ is a right angled triangle.

It is given that

$$OQ = 25 \text{ cm and } PQ = 24 \text{ cm}$$

By using Pythagoras theorem in $\triangle OPQ$,

$$OQ^2 = OP^2 + PQ^2$$

$$\Rightarrow (25)^2 = OP^2 + (24)^2$$

$$\Rightarrow OP^2 = 625 - 576$$

$$\Rightarrow OP^2 = 49$$

$$\Rightarrow OP = 7 \text{ cm}$$

So, option A i.e. 7 cm is the radius of the given circle.

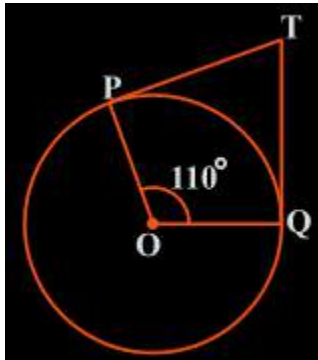
2. In Fig. 10.11, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to

- (A) 60°

- (B) 70°
- (C) 80°
- (D) 90°

Answer:

From the question, it is clear that OP is the radius of the circle to the tangent PT and OQ is the radius to the tangents TQ.



So, $OP \perp PT$ and $TQ \perp OQ$

$$\therefore \angle OPT = \angle OQT = 90^\circ$$

Now, in the quadrilateral POQT, we know that the sum of the interior angles is 360°

$$\text{So, } \angle PTQ + \angle POQ + \angle OPT + \angle OQT = 360^\circ$$

Now, by putting the respective values we get,

$$\Rightarrow \angle PTQ + 90^\circ + 110^\circ + 90^\circ = 360^\circ$$

$$\Rightarrow \angle PTQ = 70^\circ$$

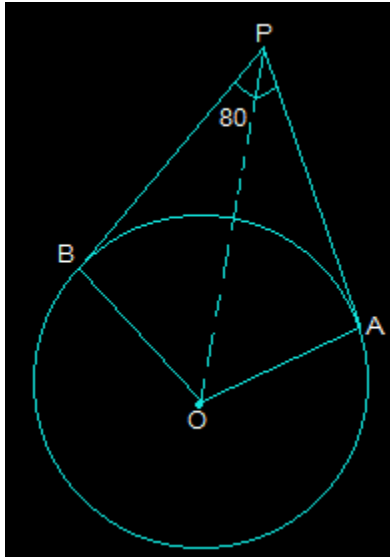
So, $\angle PTQ$ is 70° which is option B.

3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80° , then $\angle POA$ is equal to

- (A) 50°
- (B) 60°
- (C) 70°
- (D) 80°

Answer:

First, draw the diagram according to the given statement.



Now, in the above diagram, OA is the radius to tangent PA and OB is the radius to tangents PB.

So, OA is perpendicular to PA and OB is perpendicular to PB i.e. $OA \perp PA$ and $OB \perp PB$

So, $\angle OBP = \angle OAP = 90^\circ$

Now, in the quadrilateral AOBP,

The sum of all the interior angles will be 360°

So, $\angle AOB + \angle OAP + \angle OBP + \angle APB = 360^\circ$

Putting their values, we get,

$$\Rightarrow \angle AOB + 260^\circ = 360^\circ$$

$$\Rightarrow \angle AOB = 100^\circ$$

Now, consider the triangles $\triangle OPB$ and $\triangle OPA$. Here,

$AP = BP$ (Since the tangents from a point are always equal)

$OA = OB$ (Which are the radii of the circle)

$OP = OP$ (It is the common side)

Now, we can say that triangles OPB and OPA are similar using SSS congruency.

$$\therefore \triangle OPB \cong \triangle OPA$$

So, $\angle POB = \angle POA$

$$\Rightarrow \angle AOB = \angle POA + \angle POB$$

$$\Rightarrow 2(\angle POA) = \angle AOB$$

By putting the respective values, we get,

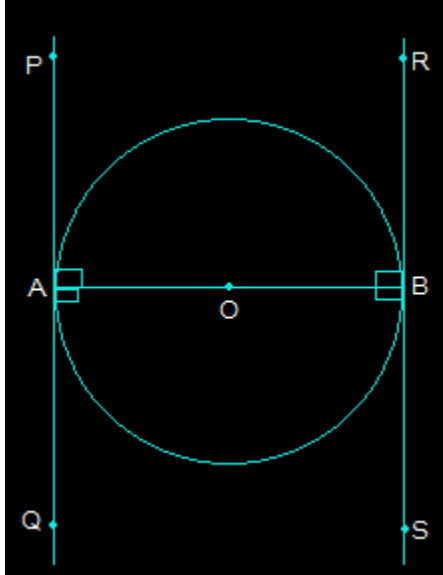
$$\Rightarrow \angle POA = 100^\circ / 2 = 50^\circ$$

As angle $\angle POA$ is 50° option A is the correct option.

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer:

First, draw a circle and connect two points A and B such that AB becomes the diameter of the circle. Now, draw two tangents PQ and RS at points A and B respectively.



Now, both radii i.e. AO and OB are perpendicular to the tangents.

So, OB is perpendicular to RS and OA perpendicular to PQ

So, $\angle OAP = \angle OAQ = \angle OBR = \angle OBS = 90^\circ$

From the above figure, angles OBR and OAQ are alternate interior angles.

Also, $\angle OBR = \angle OAQ$ and $\angle OBS = \angle OAP$ (Since they are also alternate interior angles)

So, it can be said that line PQ and the line RS will be parallel to each other. (Hence Proved).

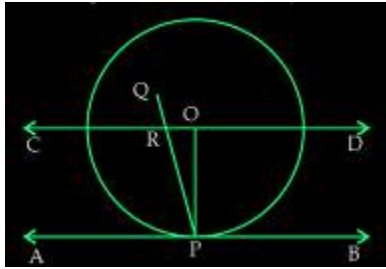
5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the center.

Solution:

First, draw a circle with center O and draw a tangent AB which touches the radius of the circle at point P.

To Proof: PQ passes through point O.

Now, let us consider that PQ doesn't pass through point O. Also, draw a CD parallel to AB through O. Here, CD is a straight line and AB is the tangent. Refer the diagram now.



From the above diagram, PQ intersects CD and AB at R and P respectively.

AS, $CD \parallel AB$,

Here, the line segment PQ is the line of intersection.

Now angles ORP and RPA are equal as they are alternate interior angles

So, $\angle ORP = \angle RPA$

And,

$\angle RPA = 90^\circ$ (Since, PQ is perpendicular to AB)

$\Rightarrow \angle ORP = 90^\circ$

Now, $\angle ROP + \angle OPA = 180^\circ$ (Since they are co-interior angles)

$\Rightarrow \angle ROP + 90^\circ = 180^\circ$

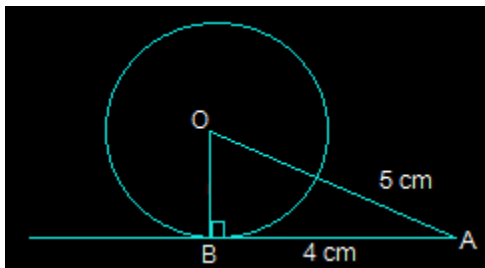
$\Rightarrow \angle ROP = 90^\circ$

Now, it is seen that the $\triangle ORP$ has two right angles which are $\angle ORP$ and $\angle ROP$. Since this condition is impossible, it can be said the supposition we took is wrong.

6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer:

Draw the diagram as shown below.



Here, AB is the tangent that is drawn on the circle from a point A.

So, the radius OB will be perpendicular to AB i.e. $OB \perp AB$

We know, $OA = 5\text{ cm}$ and $AB = 4\text{ cm}$

Now, In $\triangle ABO$,

$OA^2 = AB^2 + BO^2$ (Using Pythagoras theorem)

$$\Rightarrow 5^2 = 4^2 + BO^2$$

$$\Rightarrow BO^2 = 25 - 16$$

$$\Rightarrow BO^2 = 9$$

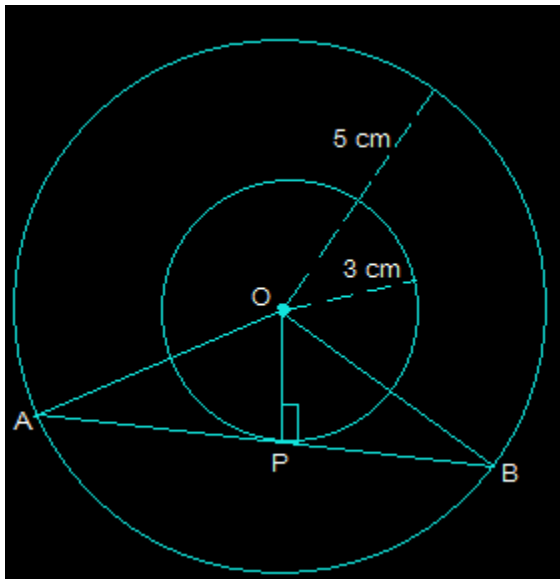
$$\Rightarrow BO = 3$$

So, the radius of the given circle i.e. BO is 3 cm.

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer:

Draw two concentric circles with the center O. Now, draw a chord AB in the larger circle which touches the smaller circle at a point P as shown in the figure below.



From the above diagram, AB is tangent to the smaller circle to point P.

$$\therefore OP \perp AB$$

Using Pythagoras theorem in triangle OPA,

$$OA^2 = AP^2 + OP^2$$

$$\Rightarrow 5^2 = AP^2 + 3^2$$

$$\Rightarrow AP^2 = 25 - 9$$

$$\Rightarrow AP = 4$$

Now, as $OP \perp AB$,

Since the perpendicular from the center of the circle bisects the chord, AP will be equal to PB

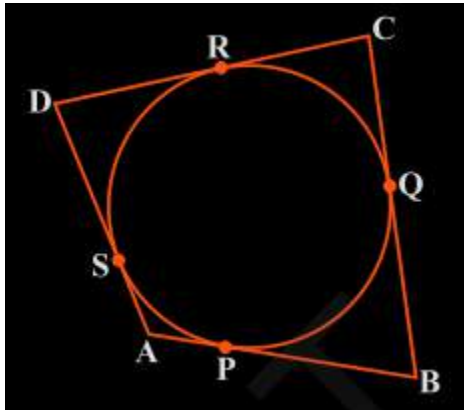
$$\text{So, } AB = 2AP = 2 \times 4 = 8 \text{ cm}$$

So, the length of the chord of the larger circle is 8 cm.

8. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig. 10.12). Prove that $AB + CD = AD + BC$

Answer:

The figure given is:



From this figure we can conclude a few points which are:

- (i) $DR = DS$
- (ii) $BP = BQ$
- (iii) $AP = AS$
- (iv) $CR = CQ$

Since they are tangents on the circle from points D, B, A, and C respectively.

Now, adding the LHS and RHS of the above equations we get,

$$DR + BP + AP + CR = DS + BQ + AS + CQ$$

By rearranging them we get,

$$(DR + CR) + (BP + AP) = (CQ + BQ) + (DS + AS)$$

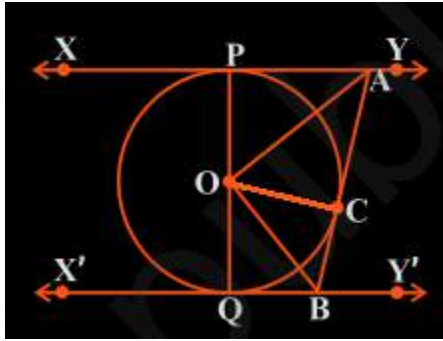
By simplifying,

$$AD + BC = CD + AB$$

9. In Fig. 10.13, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.

Answer:

From the figure given in the textbook, join OC. Now, the diagram will be as-



Now the triangles $\triangle OPA$ and $\triangle OCA$ are similar using SSS congruency as:

- (i) $OP = OC \rightarrow$ They are the radii of the same circle
- (ii) $AO = AO \rightarrow$ It is the common side
- (iii) $AP = AC \rightarrow$ These are the tangents from point A

So, $\triangle OPA \cong \triangle OCA$

Similarly,

$\triangle OQB \cong \triangle OCB$

So,

$\angle POA = \angle COA \dots$ (Equation i)

And, $\angle QOB = \angle COB \dots$ (Equation ii)

Since the line POQ is a straight line, it can be considered as a diameter of the circle.

So, $\angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$

Now, from equations (i) and equation (ii) we get,

$$2\angle COA + 2\angle COB = 180^\circ$$

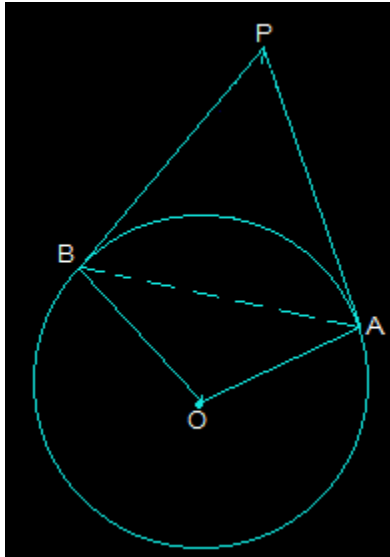
$$\Rightarrow \angle COA + \angle COB = 90^\circ$$

$$\therefore \angle AOB = 90^\circ$$

10. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the center.

Answer:

First, draw a circle with centre O . Choose an external point P and draw two tangents PA and PB at point A and point B respectively. Now, join A and B to make AB in a way that it subtends $\angle AOB$ at the center of the circle. The diagram is as follows:



From the above diagram, it is seen that the line segments OA and PA are perpendicular.

So, $\angle OAP = 90^\circ$

In a similar way, the line segments $OB \perp PB$ and so, $\angle OBP = 90^\circ$

Now, in the quadrilateral OAPB,

$\therefore \angle APB + \angle OAP + \angle PBO + \angle BOA = 360^\circ$ (since the sum of all interior angles will be 360°)

By putting the values we get,

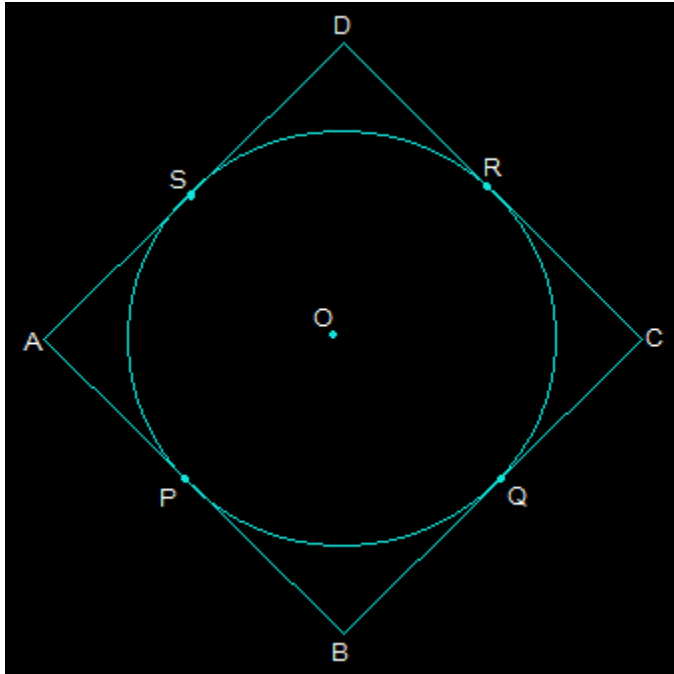
$$\Rightarrow \angle APB + 180^\circ + \angle BOA = 360^\circ$$

So, $\angle APB + \angle BOA = 180^\circ$ (Hence proved).

11. Prove that the parallelogram circumscribing a circle is a rhombus.

Answer:

Consider a parallelogram ABCD which is circumscribing a circle with a center O. Now, since ABCD is a parallelogram, $AB = CD$ and $BC = AD$.



From the above figure, it is seen that,

- (i) $DR = DS$
- (ii) $BP = BQ$
- (iii) $CR = CQ$
- (iv) $AP = AS$

These are the tangents to the circle at D, B, C, and A respectively.

Adding all these we get,

$$DR + BP + CR + AP = DS + BQ + CQ + AS$$

By rearranging them we get,

$$\Rightarrow (BP + AP) + (DR + CR) = (CQ + BQ) + (DS + AS)$$

Again by rearranging them we get,

$$\Rightarrow AB + CD = BC + AD$$

Now, since $AB = CD$ and $BC = AD$, the above equation becomes

$$2AB = 2BC$$

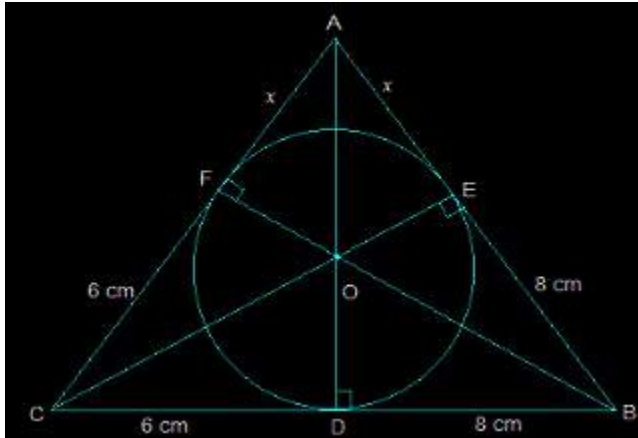
$$\therefore AB = BC$$

Since $AB = BC = CD = DA$, it can be said that ABCD is a rhombus.

12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see Fig. 10.14). Find the sides AB and AC.

Answer:

The figure given is as follows:



Consider the triangle ABC,

We know that the length of any two tangents which are drawn from the same point to the circle is equal.

So,

(i) $CF = CD = 6 \text{ cm}$

(ii) $BE = BD = 8 \text{ cm}$

(iii) $AE = AF = x$

Now, it can be observed that,

(i) $AB = EB + AE = 8 + x$

(ii) $CA = CF + FA = 6 + x$

(iii) $BC = DC + BD = 6 + 8 = 14$

Now the semi perimeter "s" will be calculated as follows

$$\Rightarrow 2s = AB + CA + BC$$

By putting the respective values we get,

$$2s = 28 + 2x$$

$$\Rightarrow s = 14 + x$$

$$\text{Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

By solving this we get,

$$= \sqrt{(14+x)48x} \dots\dots\dots (i)$$

Again, the area of $\triangle ABC = 2 \times$ area of $(\triangle AOF + \triangle COD + \triangle DOB)$

$$= 2 \times [(\frac{1}{2} \times OF \times AF) + (\frac{1}{2} \times CD \times OD) + (\frac{1}{2} \times DB \times OD)]$$

$$= 2 \times \frac{1}{2} (4x + 24 + 32) = 56 + 4x \dots\dots\dots (ii)$$

Now from (i) and (ii) we get,

$$\sqrt{(14+x)48x} = 56 + 4x$$

Now, square both the sides,

$$48x(14+x) = (56+4x)^2$$

$$\Rightarrow 48x = [4(14+x)]^2 / (14+x)$$

$$\Rightarrow 48x = 16(14+x)$$

$$\Rightarrow 48x = 224 + 16x$$

$$\Rightarrow 32x = 224$$

$$\Rightarrow x = 7 \text{ cm}$$

$$\text{So, } AB = 8+x$$

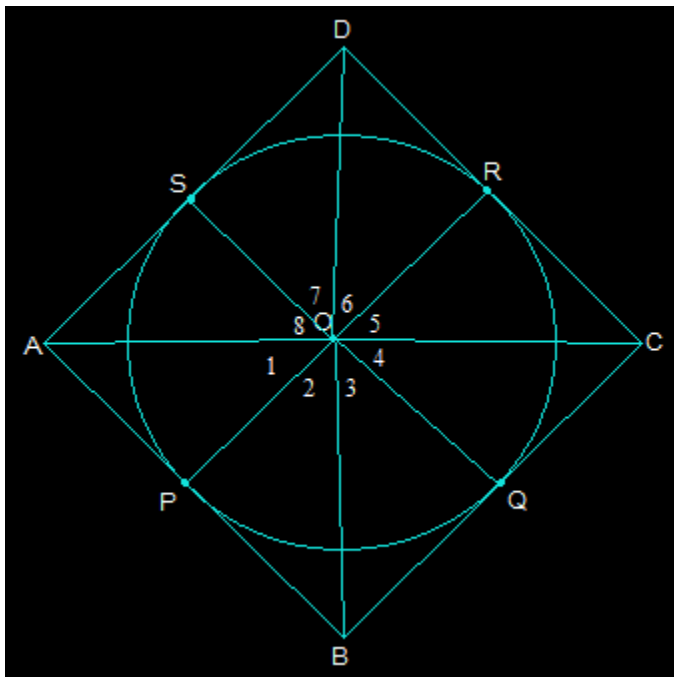
$$\text{i.e. } AB = 15 \text{ cm}$$

$$\text{And, } CA = x+6 = 13 \text{ cm.}$$

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer:

First draw a quadrilateral ABCD which will circumscribe a circle with its centre O in a way that it touches the circle at point P, Q, R, and S. Now, after joining the vertices of ABCD we get the following figure:



Now, consider the triangles OAP and OAS,
 $AP = AS$ (They are the tangents from the same point A)
 $OA = OA$ (It is the common side)
 $OP = OS$ (They are the radii of the circle)
 So, by SSS congruency $\triangle OAP \cong \triangle OAS$

So, $\angle POA = \angle AOS$

Which implies that $\angle 1 = \angle 8$

Similarly, other angles will be,

$$\angle 4 = \angle 5$$

$$\angle 2 = \angle 3$$

$$\angle 6 = \angle 7$$

Now by adding these angles we get,

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

Now by rearranging,

$$\Rightarrow (\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^\circ$$

$$\Rightarrow 2\angle 1 + 2\angle 2 + 2\angle 3 + 2\angle 4 = 360^\circ$$

Taking 2 as common and solving we get,

$$(\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180^\circ$$

$$\text{Thus, } \angle AOB + \angle COD = 180^\circ$$

Similarly, it can be proved that $\angle BOC + \angle DOA = 180^\circ$

Therefore, the opposite sides of any quadrilateral which is circumscribing a given circle will subtend supplementary angles at the center of the circle.